

AN
APPENDIX
TO THE
ELEMENTS OF EUCLID,
IN SEVEN BOOKS.

CONTAINING

FORTY-TWO moveable Schemes for forming the various Kinds of Solids, and their Sections, by which the Doctrine of Solids in the Eleventh, Twelfth, and Fifteenth Books of EUCLID is illustrated, and rendered more easy to Learners than heretofore.

BOOK

- | | |
|--|--|
| I. Contains the Five regular Solids. | IV. Contains sundry Sorts of Prisms. |
| II. Shews the Inscription and Circumscription thereof, as set forth in the Fifteenth Book of the Elements. | V. Various Kinds of Pyramids, and Frustrums thereof. |
| III. Exhibits a great Variety of irregular Solids. | VI. Some difficult Propositions in the Eleventh and Twelfth Books. |
| | VII. The Cone and its several Sections. |

SECOND EDITION.

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L O N D O N :

SOLD BY T. CADELL, BOOKSELLER IN THE STRAND.

P R E F A C E.

THE approbation shewn to the first edition of this work, and the many applications made for it while out of print, are motives that have encouraged me not only to issue this second edition, but also to make fundry additions and improvements to it, in which the Solids contained in the first and third books hereof, with the addition of their section-planes, are shewn by a new and more expressive method of exhibiting them; the number of irregular solids is also considerably increased by the addition of others, whose formations, far as I know, are wholly new; and the sixth book is made more extensively useful by a farther consideration of the doctrine of Planes described in the definitions and propositions of the eleventh and twelfth books of Euclid, the same being more particularly illustrated and explained by moveable diagrams having the same lines and letters thereon as are in the planschemes belonging to those books, which being duly elevated, the several planes and lines heretofore described in plano appear in the very positions they are to be considered, or conceived in the mind of the learner, principally adapted to the schemes given of those books in the edition by professor Simson.

P R E F A C E.

of Glasgow, together with others suited to those in the seventh and eighth books of the elements of Geometry by my predecessor; but as these things would augment the present work too much, and that those who have the first Edition hereof may have an opportunity of obtaining these additions without detriment to their former purchase, they are reserved for a second or supplemental volume to what is herein contained.

J. L. C.

APPENDIX to the Elements of EUCLID.

B O O K I.

Of the Five regular Solids.

TO form the Solids described in this book.

Raise up the moveable parts of the scheme, and fold them around the part that is shaded.

I. *Of the Tetraedron, Plate 1.*

The Tetraedron is a solid bounded by four equal equilateral triangles.

II. *Of the Hexaedron, or Cube, Plate 2.*

The Hexaedron, or Cube, is bounded by six equal squares.

III. *Of the Octoedron, Plate 3.*

The Octoedron is bounded by eight equal equilateral triangles.

IV. *Of the Dodecaedron, Plate 4.*

The Dodecaedron is bounded by twelve equal regular pentagons.

V. *Of the Icofaedron, Plate 5.*

The Icofaedron is bounded by twenty equal equilateral triangles.

There

Of the Five regular Solids.

There are particular rules for finding either the superficial or solid content of each of those bodies; but the same may be more readily obtained by the following table.

A T A B L E,

shewing the superficial and solid content of each of the five regular solids, each side * whereof is unity.

Names.	Superficies.	Solidity.
Tetraedron.	1,732051	0,1178511
Hexaedron.	6,000000	1,0000000
Octoedron.	3,464102	0,4714045
Dodecaedron.	20,645729	7,663119
Icosaedron.	8,660254	2,181695

Use of the above table.

To find the superficial content of either of the above-named solids, one side thereof being given.

Multiply the tabular number that stands under *Superficies* in a line with the respective solid, by the *square* of the given side thereof; and

For the solid content,

Multiply the tabular number that stands accordingly under *Solidity* by the *cube* of the given side, the former of those products is the *superficial*, and the latter is the *solid* content required.

* By the word *side* is here to be understood, the line in which any two sides of the bounding planes meet each other.

EXAMPLE

Of the Five regular Solids.

E X A M P L E I.

What is the content superficial and folid of a Tetraedron, each side being 9 inches?

<i>Superficies.</i>		<i>Solidity.</i>
1,732051 <u>81</u>		0,1178511 <u>729</u>
1732051 13856408		10606599 2357022 <u>8249577</u>
Area <u>140,296131</u> Sup.		Area <u>85,9134519</u> Solid.

E X A M P L E II.

Suppose each side of the Hexaedron, or Cube, 9 inches?

<i>Superficies.</i>		<i>Solidity.</i>
81 <u>6,</u>		729 <u>1</u>
<u>486,</u>		<u>729</u>

E X A M P L E III.

Suppose each side of the Octoedron 9 inches?

<i>Superficies.</i>		<i>Solidity.</i>
3,464102 <u>81</u>		,4714 <u>729</u>
3464102 27712816		42426 9428 <u>32998</u>
<u>280,592262</u>		<u>343,6506</u>

B

E X A M P L E

Of the Five regular Solids.

EXAMPLE IV.

Suppose each side of the Dodecaedron 9 inches?

<i>Superficies.</i>	<i>Solidity.</i>
20,645729 81	7,663119 729
<hr/> 20645729	<hr/> 689,68071
165165832	1532,6238
<hr/> 1672,304049	<hr/> 53641,833
	<hr/> 5586,413751

EXAMPLE V.

Suppose each side of the Icofaedron 9 inches?

<i>Superficies.</i>	<i>Solidity.</i>
8,660254 81	2,181695 729
<hr/> 8660254	<hr/> 19635255
69282032	4363390
<hr/> 701,480574	<hr/> 15271865
	<hr/> 1590,455655

B O O K I I.

Containing an illustration of EUCLID's Fifteenth Book.

Directions for folding the schemes together.

P L A T E V I.

FIRST form the Tetraedron, by folding together its several triangles around that particular one which is shaded:

Then move its vertex till the other angular point, which is moveable, is so much elevated above the plane, that the side which connects those two points of the Tetraedron becomes parallel to the plane from which the whole was raised.

The Tetraedron being in that position, bring the several parts which constitute the Hexaedron around it; thus will the four angular points of the Tetraedron fall exactly in the four angles of the Cube.

P L A T E V I I.

The Octoedron being first formed, fold the Tetraedron over it.

P L A T E V I I I.

First form the Octoedron, and raise it perpendicularly upon one of its angular points, then fold the Cube around it, and the angular points of the Octoedron will then touch the center of each side of the Cube.

P L A T E I X.

Form the Cube, and incline it a little forward till its sides bisect those of the Octoedron, when folded around it.

P L A T E X.

Form the Dodecaedron, and incline it so that its sides may bisect those of the Icofaedron, when folded around it.

B O O K III.

Containing several irregular Solids.

P R O B L E M.

TO measure an irregular solid of any form whatsoever.

G E N E R A L R U L E.

Procure a suitable vessel, as a tub, cistern, or any such as can be most easily measured; then put the body whose content is required into it, and pour into the vessel so much water as will just cover the solid: mark the side of the vessel where it is even with the surface of the water, then take out the solid, and observe well how far the water descends; for that part of the vessel contained between the highest ascent and lowest descent of the water being measured, gives the solidity of the body required.

EXAMPLE. Suppose the solidity of a very irregular piece of stone was required, and that having put it into a square cistern, whose dimensions are 50 inches, after having just covered it with water, on taking it out I find the water to descend 8 inches, from thence to find its content.

50 inches	
50	

Superficial area 2500	of the base of the cistern,
	8 its depth, or descent of the water,

Solidity 20000	of that part of the cistern possessed by the

body, and therefore the content of the body required in inches, which, divided by 1728, gives 11,516 feet, &c. the content in solid feet.

N. B. To put the water into the vessel first, and then immerse the solid therein, may, in some cases, be a more preferable way of proceeding.

To fold any of the figures contained in this book.

Bring the contiguous parts of each figure around that which is shaded, the rest will then join together, and form the solid required.

B O O K IV.

Containing various sorts of Prisms.

D E F I N I T I O N.

A PRISM is a solid figure comprehended by planes, among which two opposite ones are parallel, equal, and similar.

R E M A R K S.

- I. The sides of all prisms are parallelograms; but the ends or bases are of various forms; hence it is that prisms receive different names, and are accordingly called triangular, quadrangular, pentangular, hexangular, &c.
- II. When a prism is terminated at its ends by parallelograms, it is then most generally termed a parallelepipedon.
- III. The method of folding together the schemes contained in this book is so obvious, that nothing more need here be observed concerning it, than only to bring the contiguous parts of each figure around that particular one which is shaded.

P R O B L E M.

To measure any sort of prism.

G E N E R A L R U L E.

Measure from the center of its base or end, to the middle of any one of its sides, multiply that length by half the sum of all the sides by which the prism is bounded, and that product is the area of the base; which being multiplied by the whole length of the prism, gives the solid content.

Or thus:

Square the side of the base, and multiply it by the affixed number in the following table belonging to the figure which the base of the prism is of, and that product gives the area of the base; which being multiplied by the length of the prism gives the content as before.

C

A G E N E R A L

A GENERAL TABLE

For finding the area of regular polygons.

Sides.	Names.	Multipliers.
3	Triangle.	,433013
4	Square.	1,000000
5	Pentagon.	1,720477
6	Hexagon.	2,598076
7	Heptagon.	3,633959
8	Octagon.	4,828427

N. B. The paralleloipedon is best measured by multiplying its length, breadth, and depth into each other.

I forbear troubling the reader here with any particular examples, there being sufficient in such authors as have wrote on the subject of mensuration; for to exhibit the bodies themselves is the only design in which I am now engaged.

B O O K V.

Containing various sorts of Pyramids, and frustrums thereof.

D E F I N I T I O N S.

I. A PYRAMID is a solid having a polygon for its base, and comprehended under triangles, all meeting together in one point; which point is called its vertex.

II. The frustrum of a pyramid is that part which remains, when the top part is cut away by a plane or section passing through all its sides parallel to its base.

To fold any of the figures contained in this book.

The parts which are shaded are such evident indications, as render further directions superfluous.

P R O B L E M I.

To measure a pyramid.

G E N E R A L R U L E.

Multiply the area of its base by a third part of its perpendicular altitude, that product is the solid content.

N. B. The perpendicular altitude is measured by a line drawn from the vertex to the center of its base.

P R O B L E M II.

To measure the frustrum of a pyramid.

R U L E.

Multiply the areas of the bottom and top bases together, extract the square root of that product, and add thereto the sum of both areas, that total multiplied by one third of the frustrum's altitude, gives the solidity required.

B O O K VI.

Containing an illustration of some Theorems in EUCLID's Eleventh and Twelfth Books.

EUCL. XI. Prop. xxviii.

T H E O R E M.

A PLANE passing through the diameters of opposite planes of a parallelopipedon divides it into two equal prisms. See this illustrated in Plates XXXIV. and XXXV.

Directions for folding the schemes contrived for illustrating the above theorem.

P L A T E XXXIV.

I. Raise up one half of the figure, and fold it so that the point E may fall upon A, and the point F upon B, and raise up the parallelogram at the end; thus have you one of the prisms.

II. Then bring over the other part of the scheme, so that the corner C may likewise fall upon A, and the corner D upon B; thus will there be formed the whole parallelopipedon and its section made by the plane passing through its diagonal A B.

P L A T E XXXV.

I. Lift up the figure, and bring the point A to C, and the point B to D.

II. Fold back the rest of the figure upon the line A B, which is properly cut for that purpose; so that the two parallelograms which are separated by A B may lie close together.

III. Then bring over the remaining part of the scheme, so that the points E and F may coincide with C and D; then folding together the four triangles, the whole parallelopipedon will be formed, and also its section made according to a plane passing through the diagonal of its base or end.

EUCL.

B O O K VII.

Containing the Conic Sections.

D E F I N I T I O N S.

I. A CONE is a round solid, having a circle for its base, and may be conceived to be generated by the revolution of a right-angled triangle turning round on one of the sides, which includes the right-angle.

II. The axis of the cone is that fixed right line, around which the triangle is supposed to revolve.

III. The extreme end or point, by which the cone is terminated, is called its vertex.

IV. The frustum of a cone is the part remaining, when its top is cut off by a plane or section passing through it parallel to its base.

R E M A R K S.

I. The cone and its frustum are measured by the same rules as obtain for pyramids and their frustums.

II. As a pyramid is the third part of a prism, having the same base and altitude, so in like manner a cone is the third part of a cylinder, having the same base and altitude; but the roundness of those figures does not admit of an explanation of this truth by this mechanical method of representing them.

Directions for folding together the schemes contained in this book.

P L A T E XXXVIII.

I. Bend the arc AB evenly round the arc EF, thereby causing A to come to E, and B to F.

II. Turn about the triangle BCD upon the side BC, so as to make BD coincide with the diameter of the base EF, and AC to coincide with DC; thus will one half of the cone be formed.

III. In like manner form the other half, by bringing G to E, and H to F, the line GK even upon EF, and the sides HI and KI even to each other; so will the whole cone and the section, through its axis, both appear.

