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SYLLABUS

OF

A PROPOSED SYSTEM

OF

LOGIC.

BY

AUGUSTUS DE MORGAN,

F.R.A.S. F.C.P.S.

OF TRINITY COLLEGE, CAMBRIDGE;

PROFESSOR OF MATHEMATICS IN UNIVERSITY COLLEGE, LONDON.

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PREFACE.

The matters collected in this Syllabus will be found in those of my writings* of which the titles follow:—I. *On the Structure of the Syllogism* .... (Cambridge Transactions, vol. viii, part 3, 1847). II. *Formal Logic, or the Calculus of Inference, necessary and probable* (London, Taylor and Walton, 1847, 8vo.). III. *On the Symbols of Logic, the Theory of the Syllogism, and in particular of the Copula,* .... (Cambridge Transactions, vol. ix, part 1, 1850). IV. *On the Syllogism,* No. iii, *and on Logic in general* (Cambridge Transactions, vol. x, part 1, 1858). Of these works the formulæ and notation of the first are entirely superseded; the notation only of the second (the *Formal Logic*) may be advantageously replaced (see § 24) by that of the third and fourth and of the present tract. There is very little in the first three writings on which my opinion has varied; but of all three it is to be said that they are entirely based on what I now call the *arithmetical* view of the proposition and syllogism (§ 8, 173, 174), extending this term not merely to the *numerically definite* syllogism, but to the ordinary form, to my own extension of it, and to Sir W. Hamilton's departure from it.

* The writings which oppose any of my views at length are the following, so far as my memory serves. I. *A letter to* Augustus De Morgan, Esq. ... *on his claim to an independent rediscovery of a new principle in the theory of syllogism, from* Sir William Hamilton, Bart. .... London and Edinburgh, Longman and Co., Machlachlan and Co., 1847, 8vo. II. *Review of my Formal Logic, signed J. S., in the* Biblical Review, 1848. III. *Review of my Formal Logic* (since acknowledged by Mr. Mansel, the author of the *Prolegomena Logica*) *in the* North British Review *for May* 1851, No. 29. IV. *Discussions on Philosophy* by Sir W. Hamilton; London, Longman and Co.; Edinburgh, Machlachlan and Co. (1st edition, 1852, 8vo. pp. 621*–652*; 2nd ed. 1853, 8vo. pp. 676–707); in which a letter is reprinted which first appeared in the *Athenæum* of August 24, 1850.

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The relations of my work on *Formal Logic* to the present syllabus are as follows. Chapter I, *First Notions*, may afford previous knowledge to the student who has hitherto paid no attention to the subject. Chapter III, On the *abstract form of the Proposition* may be consulted at § 93 of this syllabus. Chapters IV, *On Propositions*, and V and VI, *On the Syllogism*, are rendered more easy by the notation of this syllabus, and are partially superseded. Chapter XIV, *On the verbal Description of the Syllogism*, is entirely superseded. All the rest of the work may be read as the titles of the chapters suggest.

A syllabus deals neither in development nor in diversified example: and does not make the space occupied by any detail a measure of its importance as a part of the whole. I have omitted many subjects which are to be found in all the books, or dwelt lightly upon them: partly because more detail is contained in my *Formal Logic*, partly because any one who masters this tract will be able to judge for himself what I should have written on the omitted subjects. I have also endeavoured to remember that as a work of this kind proceeds, less detail of explanation is necessary.

I should suppose that a student of ordinary logic would find no great difficulty in understanding my meaning: and that those who are accustomed to symbolic expression, mathematical or not, would, even though unused to logical study, find no more difficulty than an ordinary student finds in Aldrich's *Compendium.* Either of these classes, I should think, would not fail to come to the point of understanding at which a reflecting mind can allow itself to meditate acceptance or rejection without latent fear of over-confidence. Whether a beginner who is conversant neither with ordinary logic nor with symbolic language will comprehend me is another question: and one on which those who try will divide into more than two classes. Such a reader, making concrete examples for himself as he goes on, and never leaving any article until he has done this, will either get through the whole tract, or will stop at the precise point at which he ought to stop, upon the principle of the next paragraph.

Every spoon has some mouths that it can feed; and some that it cannot. Every writer has some readers who are made for him, and he for them; and some between whom and himself there is a great gulph. I might prove this, in my own case, by a chain of discordant

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testimonies running through thirty years, if I had leisure and liking to hunt up extracts from reviews. I will content myself with a couple which are at hand: observing that I have no acquaintance with the authors. In 1830, I published my treatise on *Arithmetic*, and the following sentences speedily appeared in reviews of it:—

This book appears to us to mystify a very simple science.	It is as clear as Cobbett in his lucid intervals.

In 1847, I published my *Formal Logic*, and two opponents of my views wrote as follows:

Mr. De Morgan is certainly not a lucid writer.	This is an undeniably long extract, and yet we would, did our limits allow, continue it . . . . We beg the reader's notice to the exquisite precision of its language; to the definiteness of every line in the picture; for though it is a description of a profound mental process, still it is a luminous picture, the light of which does not interfuse its lineaments . . . . We are at very solemn issue with Mr. De Morgan upon this argument . . .

These antagonisms remind us of the stork and the fox, and of the failure of their attempts to entertain each other at dinner. When an author's dish and a reader's beak do not match, they must either divide the blame, or agree to throw it on that exquisite piece of atheism, *the nature of things*.

The points on which I differ from writers on logic are so many and so fundamental, that I am among them as Hobbes among the geometers, and, *mutato nomine*, may say with him of Malmesbury:— *In magno quidem periculo versari video existimationem meam, qui a* logicis *fere omnibus dissentio. Eorum enim qui de iisdem rebus mecum aliquid ediderunt, aut solus insanio ego aut solus non insanio; tertium enim non est, nisi (quod dicet forte aliquis) insaniamus omnes.*

Hobbes was in the wrong: the question of parallel or contrast must be left to time. But though some writers on logic explicitly renounce me and all my works, yet one point of those same works is adopted here and one there; so that in time it may possibly be said that

Mahometans eat up the hog.

All these differences are not about the truth or falsehood of my neologisms, but about the legitimacy of their adoption into *logic*, the study of the *laws of thought*. And I cannot imitate Hobbes so far

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as to write *contra fastum logicorum*, seeing that, *oblitis obliviscendis*, I have personally nothing but courtesy to acknowledge from all the writers of known name who have done me the honour of alluding to my speculations. In return, I endeavour to tilt at their shields and not at their faces. Should I make any one feel that I have missed my attempt, I will pray the introduction into these lists of the old practice of the playground. When I was a boy, any offence against the rules of the game was held to be nullified if the offender, before notice taken, could cry *Slips!*—as I now do over all, to meet contingencies. As to the *matter* of our differences, I neither give nor take quarter: it is

> I will lay on for Tusculum,
> Do thou lay on for Rome!

as will sufficiently appear in my notes.

Now to another topic. I produce a fragment of a well-known conversation: those who choose may fill up the chasms.

> "... I therefore dressed up three paradoxes with some ingenuity .... The whole learned world, I made no doubt, would rise to oppose my systems: but then I was prepared to oppose the whole learned world. Like the porcupine, I sat self-collected, with a quill pointed against every opposer. Well said, my boy, cried I, ... you published your paradoxes; well, and what did the learned world say to your paradoxes? Sir, replied my son, the learned world said nothing to my paradoxes; nothing at all, sir. Every man of them was employed in praising his friends and himself, or condemning his enemies; and unfortunately, as I had neither, I suffered the cruellest mortification, neglect."

A friend of the author just quoted remarked that a shuttlecock cannot be kept up unless it be struck from both ends. I was spared all the mortification of neglect by the eminence of the player who took up the other battledore. Of this celebrated opponent I can truly say that, so far as I myself was concerned, I never looked with anything but satisfaction upon certain points of procedure to which I shall only make distant allusion. For I saw from the beginning that he was playing my game, and raising the wind which was to blow about the seeds of my plant. The mathematicians who have written on logic in the last two centuries have been wholly unknown to even the far-searching inquirers of the Aristotelian world: to the late Sir William Hamilton of Edinburgh I owe it that I can present this tract to the moderately well informed elementary student of logic, as containing matters of which he is likely enough to have heard something, and may possibly be curious to hear more.

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In controversy—and controversy was to him an element of life and a spring of action—Sir William Hamilton was too much the fencer of the moment, too much the firer of to-morrow's article: his impulses sometimes leap him over the barrier which divides philosophy from philosophism. Root the proofs of this out of his pages, and we have before us a man both learned and ingenious, profound and acute, weighty and flexible, displaying a most instructive machinery of thought as well when judged right as when judged wrong, save only when cause of regret arises that Oxford did not demand of his youth two books of Euclid and simple equations. In describing a character some points of which are at tug of war—especially when liking for quotation was one of them—Greek may meet Greek. He showed more fondness than was politic in one so plainly destined to survive the grave for a too literal version of the motto

> Οὗτος κράτιστός ἐστ' ἀνὴρ ὦ Γοργία
> Ὅστις ἀδικεῖσθαι πλεῖστ' ἐπίσταται βροτῶν.

But on the other hand, though too much inclined to rule the house, he was οἰκοδεσπότης ὅστις ἐκβάλλει ἐκ τοῦ θησαυροῦ αὐτοῦ καινὰ καὶ παλαιά. And this as to words as well as things. Magnificent command of language, old terms rare enough to be new, and new terms good enough to be old, mask defects and heighten merits. In his writings against me, it delighted him to enliven the statements of the accuser by portraits of the mind of his opponent: colouring his notion of the mathematician from the darkness of his want of notion of the mathematics, his great and admitted defect as a psychologist. I dealt with the statements in my last Cambridge paper: I now oppose my sketch of him to his sketch of me, without the least misgiving as to which of the two will be pronounced the best likeness.

A. DE MORGAN.

University College, London,
November 12, 1859.

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***** The references are to the sections.**

***** The notes end the articles in which the marks of reference occur.**

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SYLLABUS,

&c. &c.

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1. Logic * analyses the *forms*, or *laws of action*, of thought.

* Logic has a tendency to correct, first, inaccuracy of thought, secondly, inaccuracy of expression. Many persons who think logically express themselves illogically, and in so doing produce the same effect upon their hearers or readers as if they had thought wrongly. This applies especially to teachers, many of whom, accurate enough in their own thoughts, nourish sophism in their pupils by illogical expression.

It is very commonly said that studies which exercise the thinking faculty, and especially mathematics, are means of cultivating logic, and may stand in place of systematic study of that science. This is true so far, that every discipline *strengthens* the logical power: that is to say, strengthens most of what it finds, be the same good or bad. It is further true that every discipline corrects *some* bad habits: but it is equally true that every discipline tends to confirm *some* bad habits. Accordingly, though every exercise of mind does much more good than harm, yet no person can be sure of avoiding the harm and retaining only the good, except by that careful examination of his own mental habits which most often takes place in a proper study of logic, and is seldom made without it. This being done, and the house being built, the scaffolding may be thrown away if the builder please: though in most cases it will be advisable to keep it at hand, for use when inspection or repairs are needed. Some persons make the argument about the utility of logic turn on the question whether disputation is or is not best conducted syllogistically: on which I hold—waiving the utter irrelevancy of the question—that those who cannot so argue need to learn, while those who can have no need to practise. It is just the same with spelling.

As to the difficult word *form*, and the variations of it, I refer to Dr. Thomson's *Outlines of the Necessary Laws of Thought*, § 11-15. By a form of thought I mean a necessary law of action, considered independently of any particular matter of thought to which it is applied.