P L A T E

EUCL. XII. *Prop.* vii.

T H E O R E M.

EVERY pyramid is the third part of a prism, having the same base and height. See this illustrated in a triangular prism by

P L A T E XXXVII.

To fold the figure contained in this plate.

I. First form the pyramid ABC, by bringing the point C to A, so that the line BC may fall upon AB.

II. Then form the pyramid BAD, by bringing D to B, making AD coincide with AB, and raise up the triangle at B.

III. Then turn back the rest of the figure upon the line AD, so that the triangle marked 7 may closely adhere to that marked 6.

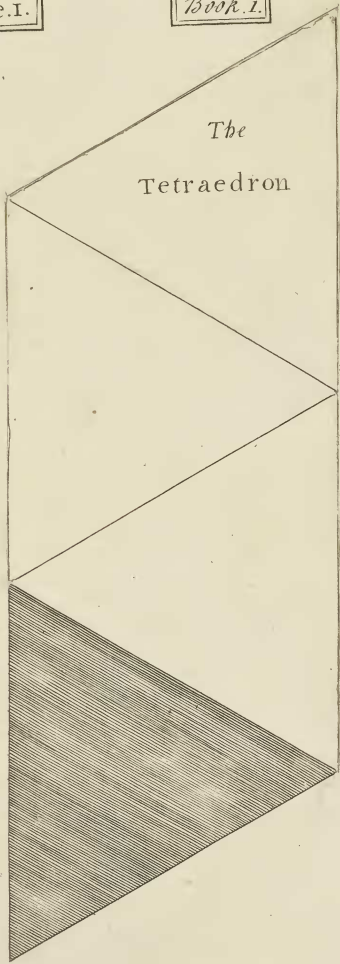
IV. Lastly, The vacuity that is now between the two pyramids formed will be filled exactly by folding together the pyramid contained under the triangles marked 8, 9, 10, in such sort that E be connected with A, and F with B; thus is the whole prism completed, and the sections above described clearly seen.

N. B. When the last-mentioned pyramid is introduced into the space remaining between the other two, by pressing a little upon that part which is the upper edge or side of the prism, the formation of the prism will be rendered more perfect.

Plate. I.

Book. I.

The
Tetraedron



P L A T E XXXIX.

I. Fold the arc AD round the base, so that A may come to B , and the arc DC in like manner, making C to unite with A at the point B .

II. Make the lines AF and CG unite together, which will form the frustrum of a cone cut parallel to its base, and shew the section to be a circle.

P L A T E XL.

Fold the arcs AB and BC round the arcs BDE , BFE , and make the sides AG and CH coincide every where together upwards from the point E , which will represent a cone cut oblique to its base, and shew the section to be an ellipsis.

P L A T E XLII.

I. Form the whole part of the cone ACB , by bringing the points A and B together, and bending it so as to become round as near as may be.

II. Bend regularly the other part of the figure, so that the arcs DE , FE , may surround the arcs FG , FH .

III. Raise up the parabola GIH , making the point I meet the coincident points A and B ; thus may you see the section made by cutting a cone parallel to one of its sides.

P L A T E XLIII.

The last directions obtain here; for by bringing E and F together, and rounding the part EDF , as was there described, and the points A and B being brought to G and H , if the hyperbola GIH be then raised up, as before directed, the section of a cone, cut parallel to its axis, will be thereby subjugated to view.

F I N I S.

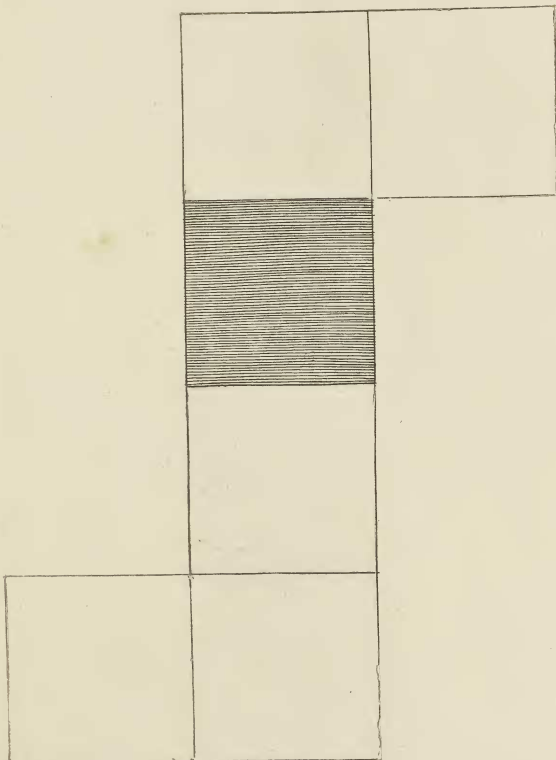
Lately Published, by the same Author,

THE THEORY OF PERSPECTIVE,
 Demonstrated and illustrated by moveable SCHEMES,
 Which shew the several Planes, Lines, and Points used in that Art in the true
 Positions in which they are to be considered.

Plate. II.

The
Hexaedron or Cube.

Book I.



EUCL. XII. *Prop.* iii.

T H E O R E M.

A TRIANGULAR pyramid may be divided into two equal triangular pyramids, having each the same base and altitude, and into two equal triangular prisms, which two prisms are together greater than the half of the whole pyramid.

Directions for folding together the scheme which explains this proposition.

P L A T E XXXVI.

I. First form the triangular pyramid ACB, by making the line BC coincide with AC.

II. Fold back upon the line BC the triangle numbered 4, so that it may lie close to that which is marked 3.

III. Then fold over the sides 5, 6, 7, bringing the line DE so that D may fall upon A, and E upon C, which, with the quadrilateral base that is shaded, forms one of the prisms.

IV. Then fold back the other part of the figure, so that the side 8 may be close to that marked 7; thus will there remain a triangular vacuity at the base of the pyramid ACB, which must be filled by folding together the sides marked 9 and 10, making the point F fall upon A, and the point G upon C; thus will the other prism be formed.

V. This done, there remains only to form the other pyramid GIH, which is effected by folding its parts back upon the line GK, and folding round the triangles marked 12, 13, 14; the partition lines whereof are all cut on the front side of the figure, which, being folded accordingly, completes the whole pyramid, and exhibits a view of the several sections described in the above theorem.

Plate.IV.

Book.1

The
Dodecaedron

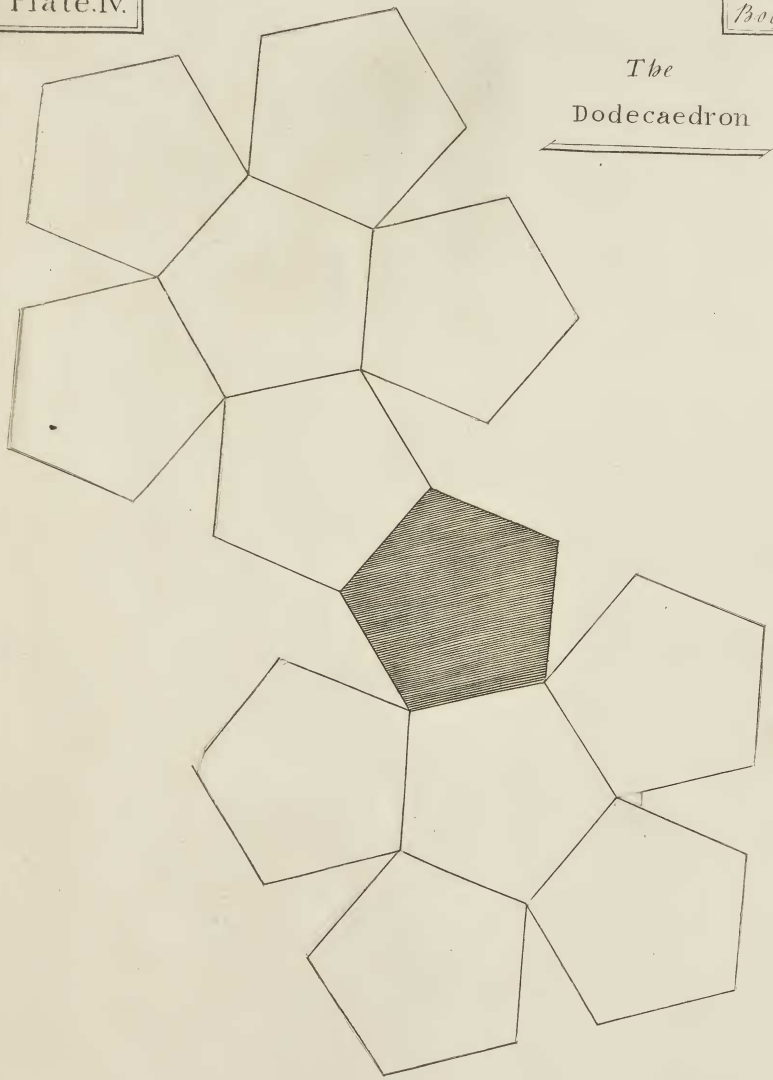
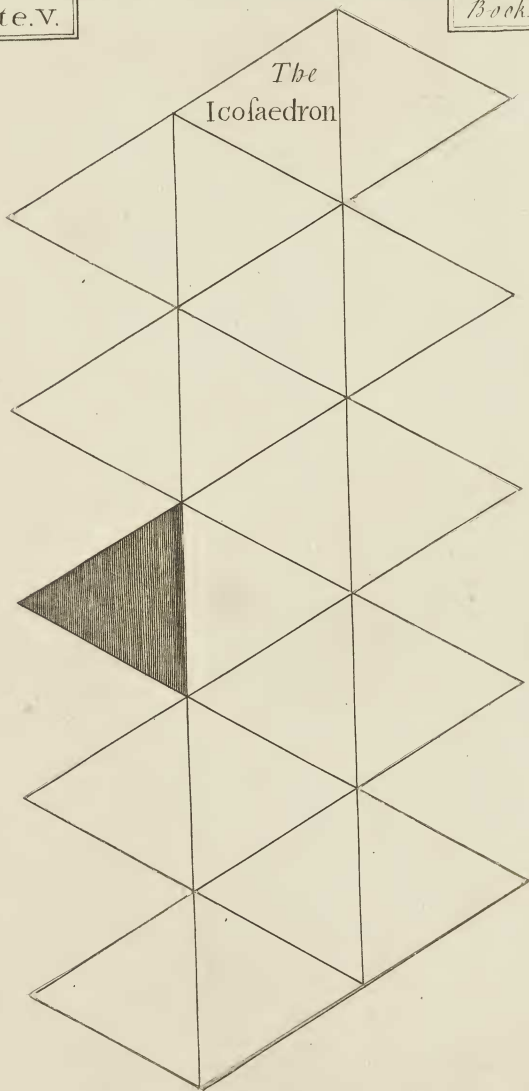
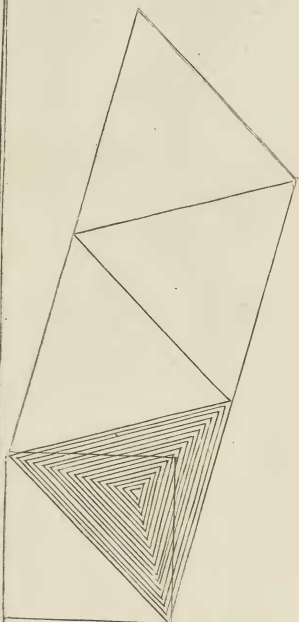


Plate.V.

Book.I.

The
Icofaedron





*The
Tetraedron.
inscribed in an
Hexaedron.*

Euclid. 15. Prop. 1.



Plate VII.

Book. 2.

The
Octoedron
inscribed in a

Tetraedron.

Euclid's Prop. 2.

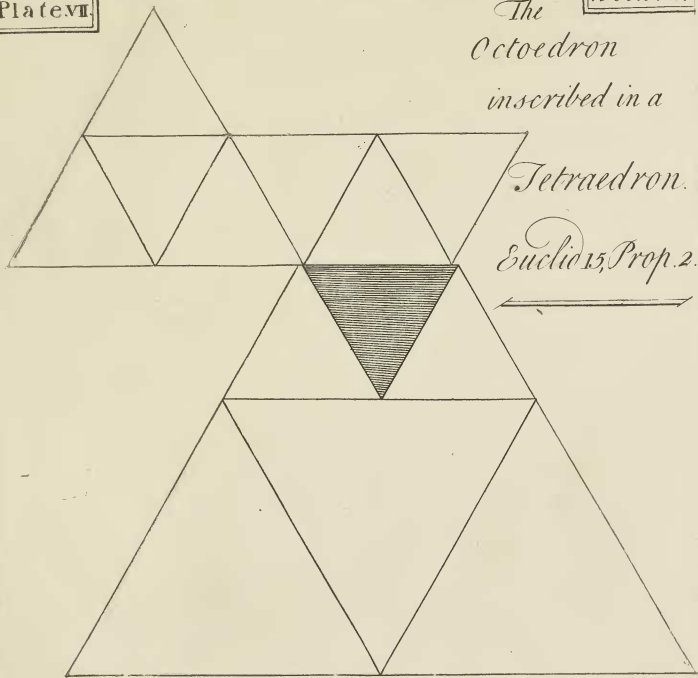
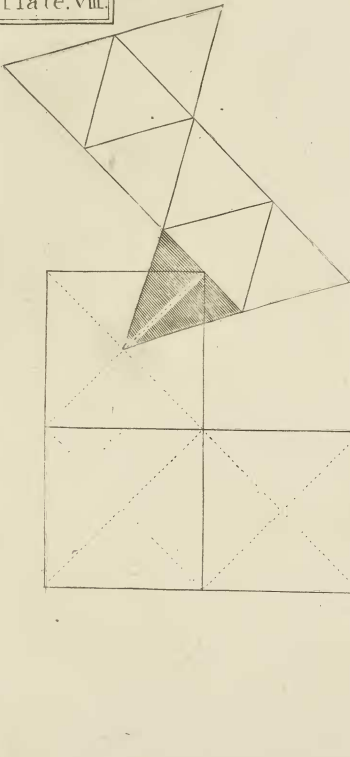


Plate. viii.

Book. 2.



The
Octoedron
inscribed in the
Hexaedron.
Euclid 15.
Prop. 5.

Plate. III.

The
Octoedron

Book. I.

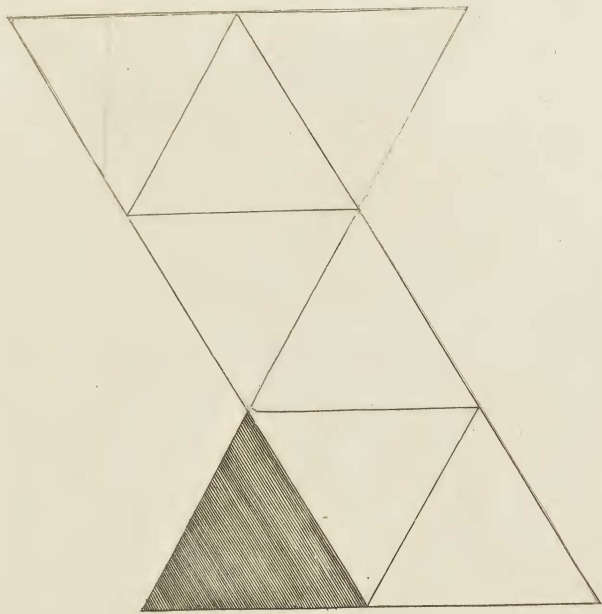
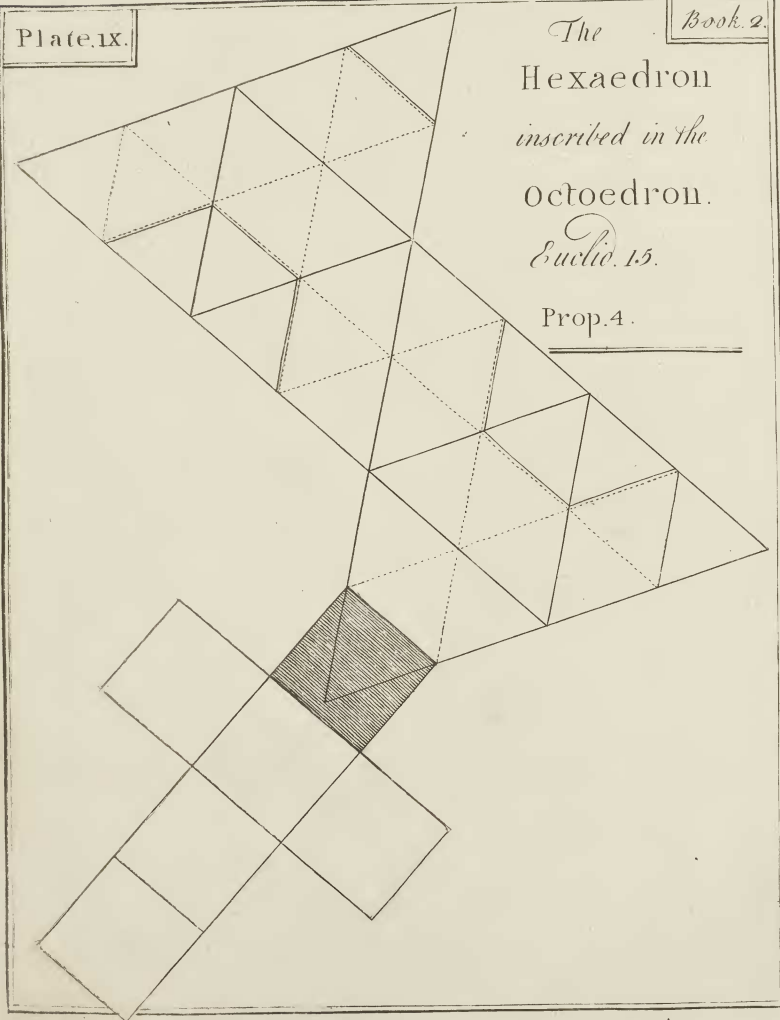
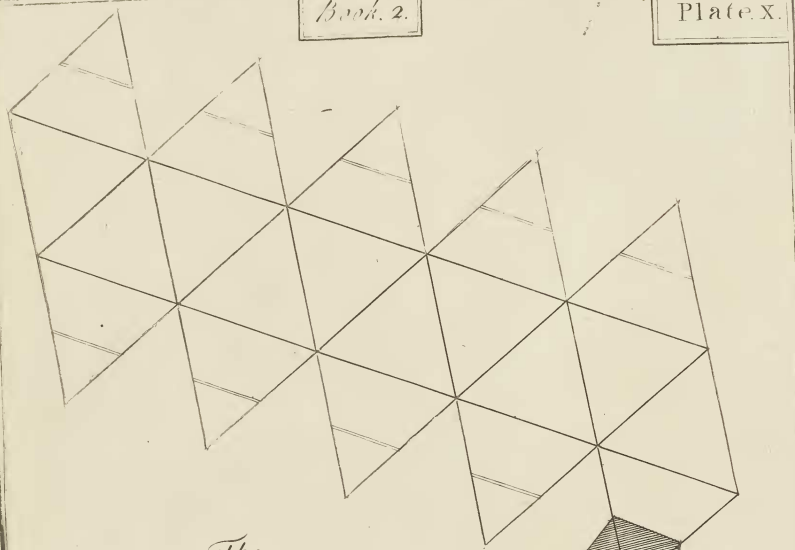


Plate. IX.

Book. 2.

The
Hexaedron
inscribed in the
Octoedron.
Euclio. 15.
Prop. 4.

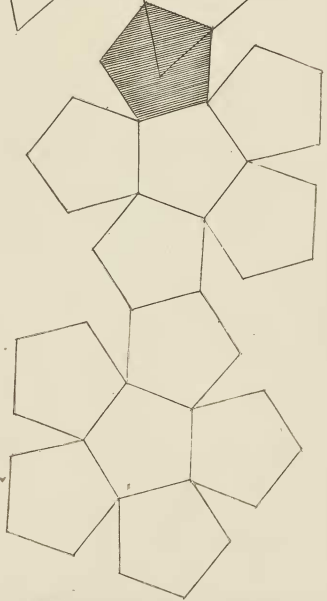




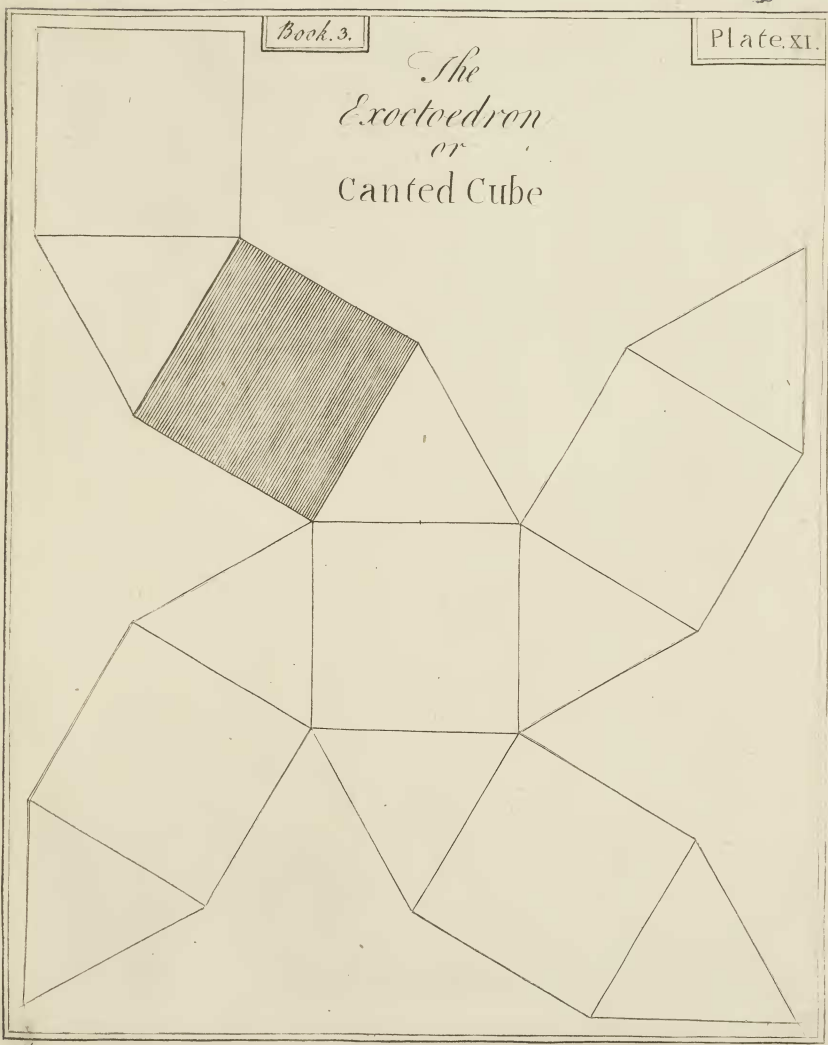
*The
Dodecaedron
incribed in an
Icosaedron.*

Euclio 15.

Prop. 5.



The
Exoctaedron
or
Canted Cube

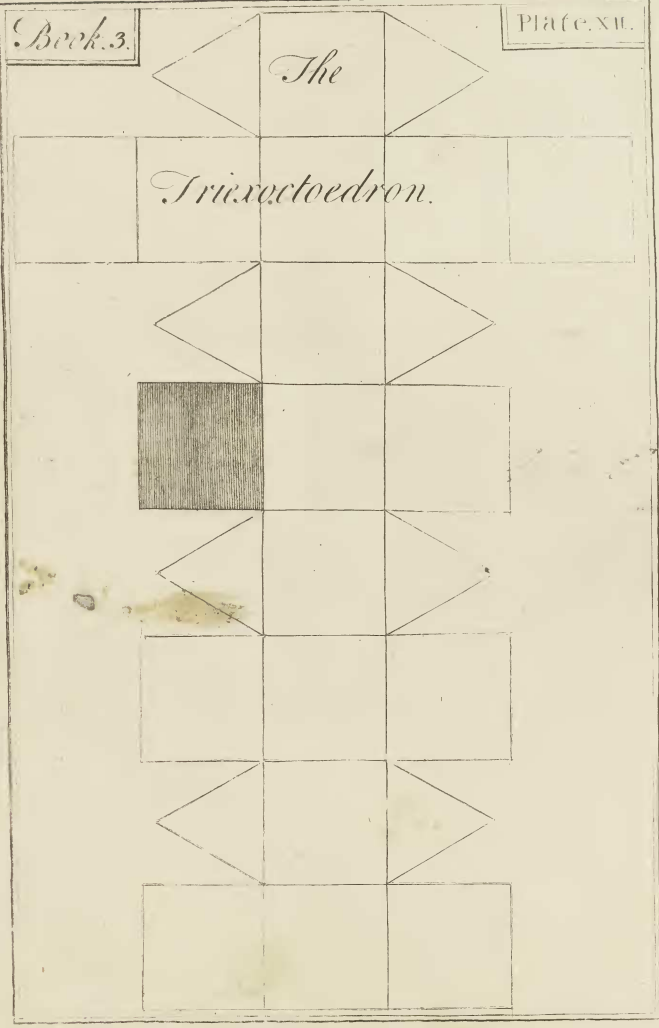


Book. 3.