2. Logic is *formal*, not *material:* it considers the law of action, apart from the matter acted on. It is not *psychological*, not *metaphysical:* it considers neither the mind in itself, nor the nature of things in itself; but the mind in relation to things, and things in relation to the mind. Nevertheless, it is so far *psycho-*

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logical as it is concerned with the results of the constitution of the mind: and so far *metaphysical* as it is concerned with the right use of notions about the nature and dependence of things which, be they true or be they false, as representations of real existence, enter into the common modes of thinking of all men.

3. The study of elementary logic includes the especial consideration of—

1. The *term* or *name*, the written or spoken sign of an object of thought, or of a mode of thinking.
2. The *copula* or *relation*, the connexion under which terms are thought of together.
3. The *proposition*, terms in relation with one another; and the *judgment*, the decision of the mind upon a proposition: usually joined in one, under one or other of the names.
4. The *syllogism*, deduction of relation by combination of other relations.

4. The thing which is not of the mind, and can be imagined to exist without the mind, is the *object*: the mind itself is called the *subject* of that object. Thus even a relation between two minds may be an *object* to a third mind. Logic considers only the connexion of the subjective and objective: it treats of things *non secundum se, sed secundum esse quod habent in anima.*

5. The consideration of names, *as names*, may be made to furnish the key to the mechanical, or instrumental, treatment of the ordinary proposition and the ordinary syllogism.

6. For this purpose a name is a mere mark, attached to an object: a letter painted on a post would do as well for explanation as the name of a notion or concept in the mind attached in thought to an external object. In this part of the subject, the assertion 'Every man is an animal' is treated as if it were merely 'Every object which has the name m-a-n has also the name a-n-i-m-a-l.' It answers* to 'Every post on which X is painted has also Y painted on it.'

* Remember that in producing a name, the *existence* of things to which it applies is *predicated*, i. e. asserted: and (§ 16) existence in the universe of the propositions. There is no *conditional* proposition intended, as Every X is Y (if Y exist). Every proposition is a conceivable or imaginable truth, when its terms are conceivable or imaginable, except only when it announces a contradiction, as 'Some men are not men,' or when it is its own subject-matter and denies itself, as 'What I now say is false,' a proposition which is false if it be true, and true if it be false.

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7. The *proposition,* in this view, is no more than the connexion of name with name, as marks of the same object: the *judgment* is no more than assertion or denial of that connexion. The word *is* asserts the connexion: the words *is not* deny it.

8. This kind of proposition belongs to the *arithmetical* view of logic: there is in it result of enumeration of similar instances: as in 'Every X is Y', '50 Xs are not Ys', i.e. there are 50 (or more) instances in which the name X occurs unassociated with the name Y.

9. The first-mentioned name is called the *subject:* the second the *predicate.*

10. The logical *quantity* (i.e. number of instances) is either *definite,* or more or less *vague.*

11. *Definite* quantity is either *absolutely* or *relatively* definite: absolutely, as in '50 Xs (and no more) are Ys'; relatively, as in '2-sevenths (and no more) of all the Xs are Ys', and in 'All the Xs are Ys', and in 'None of the Xs are Ys', which last is both absolutely and relatively definite.

12. Quantity may be definite at one end, and vague at the other: as in '50 (or more) Xs are Ys'; 'at least 2-sevenths of all the Xs are Ys'; 'most of the Xs (i.e. more than half) are Ys', which, however, is *limited* at one end, not *definite.*

13. The only perfectly definite quantities in ordinary use are *all* and *none.* The materials for other absolute or relative numerical definiteness are but seldom found in human knowledge. The words *all* and *none* are signs of *total* quantity, and make propositions *universal,* as 'All Xs are Ys', 'No Xs are Ys'.

14. The *contrary* (usually called *contradictory*) propositions of the last are 'Some Xs are not Ys', and 'Some Xs are Ys'. Here 'some' is a quantity entirely vague in one direction: it is *not-none;* one* at least; or more; all,† it may be. *Some,* in common life, often means both *not-none* and *not-all;* in logic, only *not-none.* *Some* is the mark of *partial* quantity; and the propositions which commence with it are *particular.* The *contrary* (usually *contradictory*) forces of the pairs are seen in 'Either all Xs are Ys, or some Xs are not Ys; not both'; and in 'Either no Xs are Ys, or some Xs are Ys; not both.'

* Remember that *some* does not guarantee more than *one.* There is much distinction between *none* (no one) as the logical contrary of *some* (one or more), and *nothing* as the limit of *some quantity.* In passing from the proposition 'no man can live without air' to 'there is one man at least who can live without air' we make a transition which alters the notions

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or concepts attached to man: the word man no longer represents entirely the same idea. But in passing from an indenture of apprenticeship with *no premium at all* to one with a premium of *one farthing*, we make no change of notion. An Act was once passed exempting such indentures from duty when the premium was under five pounds sterling: the Court of King's Bench held that the exemption did not apply when there was *no premium at all*, because "no premium at all" is not "a premium under five pounds." That is, the judges gave to *nothing*, as the terminus of continuous quantity, the force of *none*, as the logical contrary of *some;* and violated to the utmost the principle of the Act, which was intended as a relief to those who could only pay small premiums, and *à fortiori*, or rather *à fortissimo*, to those who could pay none at all. Lawyers ought to be much of logicians, and something of mathematicians.

† *Some* may be *all*. In common language this is or is not the case according to the speaker's state of knowledge. But in logic there are no implications which depend upon the matter. When a logician says that 'Every X is Y' he means that 'All the Xs are some Ys' and that 'some Ys are all the Xs'. Whether he have or have not exhausted all the Ys he does not here profess to state, even if he know. Again, when he says 'Some Xs are Ys' he does not mean to imply 'Some Xs are not Ys': that is, his *some* may be *all*, for anything he asserts to the contrary. But when two propositions, each of which contains the vague *some*, are conjoined, the mere meaning may render the conjunction an absurdity unless *some* take the force of *all*. Just as in algebra an equation having two unknown quantities has the values of those quantities vague; but when two such equations are conjoined, those values become definite: so in logic, in which the same thing occurs. Thus the two propositions 'All Xs are some Ys', and 'All Ys are some Xs', when true together, force the inference that *some* must in both cases be *all*. Forget that *some* is that which *may be* all, and these two propositions appear to contradict one another: very distinguished logicians have asserted that they do so. Sir W. Hamilton (*Discussions*, 2nd edition, p. 688) calls them *incompossible* propositions, meaning propositions which cannot be true together. The word is an excellent one, and much wanted: but not here. These two propositions are not *incompossible*, unless *some* take into its meaning *not-all* as well as *not-none:* and *some* is never allowed by logicians to mean *not-all*. It is needless to argue things so plain: it would have been needless to state them twice, except for the eminence of the writers who deny them upon occasion.

15. In 'All Xs are Ys', Y is partially spoken of; there may or may not be more Ys besides: the same of 'Some Xs are Ys'. In 'No Xs are Ys', and in 'Some Xs are not Ys', Y is totally spoken of; each X spoken of is not any one of all the Ys.

16. By the *universe* (of a proposition) is meant the collection of all objects which are contemplated as objects about which assertion or denial may take place. *Let every name which belongs to the whole universe be excluded as needless:* this must be particularly remembered. Let every object which has not the name

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X (*of which there are always some*) be conceived as therefore marked with the name x, meaning not-X, and called the *contrary* of X. Thus every thing is either X or x; nothing is both: 'All Xs are Ys' means 'No Xs are ys': 'No Xs are Ys' means 'All Xs are ys': 'Some Xs are not Ys' means 'Some Xs are ys': 'Some Xs are Ys' means 'Some Xs are not ys'.

17. The following enlarged definitions include the definitions above given, and apply to all the uses of terms and relations in this work. Let a TERM be *total* or *partial* according as every existing instance must or need not be examined to verify the proposition. Thus in 'Everything is either X or Y', X and Y are both partial: an object being examined and found to be X, the proposition is made good so far as that object is concerned; that object may also be Y, but if so, it need not be ascertained: consequently, Y is partially spoken of; and the same may be said of X. But in 'Some things are neither Xs nor Ys', X and Y are both total: we can only verify it by an object which is not any one of the Xs and not any one of the Ys.

18. Let a PROPOSITION be *universal* or *particular*, according as the whole universe of objects must or need not be examined to verify it. Thus 'Everything is either X or Y' is plainly universal: but 'Some things are neither Xs nor Ys' is particular: the first object examined may settle the truth of the proposition.

19. Let a PROPOSITION be *affirmative* which is true of X and X, false of X and not-X or x; *negative*, which is true of X and x, false of X and X. Thus 'Every X is Y' is affirmative: 'Every X is X' is true; 'Every X is x' is false. But 'Some things are neither Xs nor Ys' is also *affirmative*, though in the form* of a *denial*: 'Some things are neither Xs nor Xs' is *true*, though superfluous in expression; 'Some things are neither Xs nor xs' is false. Again, 'Everything is either X or Y' is *negative*, though in the form of an *assertion*: 'Everything is either X or X' is false; 'Everything is either X or x' is true.

* When contrary terms are introduced, it is impossible to define the opposition of quality by assertion or denial: for every assertion is a denial, and every denial is an assertion. The denial 'No X is Y' is the assertion 'All Xs are ys.' The necessary distinction between *affirmative* and *negative* is therefore drawn as in the text: these technical terms are retained, though perhaps they are hardly the right ones for *me* to use.

20. Affirmative and negative propositions are said to be of different *quality*.

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21. Let X, totally spoken of, be X) or (X: let X, partially spoken of, be)X or X(. Let a negative proposition be denoted by one dot; an affirmative proposition by two dots or none, at pleasure. I follow Sir William Hamilton in calling this notation *spicular* (see § 216, note). So far as yet appears, we have propositions with the following symbols,—

Contraries	A	Universal Affirmative	X))Y	All Xs are some Ys.
	O	Particular Negative	X(·(Y	Some Xs are not (all) Ys.
Contraries	E	Universal Negative	X)·(Y	All Xs are not (all) Ys.
	I	Particular Affirmative	X()Y	Some Xs are some Ys.

22. These forms have been denoted by the letters A, O, E, I, for many centuries; A and I from the vowels in *AffIrmo;* E and O from the vowels in *nEgo*. The word (all), in parentheses, is not grammatical: the word *any* should be substituted for *all.* The reason why, for the present, I do not use '*any*' will appear in the sequel.

23. Take the four pairs, X, Y; X, y; x, Y; x, y; and apply the four forms above to all four. Sixteen results appear; of which eight are but a repetition of the other eight. Of the eight* which are distinct, we have four written above: the remaining four appear among the following,—

From)) we have X))Y, X))y, x))Y, and x))y. Of these X))y is obviously X)·(Y. And x))Y is x)·(y, a new form: no not-X is not-Y; nothing is both not-X and not-Y; everything is either X or Y. This being a universal proposition with both terms partial, and also a negative proposition, let it be marked X(·)Y. Again, x))y is x)·(Y, or Y)·(x, or no Y is not-X, or every Y is X, or some Xs are all Ys, or X((Y.

From () we have X()Y, X()y, x()Y, x()y. Here X()y is X(·(Y; and x()Y is Y()x, some Ys are not Xs, or all Xs are not some Ys, or X)·)Y. And x()y is a new form, Some not-Xs are not-Ys; some things are neither Xs nor Ys. This is a particular proposition, affirmative, with X and Y both total: let it be marked X)(Y.