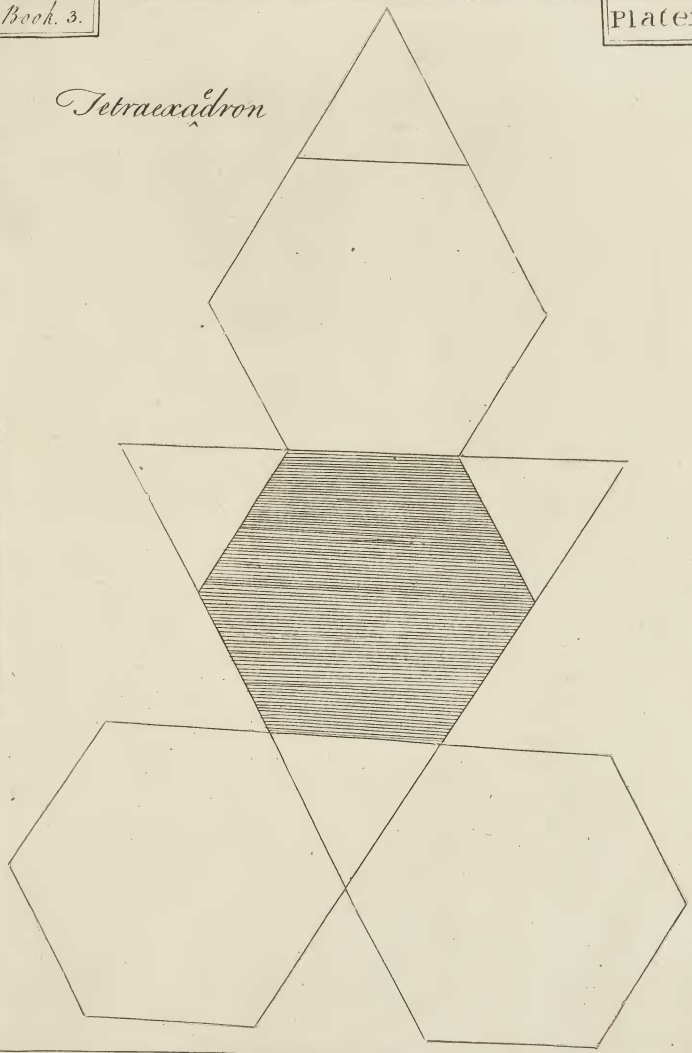
Plate. XII.

The

Triaxoctaedron.

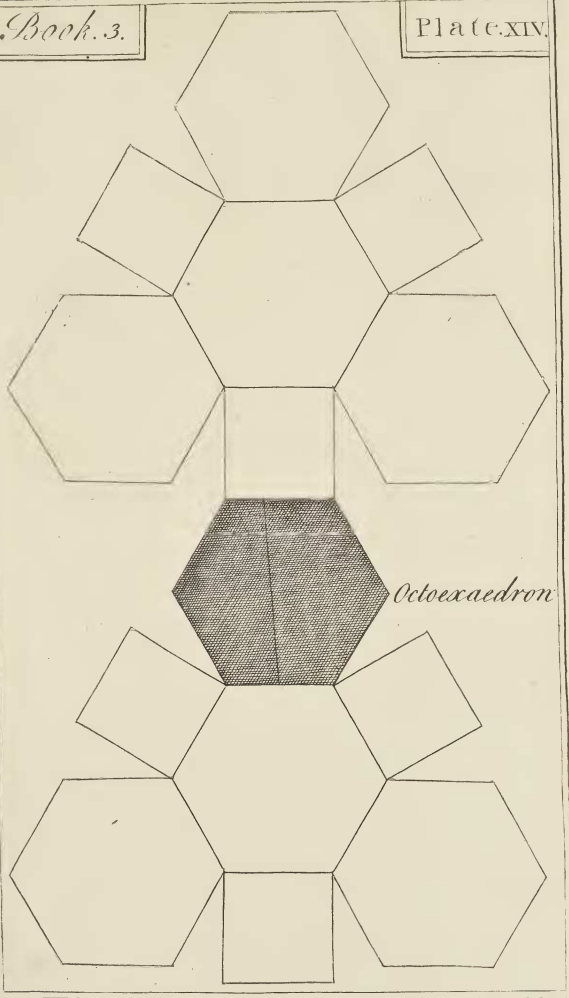


Tetraexáδron



Book. 3.

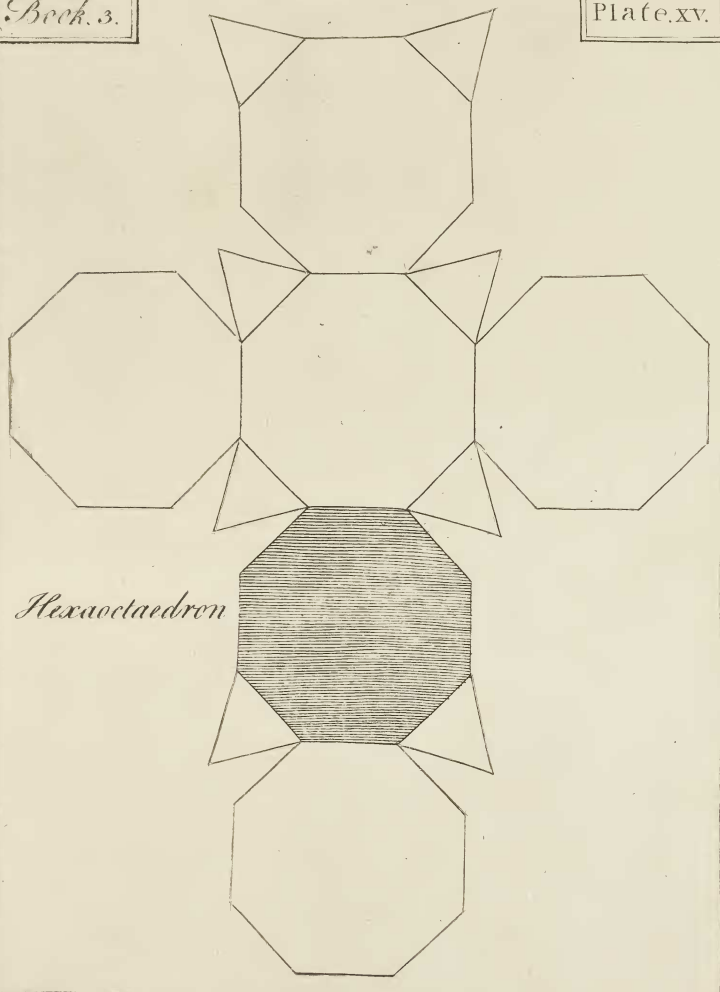
Plate. XIV.



Octoexaedron

Book. 3.

Plate. xv.

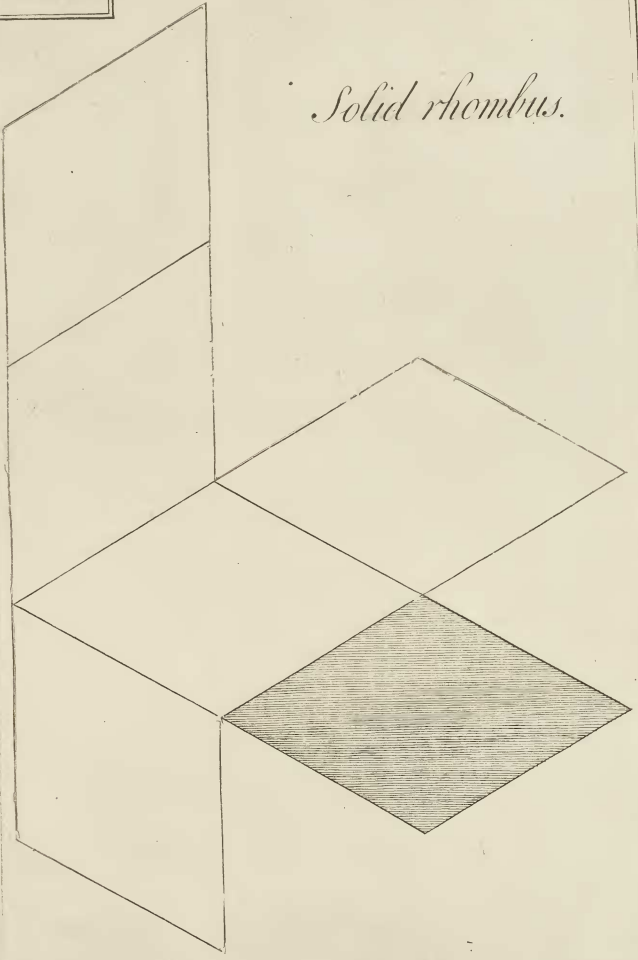


Hexa-octaedron

Book. 3.

Platexvi

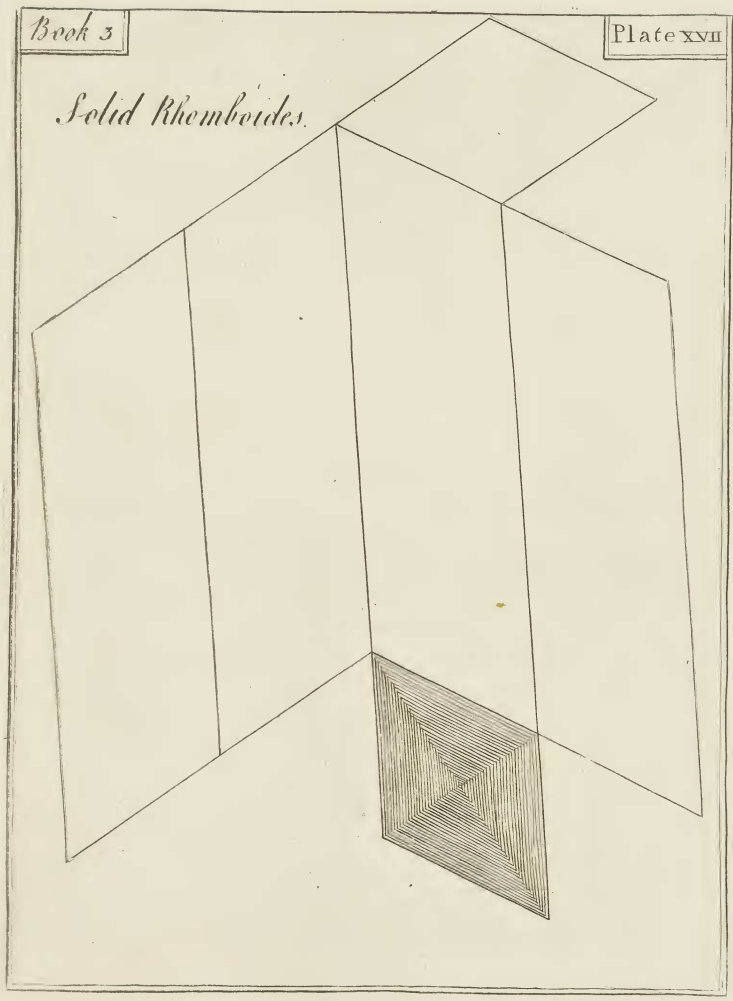
Solid rhombus.



Book 3

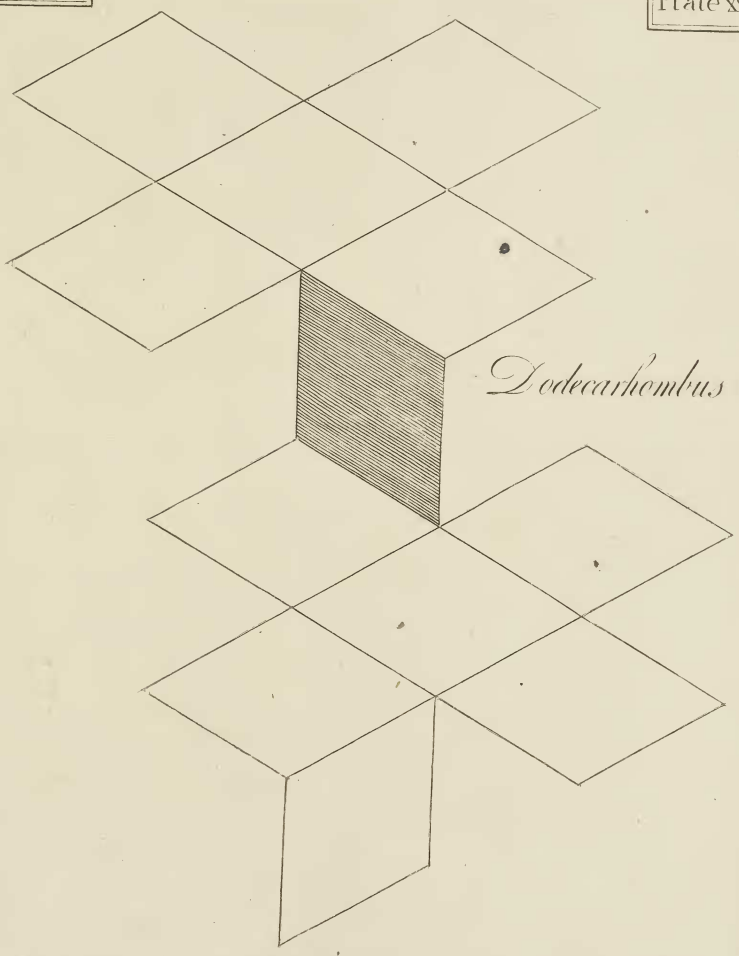
Plate XVII

Solid Rhomboides.



Book 3.

Plate XVIII

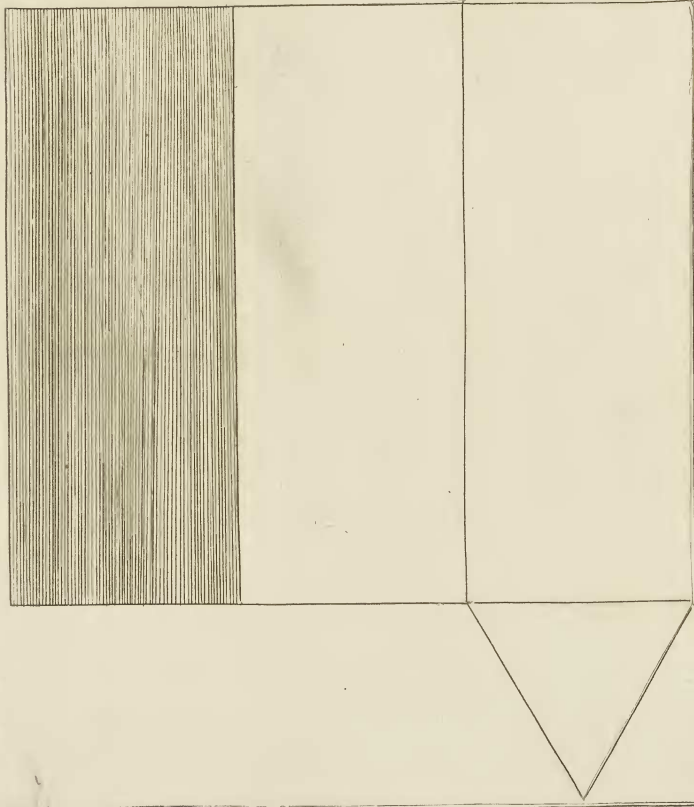


Dodecahedrus

Platexix

Book 4

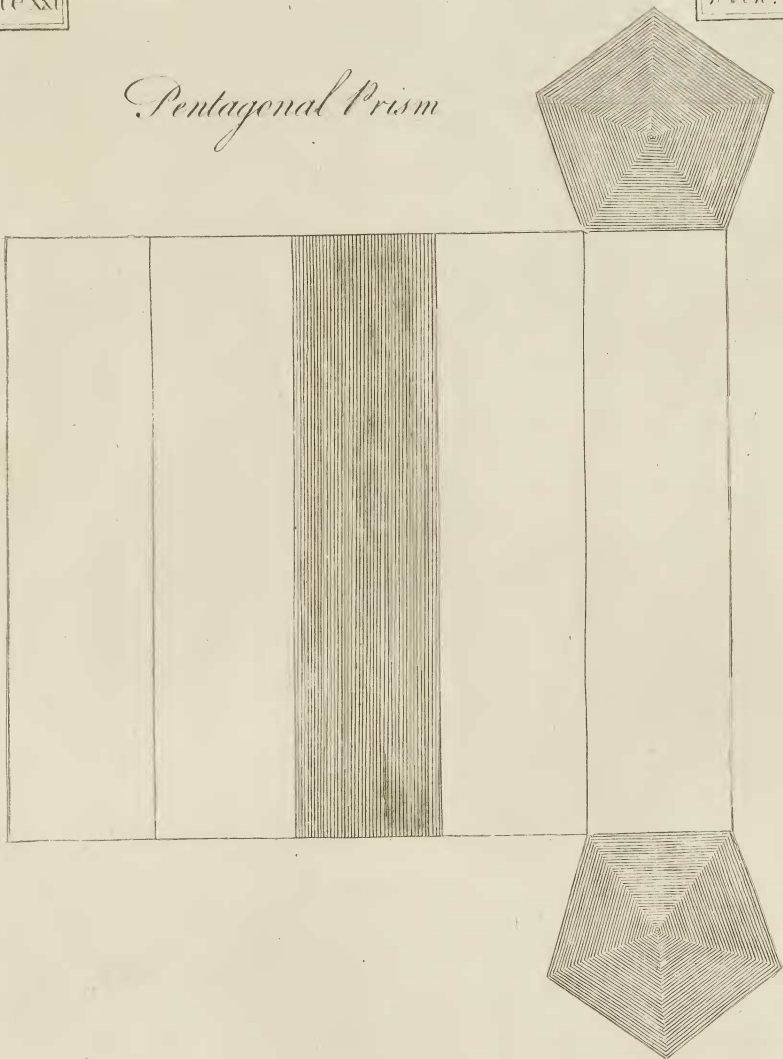
Triangular Prism



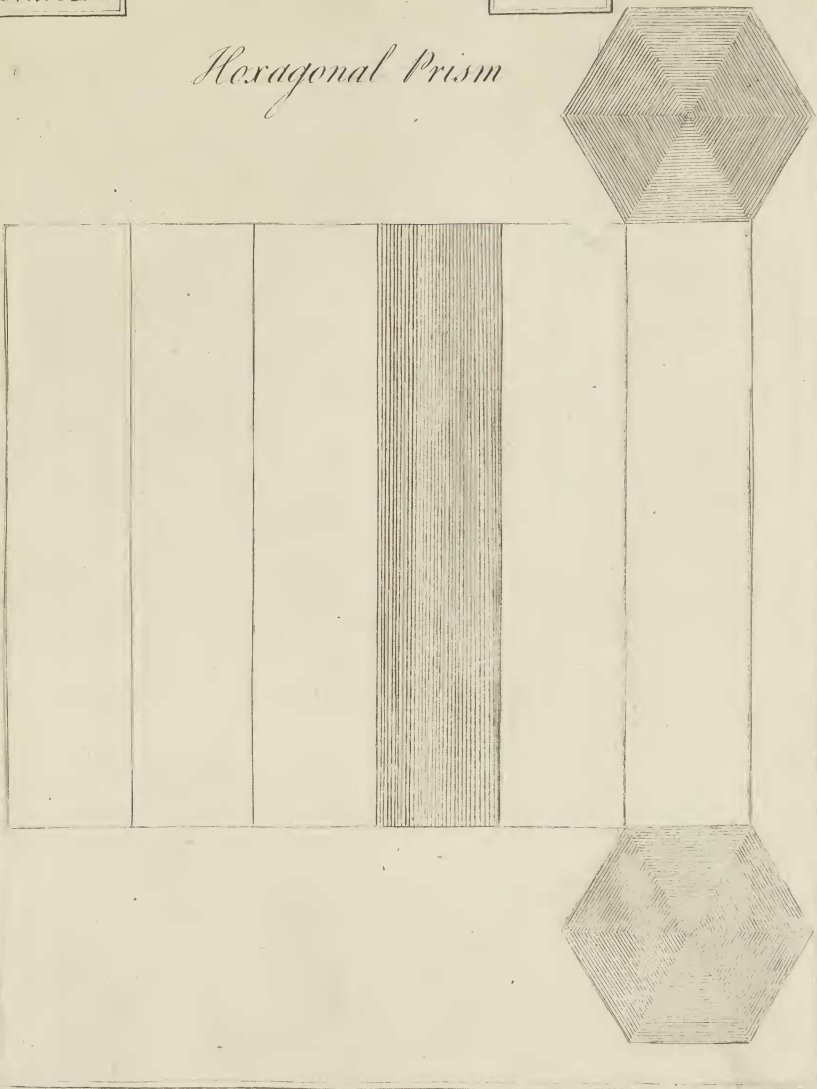
Parallelepipedon

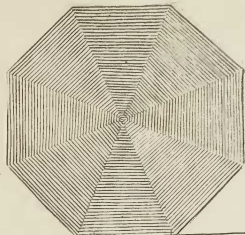


Pentagonal Prism

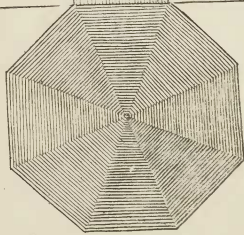
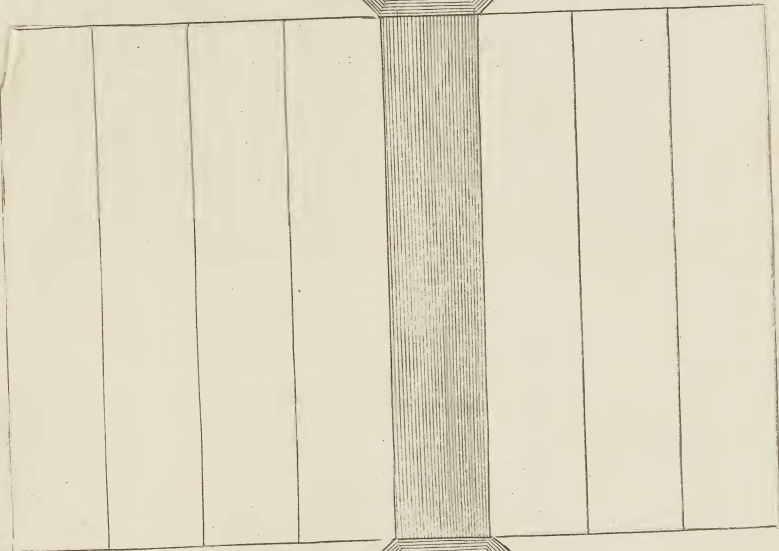


Hexagonal Prism

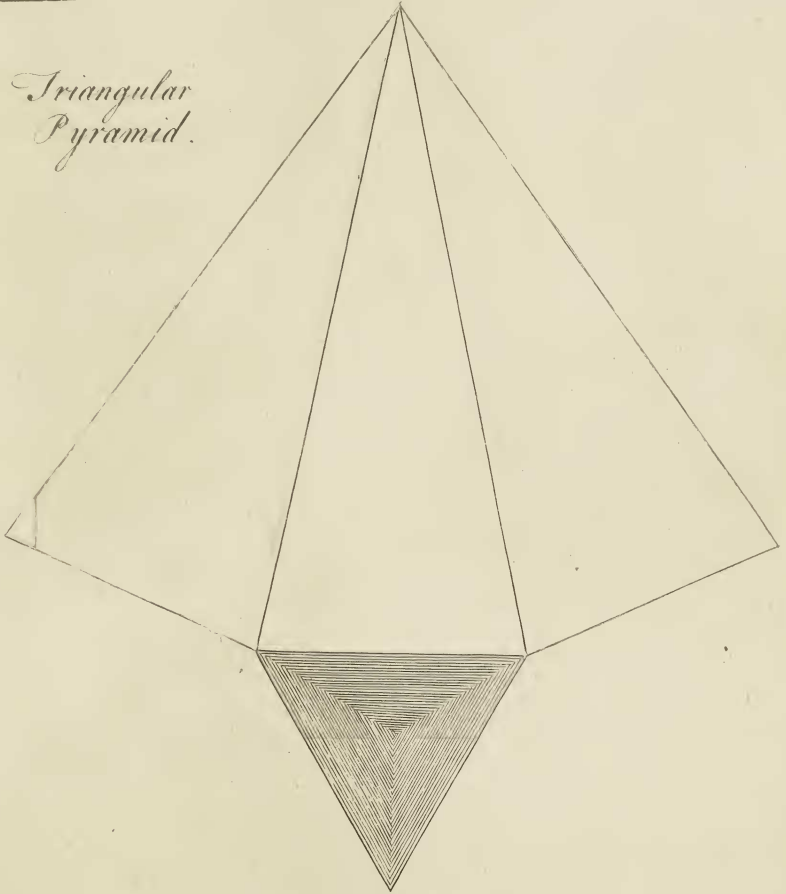


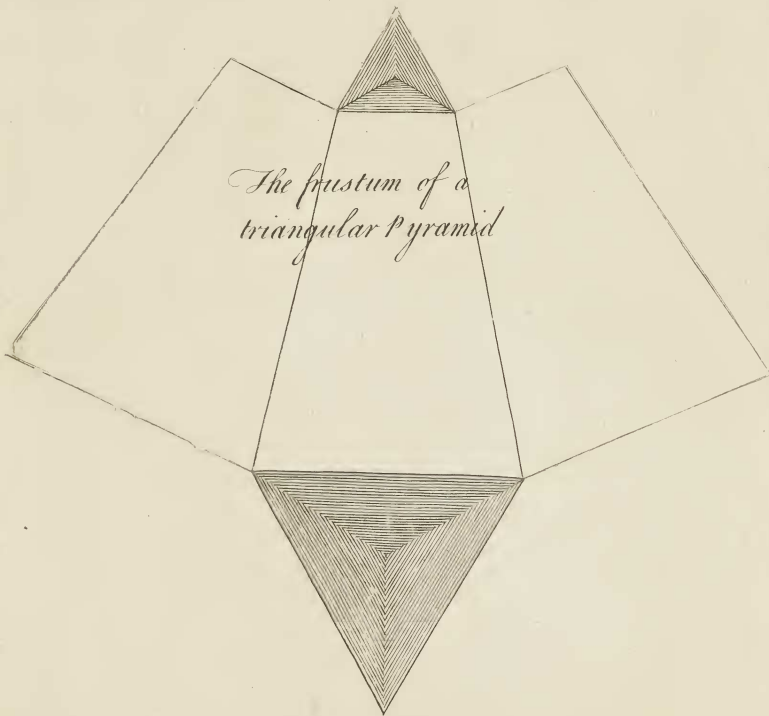


Octogonal Prism.