* Any one who wishes to test himself and his friends upon the question whether analysis of the forms of enunciation would be useful or not, may try himself and them on the following question:—Is either of the following propositions true, and if either, which? 1. All Englishmen who do not take snuff are to be found among Europeans who do not use tobacco. 2. All Englishmen who do not use tobacco are to be found among Europeans who do not take snuff. Required immediate answer and demonstration.

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24. The eight distinct* forms in which X and Y appear are as follows; the ungrammatical introduction (all) being made as before,—

	Universal propositions		*Contrary particular propositions*
X))Y	All Xs are some Ys	X(·(Y	Some Xs are not (all) Ys
X)·(Y	All Xs are not (all) Ys	X()Y	Some Xs are some Ys
X(·)Y	Everything is either some X or some Y (or both)	X)(Y	Some things are not either (all) Xs nor (all) Ys
X((Y	Some Xs are all Ys	X)·)Y	All Xs are not some Ys

For symmetry, X)·(Y might be read 'Everything is either not (all) X, or not (all) Y'; and X()Y as 'Some things are both some Xs and some Ys'. This will be better seen when we come to § 206. At present, however, I preserve the ordinary reading.

* The following is the comparison of the notation in my *Formal Logic* with that used in my second and third papers in the *Cambridge Transactions*, and in this syllabus.

For X))Y, X)Y and A,	For X()Y, XY and I,
For X((Y, X(Y and A′	For X)(Y, xy and I′
For X)·(Y, X.Y and E,	For X(·(Y, X:Y and O,
For X(·)Y, x.y and E′	For X)·)Y, Y:X and O′
For X)∘)Y, D, and A, + O′	For X)∘(Y, C, and E, + I′
For X ‖ Y, D and A, + A′	For X |·| Y, C and E, + E′
For X(∘(Y, D′ and A′ + O,	For X(∘)Y, C′ and E′ + I,

These comparative notations being fixed in the mind, any part of my *Formal Logic* may be read in illustration of the present work. And the *detailed character* (§ 64, note) of the latest notation is, if I may judge, so much of a facilitation, that any reader of the *Formal Logic* will find it easier to translate the notation as he goes on than to confine himself entirely to the notation of that work: and this especially as to the tests of validity and the assignment of the symbol of inference.

25. The thirty-two forms which arise from application of contraries are as now written, all the eight cases above being used: the four in each line are of identical meaning.

Universals				*Particulars*			
X))Y ,	X)·(y ,	x(·)Y ,	x((y	X(·(Y ,	X()y ,	x)(Y ;	x)·)y
X)·(Y	X))y	x((Y	x(·)y	X()Y ,	X(·(y ,	x)·)Y ,	x)(y
X(·)Y	X((y	x))Y	x)·(y	X)(Y ,	X)·)y ,	x(·(Y ,	x()y
X((Y	X(·)y	x)·(Y	x))y	X)·)Y ,	X)(y ,	x()Y ,	x(·(y

26. The rule of *contraversion*—changing a name into its contrary without altering the import of the proposition—is, Change also the *quantity* of the term, and the *quality* of the proposition. Thus X))Y is X)·(y and x(·)Y. When both names are contra-

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verted, change both quantities, and preserve the quality: thus X(·)Y is x)·(y.

27. The rule of *conversion**—making the names change places, without altering the import of the proposition—is, Write or read the proposition backwards. Thus X))Y is Y((X; or X))Y may be read backwards, Some Ys are all Xs. That is, make both the terms and their quantities change places.

* Writers on logic have nearly always meant by *conversion* merely the change of place in the terms, without change of place in the quantities. Accordingly, when the quantities are different, (common) logical conversion is illegitimate. Thus X))Y and Y))X are not the same: but X()Y and Y()X are the same. There is this difficulty in the way of using the word *conversion* in the sense proposed in the text: namely, that common logic has rooted it in common language that 'Every X is Y' is the converse (true or false as the case may be) of 'Every Y is X.' Leaving the common idioms for the student to do as he likes with, I shall, if I have occasion to speak of a proposition in which terms only are converted, and not quantities, call it a *term-converse*.

28. Each universal is inconsistent with the universals of different qualities, and indifferent to the universal of different quantities. Thus X))Y is inconsistent with X)·(Y and X(·)Y, and neither affirms nor denies X((Y. Each universal affirms the particulars of the same quality, contradicts the particular of different quantities, and is indifferent to the particular of the same quantities. Thus X))Y affirms X()Y and X)(Y, contradicts X(·(Y, and neither affirms nor denies X)·)Y.

	Affirms		Contradicts	Is inconsistent with		Neither affirms nor denies	
))	()	)(	(·(	)·(	(·)	((	)·)
)·(	)·)	(·(	()	))	((	(·)	)(
(·)	(·(	)·)	)(	((	))	)·(	()
((	)(	()	)·)	(·)	)·(	))	(·(
	Is affirmed by		Contradicts	Is neither affirmed nor denied by			
(·(	)·(	(·)	))	((	)·)	()	)(
()	((	))	)·(	(·)	)(	(·(	)·)
)(	))	((	(·)	)·(	()	)·)	(·(
)·)	(·)	)·(	((	))	(·(	)(	()

29. Contrary names, in identical propositions, always appear with different quantities. We cannot speak of *some* Xs without speaking about *all* xs; nor of *all* Xs without speaking about *some* xs.

30. A particular proposition is *strengthened* into a universal which affirms it (and more, may be) by altering *one* of the quantities: thus)·) is affirmed in (·) and in)·(Remember § 16.

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31. In a universal proposition, if *one* term be partial, it has the amount, not the character, of the quantity of the other: if *both*, the quantities of the two terms together make up the whole universe, with the part common to both, if any, repeated twice.

32. In a particular proposition, the quantity of a partial term is vague, but remains the same through all forms. And when both terms are total, the partial quantity still remains expressed: as in X)(Y, or *Some things* are neither Xs nor Ys; which some things are as many as the xs or ys in the equivalents x()y, X)·)y, and x(·(Y.

33. If a proposition containing X and Y be joined with a proposition containing Y and Z, a third proposition containing X and Z may necessarily follow. In this case the two first propositions (*premises*) and the proposition which follows from them (*conclusion*) form a *syllogism.*

34. If an X be a Y, if that same Y be a Z, then the X is the Z. This is the *unit-syllogism* from collections of which all the syllogisms of this mode of treating propositions must be formed. At first sight it seems as if there were another: if an X be a Y, if that same Y be not any Z, then the X is not any Z. But this comes under the first, as follows: the X is a Y, that Y is a z, therefore the X is a z, that is, is not any Z. The introduction of contraries brings all denials under assertions.

35. Two premises have a valid conclusion when, and only when, they necessarily contain unit-syllogisms; and the conclusion has one item of quantity for every unit-syllogism so necessarily contained.

36. And all syllogisms may be derived from the following combinations:—

)))) or X))Y Y))Z, or All Xs are Ys and all Ys are Zs. The conclusion is X))Z, All Xs are Zs: there is the unit-syllogism, This X is a Y, that same Y is a Z, repeated as often as there are Xs in existence in the universe. Or, X))Y))Z gives X))Z, or)))) gives)).

())) or X()Y Y))Z, or Some Xs are Ys, all Ys are Zs. The conclusion is X()Z, Some Xs are Zs: there is the unit-syllogism so often as there are Xs in the first premise. Or, X()Y))Z gives X()Z, or ())) gives ().

((() or X((Y Y()Z, or Some Xs are all Ys, some Ys are Zs. The conclusion is X()Z, Some Xs are Zs: this case is, as to form,

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nothing but the last form inverted. Or, X((Y( )Z gives X( )Z or ((( ) gives ( ).

(( )) or X((Y Y))Z or Some Xs are all Ys, All Ys are Zs. The conclusion is X( )Z, Some Xs are Zs, as many as there are Ys in the universe. Or, X((Y))Z gives X( )Z, or (( )) gives ( ). But this case gives no stronger conclusion than ( ))) or than ((( ), though it has *both* premises universal.

These are all the ways in which affirmative premises produce a conclusion in a manner which has no need to take cognisance of the existence of contrary terms. And since all negations are contained among affirmations about contraries, we may expect that application of these cases to all combinations of direct and contrary will produce all possible valid syllogisms.

37. Apply the form)))) to the eight varieties XYZ, xYZ, xyZ, xyz, xYz, XYz, Xyz, XyZ, and contravert x, y, z, whenever they appear. Thus)))) applied to x y Z is x))y))Z, or X((Y combined with Y(·)Z or X((Y(·)Z or (((·). The conclusion is x))Z or X(·)Z. That is, X((Y(·)Z gives X(·)Z; or, If some Xs be all [make up all the] Ys, and everything be either Y or Z, then everything is either X or Z. This process applied to the eight varieties gives the following eight forms of *universal* syllogism, that is, *universal premises with universal conclusion.*

)))) (·))) (((·) (((((·))·( )))·( )·((( )·((·)

Here are all the ways in which two *universals* give *different* quantities to the middle term.

38. Apply ( ))) to the eight varieties and we have eight *minor-particular* syllogisms, *particular conclusion with the* minor (or first) *premise particular.*

( ))))·))))((·))((( )·))·(( ))·((·((((·((·)

Here are all the ways in which a particular followed by a universal give *different* quantities to the middle term.

39. Apply ((( ), and we have eight *major-particular* syllogisms, *particular conclusion with the* major (or second) *premise particular.*

((( ))·(( ))))·))))( )·((·((((·((·))((·))·)

Here are all the ways in which a universal followed by a particular gives *different* quantities to the middle term.

40. Apply (( )) and we have eight *strengthened particular* syllogisms, *universal premises with particular conclusion.*

(( )))·( )))) (·))) (( )·( )·((( )·((·) (((·) (·)

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Here are all the ways in which two universals give *the same* quantity to the middle term.

41. There are 64 possible combinations, of which the 32 enumerated give inference. The remaining 32 may be found by applying the eight varieties to () ((,)) (), ())(and ())·): and in no case does any inference follow. Thus X()Y and Y((Z are consistent with any of the eight relations between X and Z, which should be ascertained by trial.

42. The test of validity and the rule of inference are as follows,—

There is inference 1. When both the premises are universal; 2. When, one premise only being particular, the middle term has different quantities in the two premises.

The conclusion is found by erasing the middle term and its quantities. Thus)·((·) gives)··) or)) (§ 21). That is 'No X is Y, and Everything is either Y or Z' gives 'Every X is Z'.

43. Premises of *like* quality give an *affirmative* conclusion: of *different* quality, a *negative*. A universal conclusion can only follow from universals with the middle term differently quantified in the two. From two particular premises nothing can follow.

44. A particular premise having the *concluding* term strengthened, the conclusion is also strengthened, and the syllogism is converted into a universal: having the *middle* term strengthened, the conclusion is not strengthened, and the syllogism is converted into a strengthened particular syllogism. Thus if ())), with conclusion (), have the premise () strengthened into)), the syllogism becomes)))) and yields)). But if () be strengthened into ((, the syllogism becomes (( )) and yields only (), as before.

45. A universal conclusion affirms two particulars: if either of these be substituted in the conclusion of the universal syllogism, the syllogism may be called a universal of weakened conclusion or a *weakened universal.* Thus X))Y))Z, made to yield only X()Z or X)(Z, instead of X))Z, is a universal of weakened conclusion. No further notice need be taken of this case.