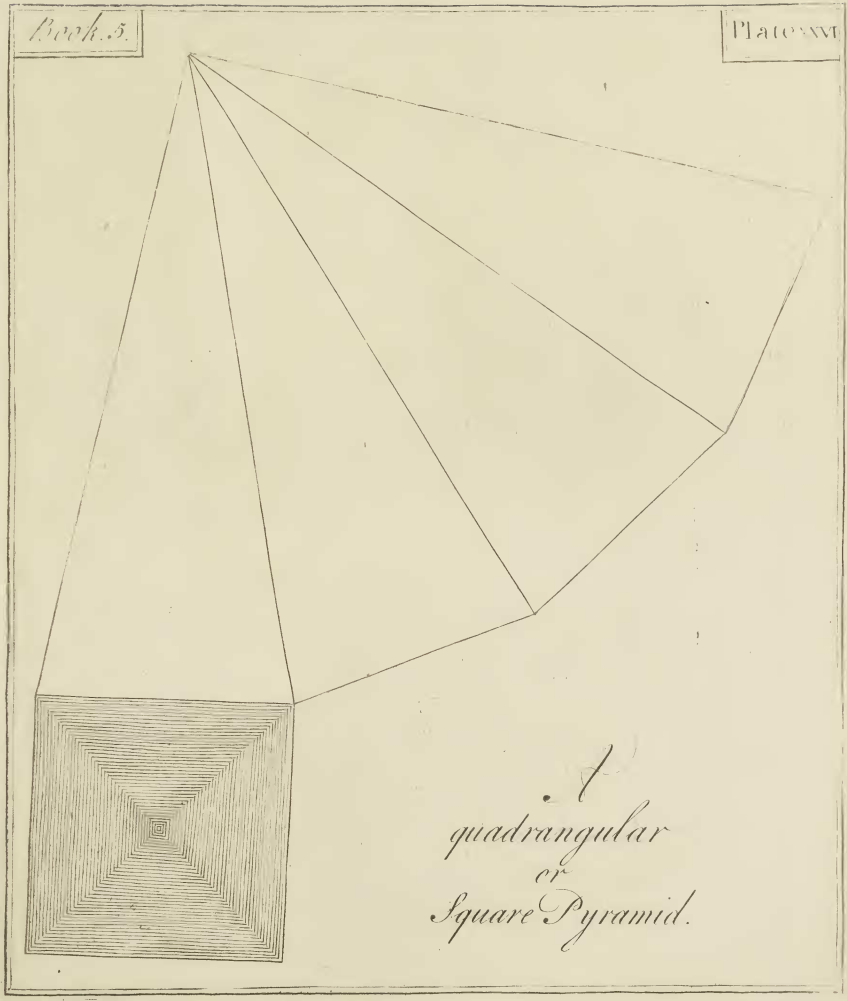


*Triangular
Pyramid.*



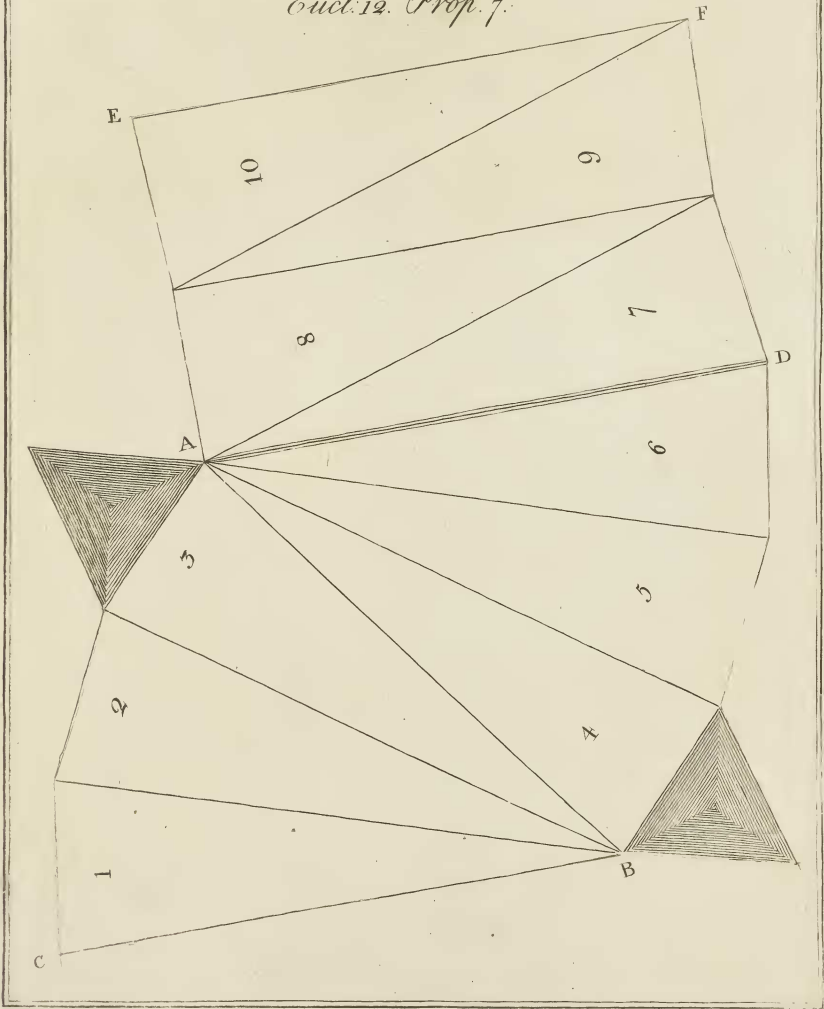


*The frustum of a
triangular Pyramid*

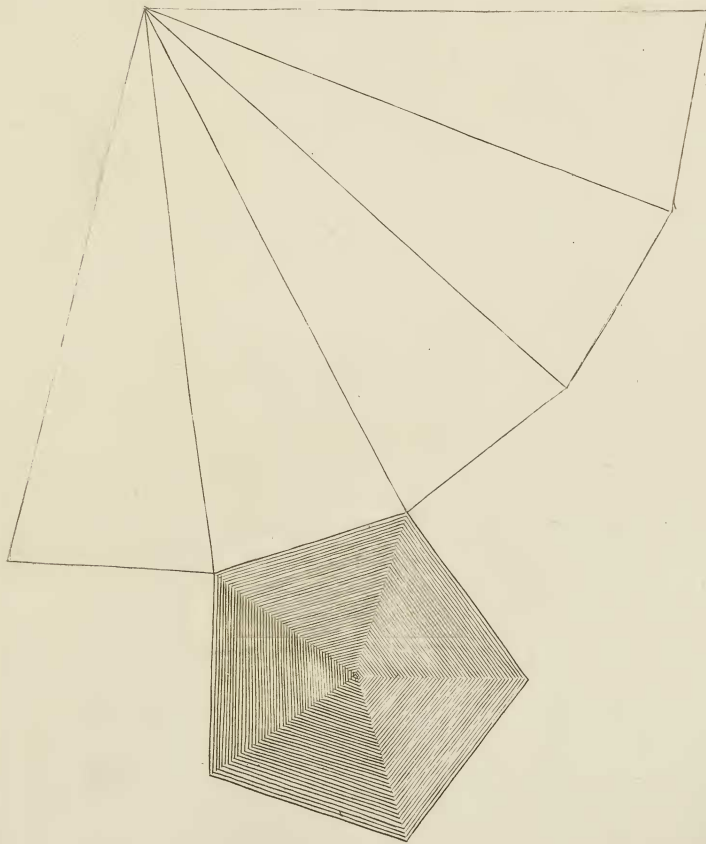


A
quadrangular
or
Square Pyramid.

Eucl. 12. Prop. 7.



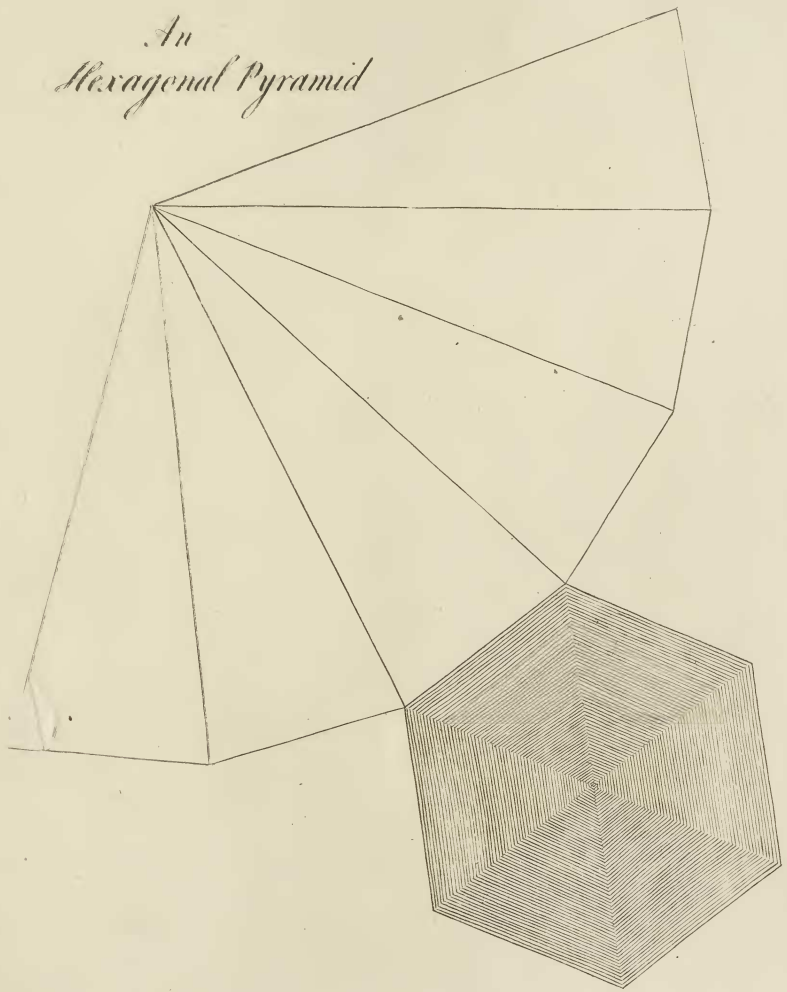
A
Pentagonal Pyramid.