46. Of the 24 syllogisms of particular conclusion, the conclusions are equally divided among (),)(, .)·), and (·(. The following table is one of many modes of arrangement of the whole.

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Premises	Strengthened Particular	Minor / Major Particular	Universal	Major / Minor Particular	Strengthened Particular
Affirmative	(( ))	())) ((()	)))) ((((	)))( )(((	)) ((
Negative	(·) (·)	(·((·) (·))·)	)·((·) (·))·(	)·((·( )·))·(	)·( )·(
Affirmative Minor	(( )·(	())·((((·(	)))·((((·)	)))·))((·)	)) (·)
Affirmative Major	(·) ((	(·((((·))(	)·((((·)))	)·(())·)))	)·( ))

The middle column contains the universals: and each universal stands horizontally between the two particulars into which it may be weakened, by weakening one of the concluding terms. And each strengthened particular stands vertically between the two particulars from which it may be formed by altering the quantity of the middle term in the particular premise only.

47. If two propositions give a third, say A and B give C; then, a, b, c, meaning the contrary propositions of A, B, C, it follows that A, B, c, cannot all be true together. Hence if A, c, be true, B must be false, or b true: that is, if A, B, give C, then A, c, give b. Or, either premise joined with the contrary of the conclusion, gives the contrary of the other premise. And thus each form of syllogism has two *opponent forms*. But the order of terms will not be correct, unless the premise which is retained be converted. If the order of the terms in the syllogism be XY YZ XZ, we shall have in one opponent XY XZ YZ, which in our mode of arrangement must be YX XZ YZ, the retained premise changing the order of its terms.

Thus the opponent forms of)·))), which gives)·), are as follows. First, (·(, retained premise converted; ((, contrary of conclusion; (·(, contrary of other premise; giving (·(((and conclusion (·(. Secondly, ((, contrary of conclusion; ((, retained premise converted; ((, contrary of other premise; giving ((((and conclusion ((.

48. The universal and particular syllogisms can be grouped by threes, each one of any three having the other two for its

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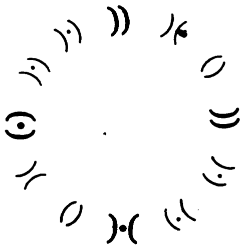
opponents. And these groups can be collected in the following *zodiac*, as it may be called.



The universal propositions at the cardinal points are so placed that any two contiguous give a universal syllogism, whether read forwards or backwards, as)·(((,)))·(. Join each of these universals with its contiguous external particular, so as to read in a contrary direction to that in which the two universals were read, and a triad is formed each member of which has the other two members for its opponent forms. As in

)·((( )·(() ())); or as in (·))))·((((·))·(.

49. The strengthened particulars have weakened universals (§ 45) for their opponent forms. Thus (( )) with the conclusion () has)))·(with the conclusion (·(and)·(((with the conclusion)·), for its opponents. And ((((with the conclusion () has)))·(with the conclusion).) and)·( )) with the conclusion)·) for its opponents.

50. The partial terms of the conclusion take quantity in the following manner,—

In universal syllogisms. If one term of the conclusion be partial, its quantity is that of the other term: if both, one has at least the quantity of the whole middle term, and the other of the whole contrary of the middle term.

51. *In fundamental particular syllogisms.* The partial term or terms of the conclusion take quantity from the particular premise.

52. *In strengthened particular syllogisms.* The partial term or terms take quantity from the whole middle term or its whole contrary, according to which is universal in both of the premises.

53. These rules run through every form of the conclusion in which there is a particular term. Thus X))Y))Z gives

1. X))Z in which Z has the quantity of X
2. x((z in which x has the quantity of z
3. x(·)Z, xs as many as ys, and Zs as many as Ys

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Again, X(·)Y)(Z gives X(·(Z, X()z, and x)·)z, in which the quantities of X(and of)z are the number of instances in the 'some things' of Y)(Z.

Thirdly, X)·(Y)·(Z gives X)(Z, x(·(Z, x()z, and X)·)z, in which the quantities of x(and)z are the number of instances in Y.

54. A *sorites* is a collection of propositions in which the major term of each is the minor term of the next, as in

X))Y)·(Z(·)T))U)·(V((W

or All Xs are Ys, and No Y is Z, and everything is either Z or T, and every T is U, and No U is V, and Some Vs are all Ws.

55. A sorites gives a valid inference, 1. *Universal,* when all the premises are universal, and each intermediate term enters once totally and once partially; 2. *Particular,* when *one* (and one only) of the two conditions just named is broken *once,* whether by contiguous universals having an intermediate of one quantity in both, or by occurrence of *one* particular without breach of the rule of quantity.

56. The inference is obtained by erasing all the intermediate terms and their quantities, and allowing an even number of dots to indicate affirmation, and an odd number of dots to indicate negation.

Thus X))Y)·(Z()T)·(U(·)V))W gives X)·)W
X(·)Y((Z((T(·)U)·(V((W gives X(·(W
X((Y(·)Z)·(T(·)U)·(V gives X((V

57. We have seen that each universal may coexist with either the universal of altered quantities or with its contrary: which is a species of *terminal ambiguity.* Thus X))Y may have either X((Y or X)·)Y true at the same time. All these coexistences may be arranged and symbolised as follows; giving propositions which, with reference to the ambiguity aforesaid, have *terminal precision.*

1. X)°)Y or both X))Y and X)·)Y — All Xs and some things besides are Ys
2. X||Y or both X))Y and X((Y — All Xs are Ys, and all Ys are Xs
3. X(°(Y or both X((Y and X(·(Y — Among Xs are all the Ys and some things besides
4. X)o(Y or both X)·(Y and X)(Y — Nothing both X and Y and some things neither
5. X|·|Y or both X)·(Y and X(·)Y — Nothing both X and Y and every thing one or the other
6. X(o)Y or both X(·)Y and X()Y — Every thing either X or Y and some things both.

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58. If any two be joined, each of which is 1, 3, 4, or 6, with the middle term of different quantities, these premises yield a conclusion of the same kind, obtained by erasing the symbols of the middle term and one of the symbols {∘}. Thus X)∘(Y(∘)Z gives X)∘)Z: or if nothing be both X and Y and some things neither, and if every thing be either Y or Z and some things both, it follows that all Xs and *two lots* of other things are Zs.

59. In any one of these syllogisms, it follows that || may be written for)∘) or (∘(in one place, or |·| for either)∘(or (∘) in one place, without any alteration of the conclusion, except reducing the *two lots* to one. But if this be done in both places, the conclusion is reduced to || or |·|, and *both lots* disappear. Let the reader examine for himself the cases in which one of the premises is cut down to a simple universal.

60. The rules of contraversion remain unaltered: thus X(∘)Y)∘(Z is the same as X(∘(y(∘(Z &c.

61. The following exercises will exemplify what precedes. Letters written under one another are names of the same object. Here is a universe of 12 instances of which 3 are Xs and the remainder Ps; 5 are Ys and the remainder Qs; 7 are Zs and the remainder Rs.

X	X	X	P	P	P	P	P	P	P	P	P
Y	Y	Y	Y	Y	Q	Q	Q	Q	Q	Q	Q
Z	Z	Z	Z	Z	Z	Z	R	R	R	R	R

We can thus verify the eight complex syllogisms

X)°)Y)°)Z   P(°)Y)°)Z   P(°(Q(°)Z   P(°(Q(°(R
P(°)Y)°(R   X)°)Y)°(R   X)°(Q(°(R   X)°(Q(°)Z

In every case it will be seen that the two lots in the middle form the quantity of the particular proposition of the conclusion.

62. The contraries of the complex propositions are as follows:

		Contraries.	
X)°)Y	Both X))Y and X)·)Y	X(·(Y or X((Y or both	X(,(Y
X||Y	—— X))Y — X((Y	X(·(Y - X)·)Y ——	X¦·¦Y
X(°(Y	—— X((Y — X(·(Y	X)·)Y - X))Y ——	X),)Y
X)°(Y	—— X)·(Y — X)(Y	X()Y - X(·)Y ——	X(,)Y
X|·|Y	—— X)·(Y — X(·)Y	X()Y - X)(Y ——	X¦,¦Y
X(°)Y	—— X(·)Y — X()Y	X)(Y - X)·(Y ——	X),(Y

63. The propositions hitherto enunciated are *cumular:* each one is a *collection* of individual propositions, or of propositions about individuals; X))Y is '*All* Xs are some Ys'. This proposition is an aggregate of *singular* propositions.

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64. There is a choice between this *cumular* mode of conception and one which may* be called *exemplar;* in which each proposition is the premise of a *unit-syllogism:* as 'this X is one Y', 'this X is not any Y'. The distinction is seen in '*All* men are animals' and '*Every* man is *an* animal', propositions of the same import, of which the first sums up, the second tells off instance by instance. In the second, *every* is synonymous with *each* and with *any.*

* The late Sir William Hamilton entertained the idea of completing the system of enunciation by making the words *all* (or when grammatically necessary, *any*) and *some* do every kind of duty. He thus put forward, as *the* system, the following collection:—

Affirmative		*Negative*	
1. All X is All Y	——	5. Any X is not any Y	X)·(Y
2. All X is some Y	X))Y	6. Any X is not some Y	X)·)Y
3. Some X is all Y	X((Y	7. Some X is not any Y	X(·(Y
4. Some X is some Y	X()Y	8. Some X is not some Y	——

Of the two propositions which are not in the common system (1 and 8) the first (§ 14, note †) is X||Y, compounded of X))Y and X((Y: it is contradicted by X(·(Y and X)·)Y, either or both. The second (8) is true in all cases in which either X or Y has two or more instances in existence: its contrary is 'X and Y are singular and identical; there is but one X, there is but one Y, and X is Y'. A system of propositions which mixes the simple and the complex, which compounds two of its own set to make a third in one case and one only, § 57, and which offers an assertion and denial which cannot be contradicted *in the system*, seems to me to carry its own condemnation written on its own forehead. From this system I was led to the exemplar system in the text. For Sir W. Hamilton's defence of his own views, and objections to mine, see his *Discussions on Philosophy*, &c. Appendix B. In making this reference, however, it is due to myself to warn the reader who has not access to the paper criticised that Sir W. Hamilton did not read with sufficient attention, partly no doubt from ill health. The consequence is that I must not be held answerable for all that is represented by him as coming from me. For example, speaking of my Table of exemplar propositions, he says "And mark in what terms it [the table of exemplars] is ushered in:—as '*a system* ....' Nay, so lucid does it seem to its inventor, that, after the notation is detailed, we are told that it '*needs no explanation.*'" The paragraph here criticised had *two* notations, one of which I called the *detailed* notation, because there is more *detail* in it than in the other: the other is the *old* notation, augmented. The first had been sufficiently explained in what preceded; the second was, as to the augmentations, new to the reader. Accordingly, the table being finished, I proceeded thus "The detailed notation needs no explanation. The form given to the old notation may be explained thus ....." Sir W. Hamilton represented me as saying that after the notation [all the notation, I suppose] is detailed, it [table or notation, I know not which] needs no explanation. I select this small point

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as one that can be briefly dealt with: there are many more, which I shall probably never notice, unless it be one at a time as occasion of illustration arises. A very decisive case is exposed in the postscript of my third paper in the *Cambridge Transactions.*

65. *Quantity* is now replaced by *mode of selection.* There is *unlimited selection,* expressed by the word *any one: vaguely limited* selection, expressed by *some one.* When we say *some one* we mean that we do not know it may be *any one.*

66. Let (X and X) now mean *any one* X: let)X and X(mean *some one* X.

67. The propositions are as follows: the first of each pair being a universal, the second its contrary particular.

	Exemplar form.	*Cumular form.*
X)(Y	Any one X is any one Y	X and Y singular and identical
X(·)Y	Some one X is not some one Y	Either X not singular, or Y not singular; or if both singular, not identical
X))Y	Any one X is some one Y	All Xs are some Ys
X(·(Y	Some one X is not any one Y	Some Xs are not (all) Ys
X((Y	Some one X is any one Y	Some Xs are all Ys
X)·)Y	Any one X is not some one Y	All Xs are not some Ys
X)·(Y	Any one X is not any one Y	All Xs are not (all) Ys
X()Y	Some one X is some one Y	Some Xs are some Ys.

Six of the forms of this exemplar system are identical with six of the forms of the cumular system. And these six forms are the forms of the old logic, if we take care always to read X((Y and X)·)Y backwards, and to count X)·(Y and X()Y as each a pair of propositions, by distinguishing the reading forwards from the reading backwards.

68. The two new forms of the exemplar system (the first and second above) come under the same symbols as the two new forms of the cumular system, (·) and)(: but the meanings are widely different. Both systems contain every possible combination of quantities, as well in universal as in particular propositions.

69. If the above propositions be applied to contraries, we have a more extensive system of propositions. I shall not enter on this enlargement, because the peculiar proposition of this system, X)(Y, is of infrequent* use in thought as connected with the consideration of X and Y in opposition to their contraries.

* All necessary laws of thought are part of the subject of logic: but a small syllabus cannot contain everything. The rejection *from logic,* and the rejection from a *book of logic,* are two very different things. It has not been

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uncommon to repudiate rare and unusual forms from the science itself, by calling them subtleties, or the like. This (§ 73) is not reasonable: but as to the contents of a work, especially of a syllabus, the time must come at which any one who asks for more must be answered by

Cum tibi sufficiant cyathi, cur dolia quæris?

As another example:—I have, § 16, required that no term shall be introduced which fills the whole universe. In common logic, with an unlimited universe, there is really no name as extensive as the universe except *object of thought*. But it is otherwise in the limited universes which I suppose. A short and easy chapter on names as extensive as the universe might be needed in a full work on logic, but not in a syllabus.

70. To make a syllogism of valid inference, it is enough that there be at least one unlimited selection of the middle term, and at least one affirmative proposition. And the inference is obtained by dropping all the symbols of the middle term. Thus X((Y(·)Z shows premises which give the conclusion X(·)Z: or 'Some one X is any one Y and Some one Y is not some one Z' giving 'Some one X is not some one Z'.

71. There are 36 valid* forms of syllogism, as follows, reading each symbol both backwards and forwards, but not counting it twice when it reads backwards and forwards the same, as in)( )(, (( )).

Fifteen in which)(is joined with itself or another,—

)( )(   )( ))   )(((   )( )·(   )( )·)   )((·(   )(()   )((·)

Fifteen in which the syllogism is but an exemplar reading of a cumular syllogism,—

))))   ()))   (( ))   )))·((( )·(())·(   )))·)   )·)))

Six which give the conclusion (·),—

(((·)   (( )·)   ())·)

* If Sir William Hamilton's system be taken, there are also 36 valid forms of syllogism, the same as in the text: but the law of inference is slightly modified, as follows. When both the middle spiculæ turn one way, as in)) and ((, then any spicula of universal quantity which turns the other way must itself be turned, unless it be protected by a negative point. Thus)((), which in the exemplar system gives)), in the cumular system gives ().

Exemplar system.	Cumular system.
Any one X is any one Y	All Xs are all Ys.
Some one Y is some one Z	Some Ys are some Zs.
Therefore Any one X is some one Z	Therefore *Some* Xs are some Zs.

This distinction will afford useful study. The minor premise of the exemplar instance implies that there is but one X and one Y.

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72. The exemplar proposition is not unknown. It is of very frequent use in complete demonstration. When Euclid proves that Every triangle has angles together equal to two right angles, he selects, or allows his reader to select, a triangle, and shows that *any triangle* has angles equal to two right angles: and the force of demonstration is for those who can see that the selection* is not limited by anything in the reasoning. The exemplar form of enunciation, then, is of at least as frequent use in purely deductive reasoning as any other; and is therefore fitly introduced even into a short syllabus. In any case it is a subject of logical consideration, as being an actual form of thought.

* The limitation of the selection by some detail of process is one of the errors against which the geometer has especially to guard. I remember an asserted trisection of the angle which I examined again and again and again, without being able to detect a single offence against Euclid's conditions. At last, in the details of a very complex construction, I found two requirements which were only possible *together* on the supposition of a certain triangle having its vertex upon the base. Now it happened that one of the angles at the base of this triangle was the very angle to be trisected: so that the author had indeed trisected *an* angle, but not *any* angle; he had most satisfactorily, and by no help but Euclid's geometry, divided the angle 0 into three equal parts, 0, 0, 0. A modification of his process would have been equally successful with 180°, which Euclid himself had trisected.

73. The following passage, written by Sir William Hamilton himself, should be quoted in every logical treatise: for it ought to be said, and cannot be said better. "Whatever is operative in thought, must be taken into account, and consequently be overtly expressible in logic; for logic must be, as to be it professes, an unexclusive reflex of thought, and not merely an arbitrary selection—a series of elegant extracts—out of the forms of thinking. Whether the form that it exhibits as legitimate be stronger or weaker, be more or less frequently applied;—that, as a material and contingent consideration, is beyond its purview."

74. The heads* of the numerically definite proposition and syllogism are as follows,—

Let u be the whole number of individuals in the universe. Let $x, y, z,$ be the numbers of Xs, Ys, and Zs. Then $u-x$, $u-y$, $u-z$ are the numbers of xs, ys, and zs.

* On this subject I have given only heads of result, the demonstrations of which will be found in my *Formal Logic*.

75. Let mXY mean that m or more Xs are Ys. Then mXy means that m or more Xs are ys, or not Ys. And mYX and myX have the same meanings as mXY and mXy.

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76. Let a proposition be called *spurious* when it must of necessity be true, by the constitution of the universe. Thus, in a universe of 100 instances, of which 70 are Xs and 50 are Ys, the proposition 20 XY is spurious: for at least 20 Xs must be Ys, and 20 XY cannot be denied, and need not be affirmed as that which might be denied.

77. Let every negative quantity be interpreted as 0: thus (6-10) XY means that *none or more* Xs are Ys, a spurious proposition.

78. The quantification of the predicate is useless. To say that mXs are to be found among nYs, is no more than is said in mXY. To say that mXs are not any one to be found among any lot of nYs is a spurious proposition, unless $m+n$ be greater than both x and y, in which case it is merely equivalent to both of the following, $(m+n-y)$ Xy, and $(m+n-x)$ Yx, which are equivalent to each other.

79. In mXY, the spurious part, if any, is $(x+y-u)$ XY; the part which is not spurious is $(m+u-x-y)$ XY. For each instance in the last there must be an x which is y. The following pairs of propositions are identical.

m XY and $(m+u-x-y)$ xy
m Xy and $(m+y-x)$ xY
m xY and $(m+x-y)$ Xy
m xy and $(m+x+y-u)$ XY

80.

Identical propositions.		Their contraries.	
m XY	$(m+u-x-y)$ xy	$(x+1-m)$ Xy	$(y+1-m)$ xY
m Xy	$(m+y-x)$ xY	$(x+1-m)$ XY	$(u+1-y-m)$ xy
m xY	$(m+x-y)$ Xy	$(u+1-x-m)$ xy	$(y+1-m)$ XY
m xy	$(m+x+y-u)$ XY	$(u+1-x-m)$ xY	$(u+1-y-m)$Xy

81. From mXY and nYZ we infer $(m+n-y)$ XZ, or its equivalent $(m+n+u-x-y-z)$ xz. The four following forms include all the cases of syllogism: the first two columns show the premises, the second two the identical conclusions,—

m XY	n YZ	$(m+n-y)$ XZ	$(m+n+u-x-y-z)$ xz
m Xy	n YZ	$(m+n-x)$ xZ	$(m+n-z)$ Xz
m XY	n yZ	$(m+n-z)$ Xz	$(m+n-x)$ xZ
m Xy	n yZ	$(m+n+y-x-z)$ xz	$(m+n+y-u)$XZ

82. When either of the concluding terms is changed into its contrary, the corresponding changes are made in the forms of inference. Thus to find the inference from m xy and n yz, we

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must, in the fourth form, write x for X, z for Z, X for x, Z for z, $u-x$ for x, and $u-z$ for z.

83. A spurious premise gives a spurious conclusion: and premises neither of which is spurious may give a spurious conclusion. A proposition is only spurious as it is *known* to be spurious: hence when u, x, y, z are not known, there are no spurious propositions.

84. Every proposition has two forms, one of names contrary to the other, both spurious, or neither. Whenever X( )Y is true in a manner which, by the constitution of the universe, might have been false, then x( )y, or X)(Y is also true in the same manner. The ordinary syllogism would have two such *contranominal* forms of one conclusion, and, properly speaking, *has* two such forms. When the conclusion is universal, we *know* it has them: for X))Z is x((z, X)·(Z is x(·)z, &c. These we may see to be the contranominal conclusions of the numerical syllogism. For X))Y is xXY, and Y))Z is yYZ, whence $(x+y-y)$XZ and $(x+y-y+u-x-z)$xz, or xXZ, which is X))Z, and $(u-z)$ xz, which is x((z. Again, let X( )Y be mXY, then, Y))Z being yYZ, we have $(m+y-y)$XZ, or mXZ, and $(m+u-x-z)$ xz, its equivalent. If x, z, u, be known, then if mXZ be any thing except what must be, we have $m+u>x+z$, and $(m+u-x-z)$ xz is x( )z or X)(Z. As it is, x, y, u, being unknown, we have mXZ certainly true, be it spurious or not, and we can say nothing of $(m+u-x-z)$ xz.

85. Syllogisms with numerically definite quantity rarely occur, if ever, in common thought. But syllogisms of *transposed* quantity occur, in which the number of instances of one term is the *whole* possible number of instances of another term. For example; —'For every Z there is an X which is Y; some Zs are not Ys'. Here we have zXY and nyZ; whence $(z+n-z)$Xz and $(z+n-x)$xZ. The first is nXz, a case of X(·(Z; some Xs are not Zs. Thus, 'For every man in the house there is a person who is aged; some of the men are not aged': it follows, and easily, that some persons in the house are not men; but not by any *common* form of syllogism.

86. Of terms in common use the only one which can give the syllogisms of this chapter is 'most'. As in

Most Ys are Xs; most Ys are Zs; therefore some Xs are Zs.

Most Ys are Xs; most Ys are not Zs; therefore some Xs are not Zs.

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Most Ys are not Xs; most Ys are not Zs; therefore some things are neither Xs nor Zs.