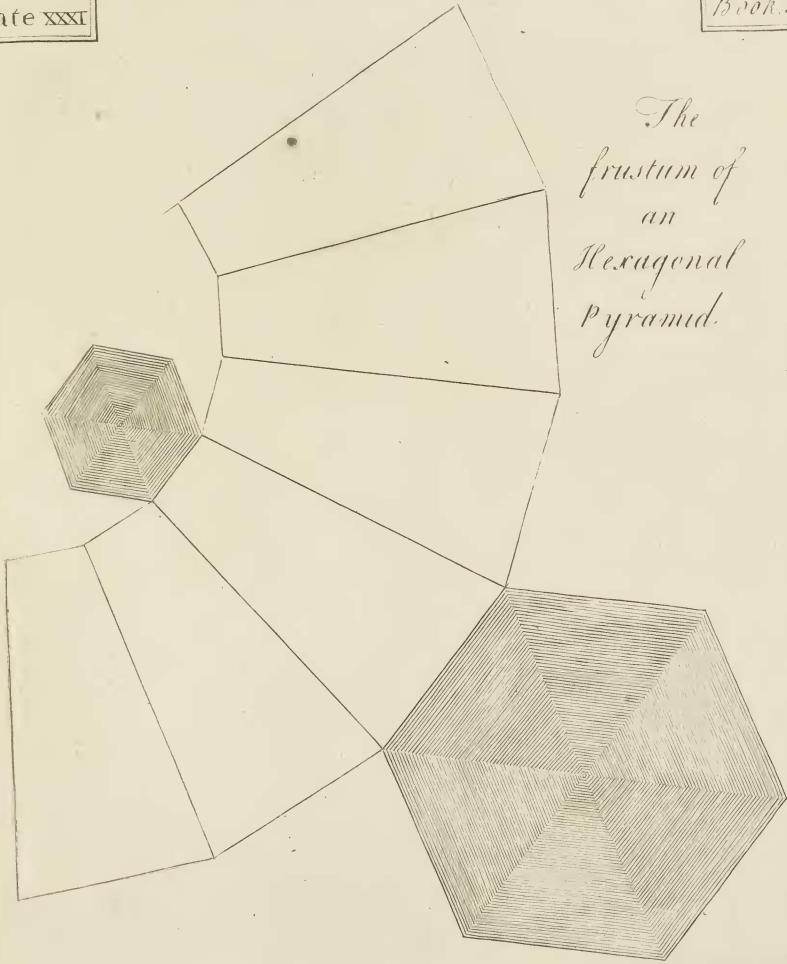
*The
frustum of a Pentagonal
Pyramid.*



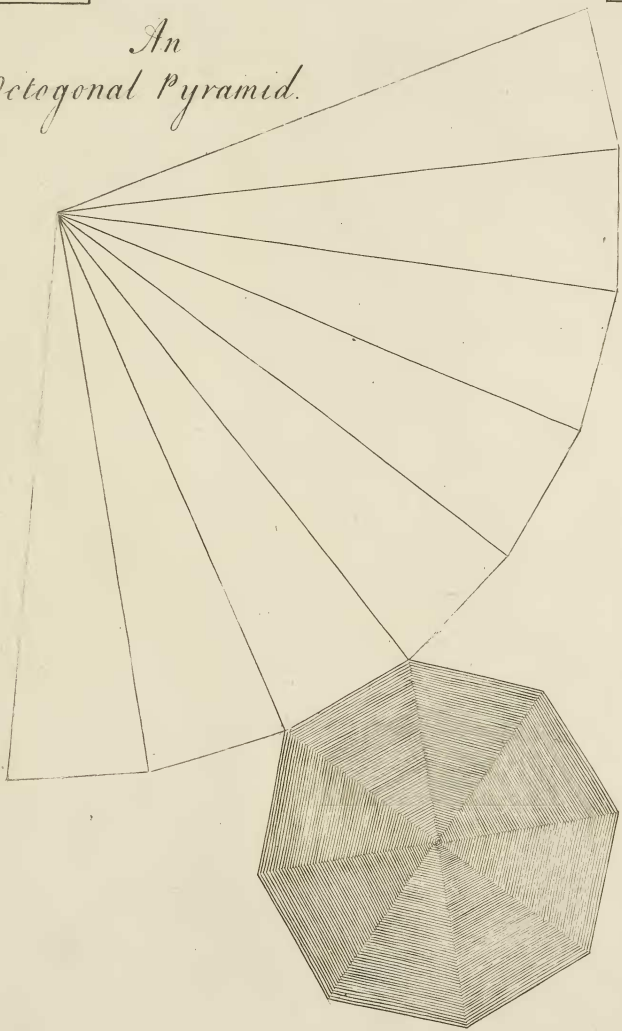
*An
Hexagonal Pyramid*



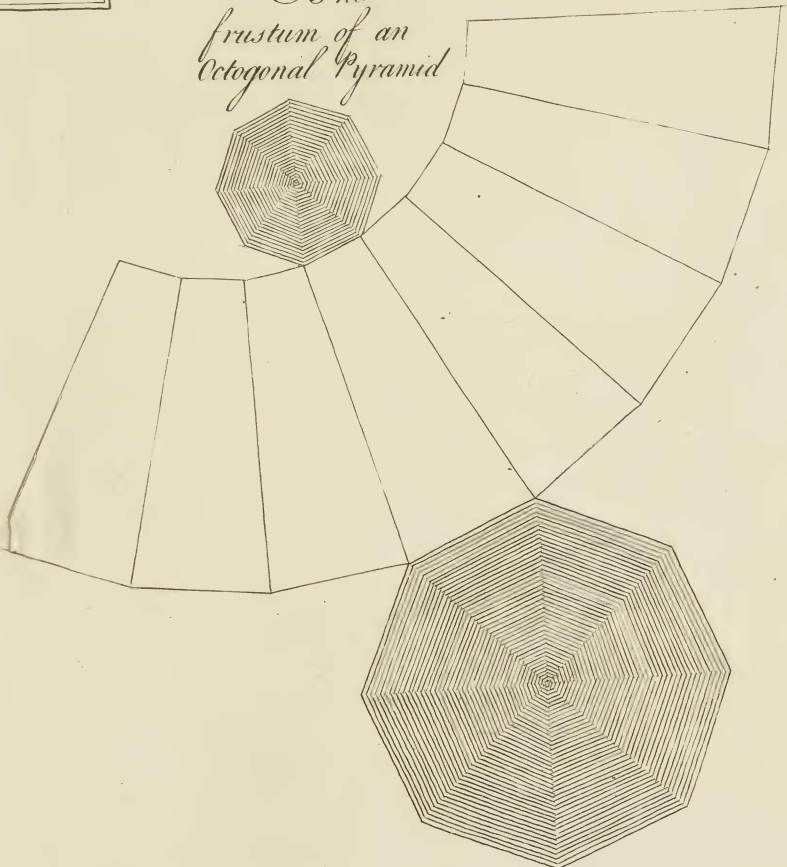
*The
frustum of
an
Hexagonal
Pyramid.*



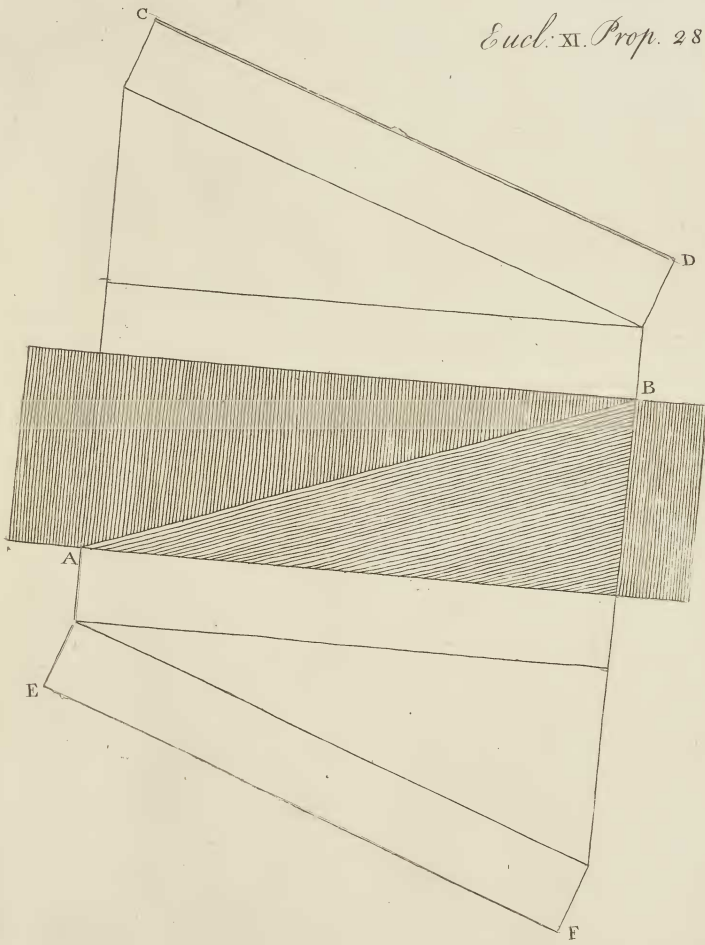
*An
Octogonal Pyramid.*



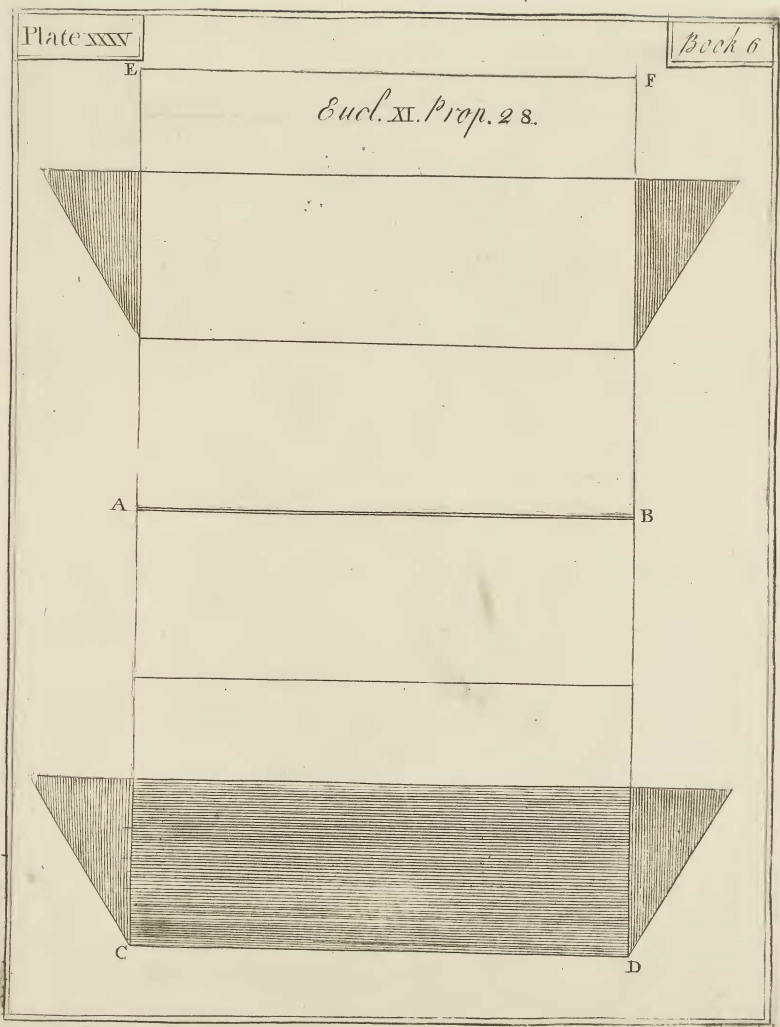
*The
frustum of an
Octogonal Pyramid*

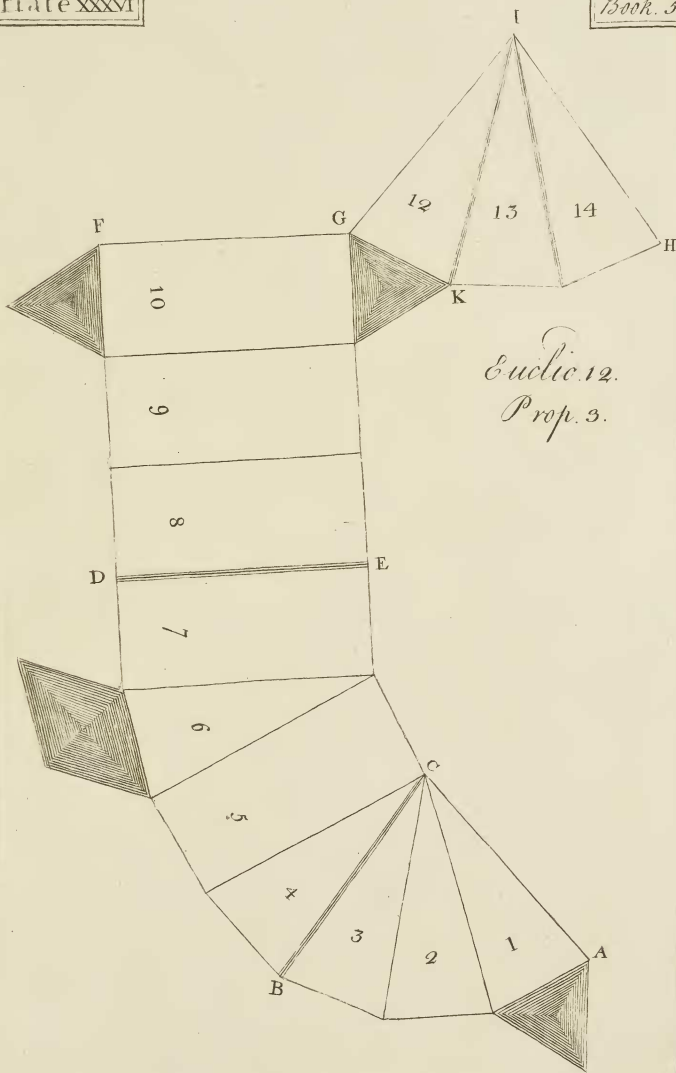


Eucl. XI. Prop. 28.