87. Each one of our syllogisms may be stated in eight different ways, each premise and the conclusion admitting two different orders. Thus X))Y, Y))Z, giving X))Z may be stated as Y((X Y))Z giving Z((X, or as X))Y, Y))Z, giving Z((X, &c. All the orders are as follows—

I.	II.	III.	IV.
XY YZ XZ	XY ZY XZ	YX YZ XZ	YX ZY XZ
YX ZY ZX	XY ZY ZX	YX YZ ZX	XY YZ ZX

88. Whenever there is a first and a second, let them be called *minor* and *major*. Write the premises so that the *minor* premise shall contain the *minor* term of the conclusion (though it has long been most common to write the major premise first), and we have

I.	II.	III.	IV.
XY YZ XZ	XY ZY XZ	YX YZ XZ	YX ZY XZ

These orders are called the four *figures*. Thus X))Y Y))Z giving X))Z is stated in the first figure; X))Y Z((Y giving X))Z is stated in the second figure; Y((X Y))Z giving X))Z is stated in the third figure; Y((X Z((Y giving X))Z is stated in the fourth figure.

89. The first figure may be called the figure of *direct transition:* the fourth, which is nothing but the first with a *converted conclusion*, the figure of *inverted transition;* the second, the figure of *reference to* (the middle term); the third, the figure of *reference from* (the middle term).

90. The first figure is the one which has been used in our symbols; and it is the most convenient. The distinction of figure is wholly useless in this tract, so far as we have yet gone: it becomes necessary when we take a wider view of the copula.

91. A *convertible* copula is one in which the copular relation exists between two names *both ways:* thus 'is fastened to' 'is joined by a road with' 'is equal to,' 'is in habit of conversation with,' &c. are *convertible* copulæ. If 'X is equal to Y' then 'Y is equal to X' &c.

92. A *transitive** copula is one in which the copular relation joins X with Z whenever it joins X with Y and Y with Z. Thus 'is fastened to' is usually understood as a transitive copula: 'X

is fastened to Y' and 'Y is fastened to Z' give 'X is fastened to Z'.

* All the copulæ used in this syllabus are *transitive*. The intransitive copula cannot be treated without more extensive consideration of the combination of relations than I have now opportunity to give: a second part of this syllabus, or an augmented edition, may contain something on this subject.

93. The junction of names by appurtenance to one object, the copula hitherto used, is both convertible and transitive: and from these qualities, *and from these alone*, it derives the whole of its functional power in syllogism. Any copula which is both transitive and convertible will give precisely the syllogisms* of our system, and no others: provided always that if contrary names be introduced, no instance of a name can, either directly or by transition, be joined by the copula with any instance of the contrary name. For example, let the copula be some transitive and convertible mode of joining or fastening together, whether of objects in space or notions in the mind, &c.: so that no X is ever joined with any x, &c. The following are two instances of syllogism.

X))Y)·(Z. Every X is joined to a Y; no Y is joined to a Z; therefore no X is joined to a Z. For if any X were joined to a Z, that Z would be joined to an X, and that X to a Y, whence that Z would be joined to a Y, which no Z is.

X(·)Y)(Z. Everything is joined either to an X or to a Y; Some things are joined neither to Ys nor to Zs; therefore Some Xs are not joined to Zs. For if every X were joined to a Z, then every thing being (by the first premise) joined either to an X or to a Y, is joined either to a Z or to a Y, which contradicts the second premise.

* The logicians are aware that many cases exist in which inference about two terms by comparison with a third is not reducible to *their* syllogism. As 'A equals B; B equals C; therefore A equals C.' This is not an instance of common syllogism: the premises are 'A *is* an equal of B; B *is* an equal of C.' So far as common syllogism is concerned, that 'an equal of B' is as good for the argument as 'B' is a *material* accident of the meaning of 'equal.' The logicians accordingly, to reduce this to a common syllogism, state the effect of composition of relation in a major premise, and declare that the case before them is an example of that composition in a minor premise. As in, A is *an equal of an equal* (of C): Every *equal of an equal* is an *equal*; therefore A is an *equal* of C. This I treat as a mere evasion. Among various sufficient answers this one is enough: *men do not think as above*. When A = B, B = C, is made to give A = C, the word *equals* is a

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copula in thought, and not a *notion attached to a predicate.* There are processes which are not those of common syllogism in the logician's major premise above: but waiving this, logic is an analysis of the form of thought, possible and actual, and the logician has no right to declare that other than the actual is actual.

94. The convertibility of the copula renders the inference altogether independent of figure.

95. Let the copula be *inconvertible,* as in 'X precedes Y' from which we cannot say that 'Y precedes X'. We must now introduce the *converse relation* 'Y follows X', and the conversion of a proposition requires the introduction of the *converse* copula.

96. This extension, *when contraries are also introduced,* is almost unknown in the common run of thought: but it may serve for exercise, and also to give an idea of one of those innumerable systems of relation with which thought unassisted by systematic* analysis would probably never become familiar.

* The uneducated acquire easy and accurate use of the very simplest cases of transformation of propositions and of syllogism. The educated, by a higher kind of practice, arrive at equally easy and accurate use of some more complicated cases: but not of all those which are treated in ordinary logic. Euclid may have been ignorant of the identity of "Every X is Y" and "Every not-Y is not-X," for any thing that appears in his writings: he makes the one follow from the other by new proof each time. The followers of Aristotle worked Aristotle's syllogism into the habits of the educated world, giving, not indeed anything that demonstrably *could not* have been acquired without system, but much that very probably *would not.* The modern logician appeals to the existing state of thought in proof of the completeness of the ordinary system: he cannot see anything in an extension except what he calls a *subtlety.* In the same manner a country whose school of arithmetical teachers had never got beyond counting with pebbles would be able to bring powerful arguments against pen, ink, and paper, the Arabic numerals, and the decimal system. They would point to society at large getting on well enough with pebbles, and able to do all their work with such means: for it is an ascertained fact that all which is done by those to whom pebbles are the highest resource, is done either with pebbles or something inferior. I have long been of opinion that the reason why common logic is lightly thought of by the mass of the educated world is that the educated world has, in a rough way, arrived at some use of those higher developments of thought which that same common logic has never taken into its compass. Kant said that the study of a legitimate subtlety (necessary but infrequent law of thought) sharpens the intellect, but is of no practical use. Sharpen the intellect with it until it is familiar, and it will then become of practical use. A law of thought, a necessary part of the machinery of our minds, of *no practical use!* Whose fault is that?

97. Let any two names be connected by transitive converse relations, for an example say *gives to* and *receives from* (under-

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standing that when X gives to Y and Y gives to Z, X gives to Z) in the following way,—

> No X gives to another X, either directly or transitively, &c.
> Every X either gives to a Y or receives from a y, but not both
> Every x either gives to a Y or receives from a y, but not both
> Every X which gives to a Y, receives from *no other* Y, &c.

The same of all combinations of names, as Y with X and x, &c.

98. The following are the propositions used, with their symbols; and in a corresponding way for any other copula which may be used,—

X)′)Y	Every X gives to a Y	X))′Y	Every X receives from a Y
X(′·(Y	Some Xs give to no Ys	X(·(′Y	Some Xs receive from no Ys
X)′·(Y	No X gives to a Y	X)·(′Y	No X receives from a Y
X(′)Y	Some Xs give to Ys	X()′Y	Some Xs receive from Ys
X(′·)Y	In every relation, something either gives to an X or receives from a Y (or both)	X(·)′Y	In every relation, something either receives from an X or gives to a Y (or both)
X)′(Y	In some relations, nothing gives to any X nor receives from any Y	X)(′Y	In some relations, nothing receives from any X nor gives to any Y
X(′(Y	Some Xs give to all the Ys	X((′Y	Some Xs receive from all Ys
X)′·)Y	All Xs do not give to some Ys	X)·)′Y	All Xs do not receive from some Ys

99. Propositions are changed into others identical with them by this addition to the rule in § 26:—When one term is contraverted, the relation is also converted: when both, the relation remains. In the following lists the four in each line are identical,—

X)′)Y	X)·(′y	x(·)′Y	x(′(y	X(′·(Y	X()′y	x)(′Y	x)′·)y
X)′·(Y	X))′y	x((′Y	x(′·)y	X(′)Y	X(·(′y	x)·)′Y	x)′(y
X(′·)Y	X((′y	x))′Y	x)′·(y	X)′(Y	X)·)′y	x(·(′Y	x(′)y
X(′(Y	X(·)′y	x)·(′Y	x)′)y	X)′·)Y	X)(′y	x()′Y	x(′·(y

The relations may be converted throughout.

100. To prove an instance, how do we know that X)′)Y is identical with x(·)′Y? If every X give to a Y, the remaining Ys, if any, do not give to any Xs, by the assigned conditions of meaning: consequently those remaining Ys receive from xs. As to ys, none of them can give to Xs, for then they would give to Ys: therefore all receive from xs. Conversely, if x(·)′Y, no X can receive from y, for then neither could that y receive from x, nor could that X give to Y: so that there would be a relation in which neither does any thing give to Y, nor receive from x. Consequently, every X gives to Y.

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101. Let the *phases* of a figure depend on the quality of the premises in the following manner: + meaning *affirmative*, and — *negative*, remember the phases in the following order,—

+ +   — +   + —   — —

102. For the four figures, let these four phases be the *first* or *primary* phases: thus + — is the primary phase of the third figure. To put the other phases in order, read backwards from the primary phases, and then forwards.

	1	2	3	4
Figure I.	+ +	— +	+ —	— —
—— II.	— +	+ +	+ —	— —
—— III.	+ —	— +	+ +	— —
—— IV.	— —	+ —	— +	+ +

Thus + — is the third phase of the second figure.

103. In the primary phases, the direct copula may be used throughout. When *one* premise departs from the primary phase in quality, the converse copula must be used in *the other;* when *both,* in the *conclusion.* This addition is all that is required in the treatment of the syllogism of inconvertible copula.

104. Thus,* the premises being X)·(Y Z))Y, we have the primary phase of the second figure, whence X)·(Z with the direct copula. That is, if no X give to a Y, and every Z give to a Y, no X gives to a Z. For if any X gave to a Z, that Z giving to a Y, that X would give to a Y, which no X does. Now contravert the middle term, and we have X))y Z)·(y, the phase of the second figure in which both premises differ from the primary phase. Hence Every X gives y, No Z gives y, yields no X *is given by* Z. For if any X were given by Z, a y would be given by that Z, which is given by no Z. But 'no X *gives* Z' will not do.

* The reader may exercise himself in the formation of more examples. The use of such a developement as the one before him is this. Every study of a generalisation or extension gives additional power over the particular form by which the generalisation is suggested. Nobody who has ever returned to quadratic equations after the study of equations of all degrees, or who has done the like, will deny my assertion that οὐ βλέπει βλέπων may be predicated of any one who studies a branch or a case, without afterwards making it part of a larger whole. Accordingly, it is always worth while to generalise, were it only to give power over the *particular.* This principle, of daily familiarity to the mathematician, is almost unknown to the logician.

105. The common system of syllogism, which being nearly complete in the writings of Aristotle may be called Aristotelian,

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is as much as may be collected out of the preceding system by the following modifications,—

1. The exclusion of all idea of a limited universe, of contrary names, and of the propositions (·) and)(. 2. The exclusion of all right to convert a proposition, except when its two terms have like quantities, as in)·(and (). Thus X))Y must not be read as 'Some Ys are all Xs'. But X))Y may undergo what is called the conversion *per accidens:* that is, X))Y affirming X()Y, which is Y()X, X))Y may be made to give Y()X. 3. The exclusion of every copula except the transitive and convertible copula. 4. The addition of the consideration of the identical pairs X)·(Y and Y)·(X, X()Y and Y()X, as perfectly distinct propositions. 5. The introduction of the distinction of figure. 6. The writing of the major and minor propositions first and second, instead of second and first: thus X))Y))Z is written 'Y))Z, X))Y, whence X))Z'.