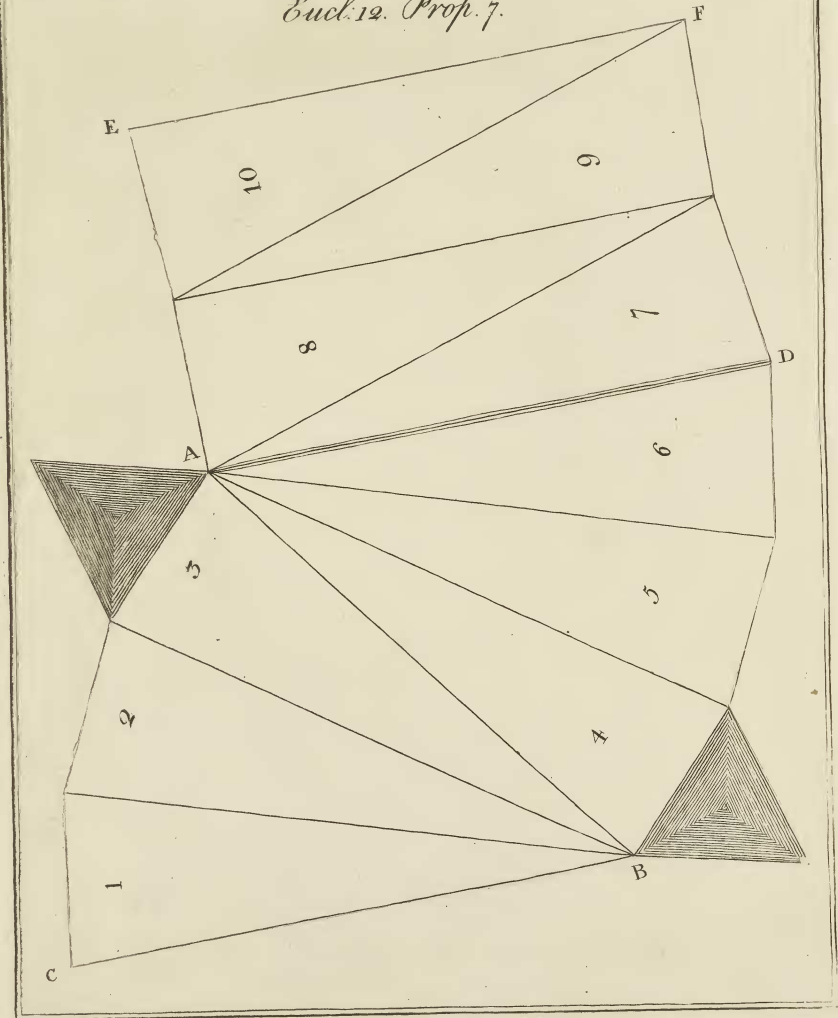


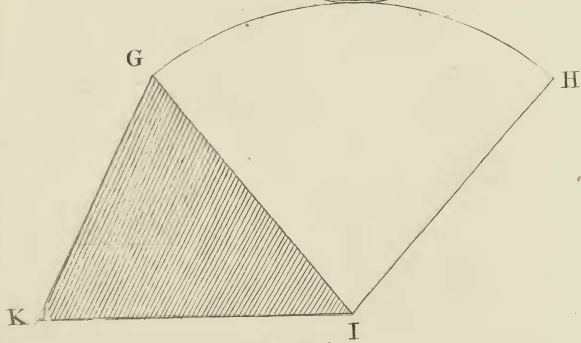
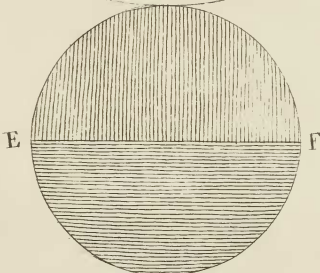
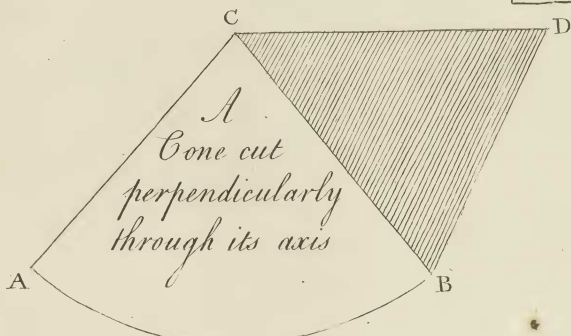
Eucl. XI. Prop. 28.

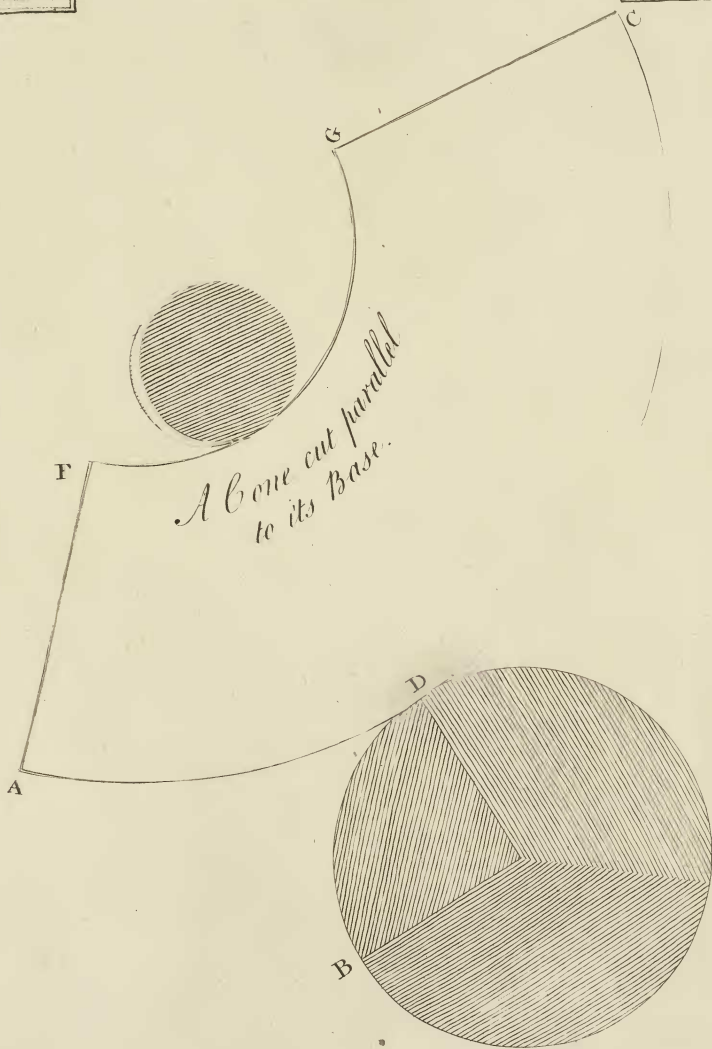




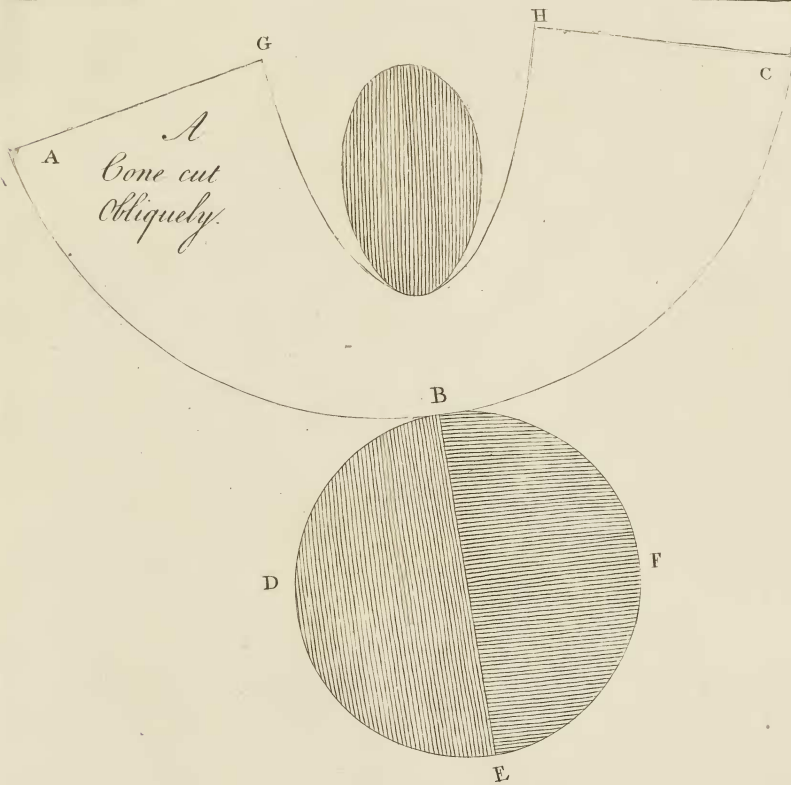
Eucl. 12. Prop. 7.

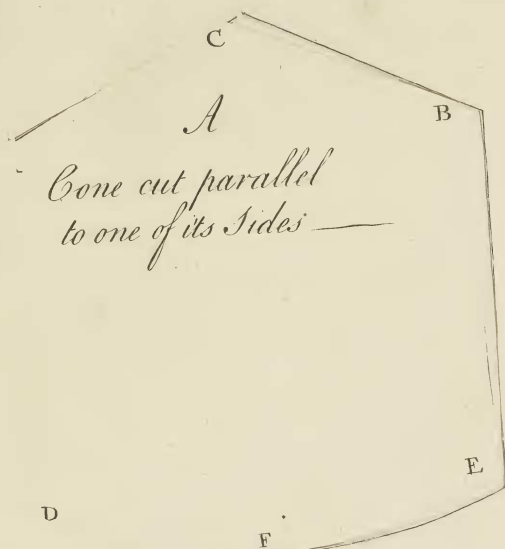




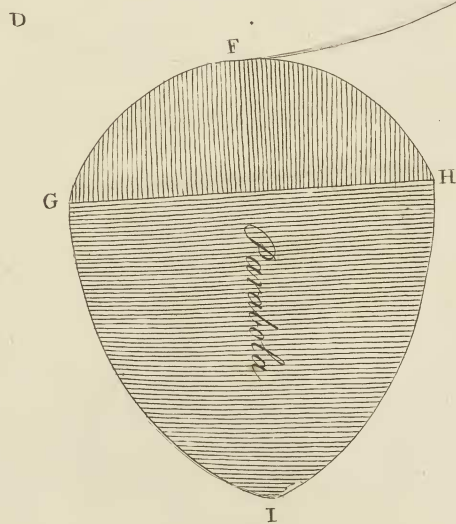


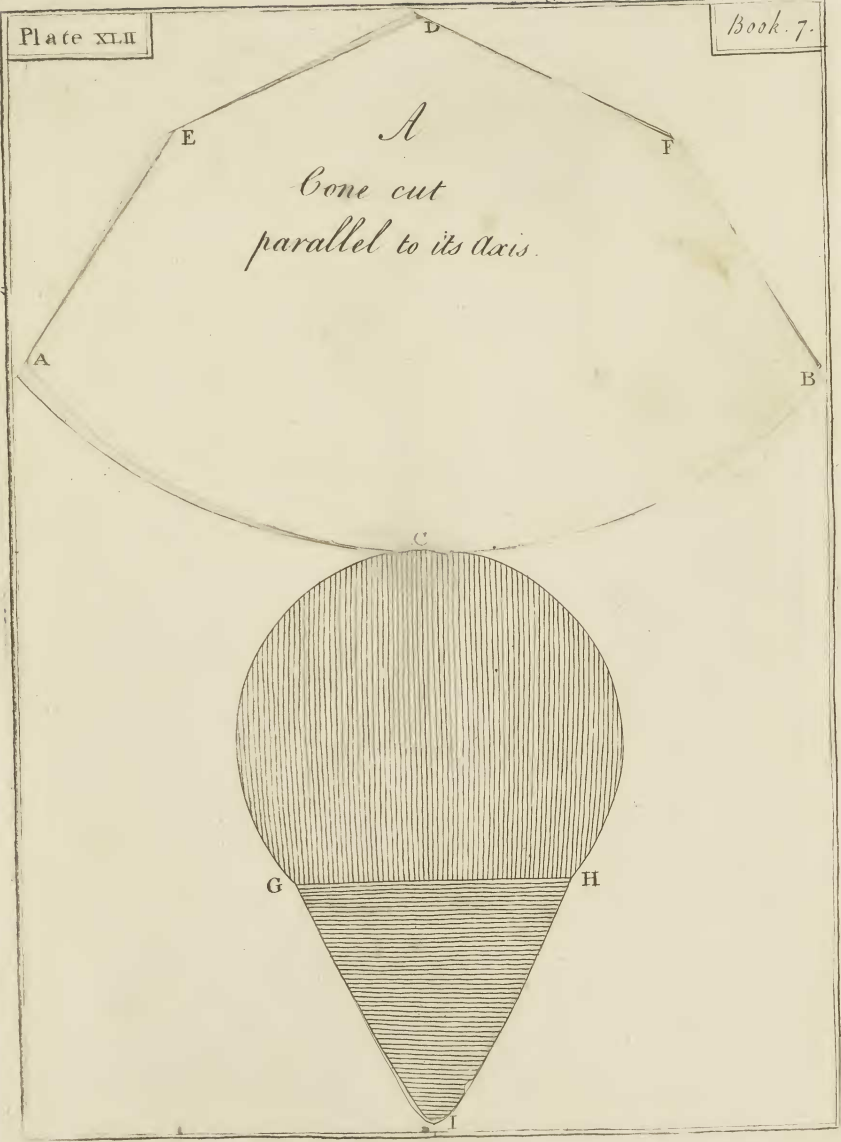
*A Cone cut parallel
to its Base.*





*Cone cut parallel
to one of its Sides*





A
Cone cut
parallel to its axis.