106. There are four forms of proposition: A, or X))Y or Y))X, not identical; E, or X)·(Y or Y)·(X, identical; I, or X()Y or Y()X, identical; O, or X(·(Y or Y(·(X, not identical.

107. There are four fundamental syllogisms in the first figure, each of which has an opponent in the second, and an opponent in the third. There are three fundamental syllogisms in the fourth figure, each of which has the other two for opponents. Altogether, fifteen *fundamental* syllogisms. There are three strengthened particular syllogisms, two in the third figure, and one in the fourth: and one weakened universal, in the fourth figure. In all, nineteen forms.

108. Every syllogism has a *word* attached to it, the vowels of which are those of its premises and conclusion. In the first figure the consonants are all unmeaning; in the other figures some of the consonants give direction as to the manner of converting into the first figure. Thus K denotes that the syllogism cannot be directly converted into the first figure, though its *opponent* in the first figure may be used to force its conclusion. S means that the premise whose vowel precedes is to be simply converted. P, which occurs in all the strengthened particulars and the weakened universal, means that the conversion *per accidens* is to be employed on the preceding member. M means that the premises must be transposed in order. Each syllogism converts into that syllogism of the first figure which has the same initial letter. *G* is an addition of my own, presently described; it must be left out when

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the old system is to be just represented. R, N, T, have no signification. The following are the names put together in memorial* verse.

* The best attainable exposition of logic in the older form, with modern criticism, is Mr. Mansel's edition of Aldrich's compendium. Should a reader of this work desire more copious specimens of old discussion, he may perhaps succeed in obtaining Crackanthorpe's *Logicæ Libri quinque* (4to, 1622 and 1677). Sanderson's Logic is highly scholastic in character. For a compendium of mediæval logic, ethics, physics, and metaphysics, I have never found anything combining brevity and completeness at all to compare with the *Precepta Doctrinæ Logicæ, Ethicæ*, &c. of John Stierius, of which seven or eight editions were published in the seventeenth century (from 1630 to 1689, or thereabouts) and several of them in London. There is a large system of the older logic in the *Institutiones Logicæ* of Burgersdicius, and a great quantity of the metaphysical discussion connected with the old logic in Brerewood *de Predicabilibus et Predicamentis*. As all these books were printed in England, there is more chance of getting them than the foreign logical works, which are very scarce in this country. For more than usual information on parts of the history of logical *quantity*, a subject now exciting much attention, see Mr. Baynes's *New Anulytic*, in which will be found much valuable history so completely forgotten that it is as new as if he had invented it himself.

109. Barbara, Cela*g*rent, Darii, Feri*g*oque prioris:
Cesare*g*, Camestres, Festino*g*, Baroko secundæ:
Tertia Darapt*g*i, Disa*g*mis, Dati*g*si, Felapton,
Bokardo, Ferison habet. Quarta insuper addit
Bramanti*g*p, Came*g*nes, Dimari*g*s, Fe*g*sapo, Fre*g*sison.

110. I leave the verification of what has been said as an exercise. As an example of reduction into the first figure take the syllogism *Camestres* from the second figure.

	II. *Camestres*		reduced into	I. *Celarent.*	
A	Every Z is Y		(m)	No Y is X	E
E	No X is Y	(s)		Every Z is Y	A
E	No X is Z	(s)		No Z is X	E

111. The letter G indicates (§ 103) the member in which, when a transitive but not simply convertible copula is used, the copula is to be the converse of the copula employed in the other members. Thus Cela*g*rent shows that the minor premise must have the converse copula. Suppose, for example, that the copulæ are *gives to* (transitively understood) and *receives from*. Then 'No Y gives to an X; every Z receives from a Y' yield 'No Z gives to an X'. For if any X received from a Z, which (second

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premise) receives from a Y, that X would receive from a Y, which contradicts the first premise.

112. In the preceding articles I have considered hardly anything but mere assertion or denial of *concomitance*, of any sort or kind whatsoever. I now proceed to more specially subjective views of logic.

113. A *term* or *name* may be in one word or in many. It describes, pictures, represents, but does not assert nor deny. Its object must exist, whether in thought only, or in external nature as well: and everything which does not contradict the laws of thought may be the object of a term. But sometimes the thinker's universe will be the whole universe of thought; sometimes only the objective universe of external reality; sometimes only a part of one or the other.

114. Terms are used in four different senses. Two *objective*, directed towards the *external* object, or to use old phrases, of *first intention*, or representing *first notions*. Two *subjective*, directed towards the *internal* mind, of *second intention*, or representing *second notions*.

115. In objective use the name represents, 1. The *individual object*, unconnected with, and unaggregated with, any other object of the same name; 2. The *individual quality*, forming part of, and residing in, the individual object. One name may, at different times, represent both: thus *animal*, the name of an object, is the name of a quality of *man*. In fact, quality is but object considered as *component* of another object. The quality *white*, a component of the notion of an ivory ball, is itself an object of thought.

116. The objective* uses of names have been considered as the bases of propositions and syllogisms, in the preceding part of this tract.

* The ordinary syllogism of the logicians, literally taken as laid down by them, is objective, of first intention, arithmetical. I call it the *logician's abacus*. When the educated man rejects its use, and laughs at the idea of introducing such *learned logic* into his daily life, I hold his refusal to be in most cases right, and his reason to be entirely wrong. He has, and his peers always have had, some command of the *subjective* syllogism, the combination of relations to which I shall come. He has no more occasion—in most cases—to have recourse to the logical abacus for his reasoning, than to the chequer-board, or arithmetical abacus, for adding up his bills. I hold the combination of relations to be the actual organ of reasoning of the world at large, and, as such, worthy of having its analysis made a part of advanced

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education; the logician's abacus being a fit and desirable occupation for childhood.

117. In subjective use the name represents, 1. A *class*, a collection of individual objects, named after a quality which is in thought as being in each one: 2. An *attribute*, the notion of quality as it exists in the mind to be given to a class. Attribute is to individual quality what class is to individual object. Between the notion of a class, and the notion of an object, though the name be the same in both cases, there is this distinction. The class-name belongs to a number of objects: the object belongs to a number of class-names; for it may be named after any one of its qualities. We have classes aggregated *of many objects*: and objects compounded *of many classes*. But in this second case the object is said to have many *qualities*. The class is a whole of one kind: the object is a whole of another kind. This distinction emerges the moment a name begins to be a *universal*, that is, belonging to more than one object.

118. *Class* and *attribute* are units of thought: a noun of multitude is not a multitude of nouns. When we are fortunate enough to get four distinct names, we readily apprehend all these distinctions. This happens in the case of our own species: the objects *men*, all having the quality *human*, give to the thought the class *mankind*, distinguished from other classes by the attribute *humanity*. Should any reader object to my account of the four uses of a name, he can, without rejection of anything else in what follows, substitute his own account of the four words *man*, *human*, *mankind*, *humanity*.

119. With grounds of classification, and reasons for nomenclature, logic has nothing to do. Any number of individuals, whether yet unclassed, or included in one class, or partly in one class and partly in another, may be constituted a new class, in right of any quality seen in all, by which an attribute is affixed to the class in the mind.

120. The term X, or Y, or Z may and does denote, at one time or another, the individual, the quality, the class, or the attribute. Any one who finds the distinction useful might think of the individual X, the quality X-ic, the class X-kind, and the attribute X-ity.

121. Identical terms are those which apply to precisely the same objects of thought, neither representing more than the other:

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so that identical terms are different names of the same class. Thus, for this earth, *man* and *rational animal* are identical terms. The symbol X||Y will be used (§ 57) to represent that X and Y are identical. When of two identical terms one, the known, is used to explain the other, the unknown, the first is called the *definition* of the second.

122. The whole extent of matter of thought under consideration I call the *universe*. In common logic, hitherto, the universe has always been the whole universe of possible thought.

123. Every term which is used (§ 16) divides the *universe* into two classes; one within the term, the other without. These I call *contraries*: and I denote the contrary class of X, the class not-X, by x. When the universe is unlimited contrary names are of little effective use; not-man, a class containing every thing except man, whether seen or thought, is almost useless. It is otherwise in a limited universe, in which contraries, by separate definiteness of meaning, cease to be mere negations each of the other, and even acquire separate* and positive names. Thus, the universe being property under English law, *real* and *personal* are contrary classes. Logic has nothing to do with the difficulties of allotment which take place near the boundary: with the decision upon those personals, for example, which, as the lawyer says, *savour of the realty*. The lawyer must determine the classification, and logic investigates the laws of thought which then apply. If the lawyer choose to make an intermediate class, between real and personal, then real and personal are no longer logical contraries.

That X and Y are contrary classes is denoted (§ 57) by X|·|Y.

* The most amusing instance which ever came within my own knowledge is as follows. A friend of mine, in the days of the Irish Church Bill, used to discuss politics with his butcher: one day he alluded to the possible fate of the Establishment. 'Do you mean do away with the church'? asked the butcher. 'Yes', said my friend, 'that is what they say'. 'Why, sir, how can that be'? was the answer; 'don't you see, sir, that if they destroy the church, we shall all have to be *dissenters*'!

124. Terms may be formed* from other terms,—

1. By *aggregation*, when the complex term stands for everything to which *any one or more* of the simple terms applies. Thus *animal* is the *aggregate* of (the *aggregants*) *man* and *brute*.

2. By *composition*, when the complex term stands but for

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everything to which *all* the simple terms apply. Thus *man* is *compounded* of (the *components*) *animal* and *rational*.

3. By mixture of these two methods of formation.

* The reader must carefully remember that we are now engaged (§ 4) especially upon the *esse quod habent in anima:* and, if not accustomed to middle Latin, he must remember also that *esse* is made a substantive, *mode of being*. *Animal* cannot be divided into *man* and *brute* except *in a mind.* Logical composition must be distinguished from physical or metaphysical. Light always consisted of the prismatic components; but, before Newton, it was not a logical quality of light that it is to be conceived as decomposible. Accordingly, a compound of qualities, though it may constitute a full distinctive *definition* of an object of thought, can never be accepted as a full *description:* there may be many more. The logician therefore must, in thinking of a compound, imitate the genial Dean Aldrich, the author of the *Compendium* of Logic to which so many have been indebted, in the structure of his fifth reason for drinking.

125. The aggregate of X, Y, Z will be represented by (X, Y, Z): the term compounded of X, Y, Z will be represented by (X-Y-Z) or (XYZ).

126. The aggregate name belongs to each of the aggregants: but the compound name does not belong to each of the components, necessarily.

127. An aggregate is not impossible if either of its aggregants be impossible, or if two of them be contradictory: but a compound is impossible in either of these cases.

128. In these and all other formulæ, care must be taken to remember that *the logical phrase implies nothing:* the phrases of ordinary conversation frequently *imply*, in addition to what they *express*. Thus 'some living men breathe' and 'every man is either animal or mineral' are colloquially false by what they imply, but logically true because the logical use implies nothing.

129. A class may be compounded of classes, as well as aggregated: thus the class *marine* is compounded of the classes *soldier* and *sailor*. An attribute may be aggregated of attributes, as well as compounded: thus Adam Smith's attribute *productive* is aggregated of land-tilling, manufacturing, &c. But composition of classes and aggregation of attributes are infrequent. Any name may be thought of either as a class, or an attribute (or *character*, as it is often called): and it is usual and convenient to think of class when aggregation is in question, and of attribute when composition is in question. So we rather say the *marine* unites the *characters* of the *soldier* and the *sailor:* the *productive* classes (as

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Adam Smith said) consist of farmers, manufacturers, &c. But all this for convenience, not of necessity: and the power of unlearning usual habits must be acquired. All modes of thought should be considered: the usual, because they are usual; the unusual, that they may become usual.

130. The words of aggregation are *either, or*: of composition, *both, and*. Thus (X, Y) is either X or Y (or both); (XY) is both X and Y.

131. The more classes aggregated, so long as each class has something not contained in any of the others, the greater the *extension** of the aggregate term. The more attributes compounded, so long as each attribute has some component not contained in the others, the greater the *intension*. *Animal* has more extension than *man*: *man* has more intension than *animal*.

* The logicians have always spoken of 'all men' as constituting the 'extent' of the term *man*: thus the whole extent of *man* is part of the extent of *animal*. They have chosen that their more and less should be referred to by phrases derived rather from the notion of area than from that of number. Hence arise certain forms of speech which, when quantity is applied to the predicate, are not idiomatic; as 'All man is some animal'.

There are savage tribes which have not sufficient idea of number even for their own purposes: among them, when a dozen or more of men are to be indicated, an *area* sufficient to contain them is marked out on the ground. The speculative philosophers of the middle ages were in something like the same position: though the mercantile world was well accustomed to large numbers, the philosophical world, excepting only *some* of the mathematicians, was very awkward at high numeration. The works on theoretical arithmetic show this well. I have been straining my eye over the twelve books of the *Arithmetica Speculativa* of Gaspar Lax of Arragon (Paris, folio, 1515) to detect, if I could, a number higher than a hundred; and I have found only one, *the date*.

132. The name of greatest extension, and of least intension, of which we speak, is the universe.

133. The contrary of an aggregate is the compound of the contraries of the aggregants: either one of the two X, Y, or both not-X and not-Y; either (X, Y) or (xy). The contrary of a compound is the aggregate of the contraries of the components; either both X and Y, or one of the two, not-X and not-Y; either (XY), or (x, y).

134. The following are exercises on complex terms,—

'Both or neither' and 'one or the other, not both', are contraries. That is (XY, xy) and (Xy, Yx) are contraries. Now

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the contrary of the first is (x, y)-(X, Y), which is (xX, xY, yX, yY), which is (xY, yX) since xX, yY, are impossible.

X||(A,B) C	gives	x||(ab, c)	X||A, B (C, D)	gives	x||a (b, cd)
X||(A,B) (C,D)	—	x||(ab,cd)	X||(A, BC) (D, EF)	—	x||(a b, c),(d e,f)
X||AB,C	—	x||(a, b) c	X||(A, B, aC)	—	x||abc

Deduce these, and explain the last.

135. A term given in extension, as (A, B, C), has its contrary given in intension, (abc); and *vice versâ*. Aggregates or components of either only enable us to deduce components or aggregates of the other.

136. A *proposition* is the presentation, for assertion or denial, of two names connected by a relation: as 'X in the relation L to Y.' A *judgment* is the sentence of the mind upon a proposition: certainly true, more or less probable, certainly false. Propositions without accompanying judgment hardly occur: so that *proposition* comes to mean, by abbreviation, *proposition accompanied by judgment*.

137. The distinction between certainty and probability is usually treated apart from logic, as a branch of mathematics. A few of the leading results, relative to authority and argument, will be afterwards given.

138. The purely formal proposition with judgment, wholly void of matter, is seen in 'There is the probability α that X is in the relation L to Y'. From the purely formal proposition no inference can follow. In all elementary logic, the terms are formal, the relation* material, and the judgment absolute assertion or denial (or, as a mathematician would say, the probabilities considered are only 1 and 0).

* The logician calls 'Every man is animal' a material instance of the formal proposition 'Every X is Y'. He will admit no relation to be *formal* except what can be expressed by the word *is*: he declares all other relations *material*. Thus he will not consider 'X equals Y' under any form except 'X *is* an equal of Y'. He has a right to confine himself to any part he pleases: but he has no right, except the right of fallacy, to call that part the whole.

139. *Contrary* propositions are a pair of which one must be true and one false: as 'he did', 'he did not'; or as 'Every X is Y', 'Some Xs are not Ys'. Contraries contradict* one another; but so do other propositions. Thus 'All men are strong' and 'all men are weak' contradict one another to the utmost: the second says there is not a particle of truth in the first. But the *contrary* merely says there is more or less falsehood: to 'all men

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are strong' the contrary is 'There are [man or] men who are not strong'.

* In the usual nomenclature of logicians, what I call the contrary is called *the* contradictory, as if it were the only one. In common language, when two persons disagree, we say they are on *contrary* sides of the question: in the usual technical language of logic, this would mean that if one should say all men are strong the other says no man is strong. But in common language, the one who *maintains the contrary* is he who advocates *anything* which the other is opposed to.

140. Every proposition has its *contrary:* there is no assertion but has its denial; no denial but has its assertion. Every logical scheme of propositions must contain a denial for every assertion, and an assertion for every denial.

141. *Inference* is the production of one proposition as the necessary consequence of one or more other propositions. Inference from one proposition may be either an *equivalent* or *identical* proposition, or an *inclusion.* If from a first proposition we can infer a second, and if from the second proposition we can also infer the first, the two propositions are *logical equivalents.* Thus 'X is the parent of Y' and 'Y is the child of X' are logical equivalents: And also 'Every X is Y' and 'Every not-Y is not-X'. But from 'Every X is Y' we can infer 'Some Xs are Ys', without being able to infer the first from the second: the second is only included in the first.

142. When inference is made from more than one proposition, the result is called a *conclusion,* and its antecedents *premises.*

143. Inference has nothing to do with the truth or falsehood of the antecedents, but only with the necessity of the consequence. When the inference from the antecedents is preceded by showing of their *truth,* the whole is called *proof* or *demonstration.*

144. *Deduction,* or *à priori* proof, is when the *compound* of the premises gives the conclusion. One false premise, and deduction wholly fails.

145. *Induction,* or *à posteriori* proof, is when the *aggregate* of the premises gives the conclusion. One false premise, and the induction partially fails.

146. *Absolute* or *mathematical* proof is when the conclusion is so established that any contradiction would be a contradiction of a *necessity** of thought.

* Logic considers the *laws of action* of thought: mathematics applies these *laws* of thought to necessary *matter* of thought. That two straight lines cannot inclose a space is a necessary way of thinking, a proposition to

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which we must assent: but it is not a law of action of thought. That if two straight lines cannot inclose a space, it follows that two lines which do inclose a space are not both straight, is an example of a rule by which thought in action must be guided.

Mathematics are concerned with *necessary matter of thought.* Let the mind conceive every thing annihilated which it can conceive annihilated, and there will remain an infinite universe of space lasting through an eternity of duration: and space and time are the fundamental ideas of mathematics. Of course then the logicians, the students of the necessary *action* of thought, are in close intellectual amity with the mathematicians, the students of the necessary *matter* of thought. It may be so: but if so, they dissemble their love by kicking each other down stairs. In very great part, the followers of either study despise the other. The logicians are wise above mathematics; the mathematicians are wise above logic: of course with casual exceptions. Each party denies to the other the power of being useful in education: at least each party affirms its own study to be a sufficient substitute for the other. Posterity will look on these purblind conclusions with the smile of the educated landholder of our day, when he reads Squire Western's fears lest the sinking fund should be sent to Hanover to corrupt the English nation. A generation will arise in which the leaders of education will know the value of logic, the value of mathematics, the value of logic in mathematics, and the value of mathematics in logic. For the mind, as for the body, Βίον πορίζου πάντοθεν πλὴν ἐκ κακῶν.

This antipathy of necessary law and necessary matter is modern. Very many of the most illustrious names in the history of logic are the names of known mathematicians, especially those of the founders of systems, and the communicators from one language or nation to another. As Aristotle, Plato, Averroes (by report), Boethius, Albertus Magnus (by report), Ramus, Melancthon, Hobbes, Descartes, Leibnitz, Wolff, Kant, &c. Locke was a competent mathematician: Bacon was deficient, for the consequences of which see a review of the recent edition of his work in the *Athenæum* for Sept. 11 and 18, 1858. The two races which have founded the mathematics, those of the Sanscrit and Greek languages, have been the two which have independently formed systems of logic.

England is the country in which the antipathy has developed itself in greatest force. Modern Oxford declared against mathematics almost to this day, and even now affords but little encouragement: modern Cambridge to this day declares against logic. These learned institutions are no fools, whence it may be surmised that possibly they would be wiser if they were brayed in a mortar; certainly, if both were placed in the same mortar, and pounded together.

147. *Moral* proof is when the conclusion is so established that any contradiction would be of that high degree of improbability which we never look to see upset in ordinary life. Among the most remarkable of moral proofs is that common case of induction in which the aggregants are innumerable, and the conclusion being proved as to very many, without a single failure, the mind

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feels confident that all the unexamined aggregants are as true as those which have been examined. This is *probable induction:* often confounded with *logical induction.*

148. A proof may be mixed: it may be *deduction* of which some components are *inductively* proved: it may be *induction,* of which some aggregants are *deductively* proved.

149. Failure of proof is not proof of the contrary.

150. If any number of premises give a conclusion, denial of the conclusion is denial of one or more of the premises. If all but one of the premises be affirmed and the conclusion denied, that one premise must be denied. These two processes, conclusion from premises, and denial of one premise by denying the conclusion and affirming all the other premises, may be called *opponents.*

151. *Repugnant alternatives* are propositions of which one must be true, and one only. If there be two sets of repugnant alternatives, of the same number of propositions in each, and if each of the first set give its own one of the second set for its necessary consequence, then each of the second set also gives its own one of the first set as a necessary consequence. Thus if A, B, C, be repugnant alternatives, and also P, Q, R, and if P be the necessary consequence of A, Q of B, R of C, then A is the necessary consequence of P, B of Q, C of R If P be true, neither B nor C can be true; for then Q or R would be true, which cannot be with P. But one of the three A, B, C, must be true: therefore A is true. And similarly for the other cases.

152. A relation is a mode of thinking two objects of thought together: a connexion or want of connexion. Denial of relation is another relation: and the two are contraries. The universe may have only a selection from all possible relations.

153. The name in relation is the *subject:* the name to which it is in relation is the *predicate.* Thus in 'mind acting upon matter' *mind* is the *subject, matter* the *predicate, acting upon* is the *relation.* When the relation is convertible, subject and predicate are distinguished only by order of writing, as in § 9.

154. All *judgments* (asserted or denied relations) may be reduced to assertion or denial of concomitance by coupling the predicate and the relation into one notion. As in 'mind *is* a thing acting on matter' or 'mind *is not* a thing acting on matter'. In all works of logic, the consideration of relation in general is

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evaded by this transformation, and the developement of the science is thereby altogether prevented.

155. If X be in some relation to Y, Y is therefore in some other relation to X. Each of these relations is the *converse* of the other. Converse relations are of identical effect, and neither exists without the other. In conversion the subject and predicate are transposed and usually change order of mention: as in 'X is master of Y; Y is servant of X'.

156. When a relation is its own converse, it is said to be *simply* convertible. As in 'X has nothing in common with Y' and 'Y has nothing in common with X'; or as in 'X is equal to Y' and 'Y is equal to X'.

157. When the subject of one relation is made the predicate of another, the first predicate may be made the predicate of a *combined* relation: as in the master* of (the-nephew-of-Y), that is, the-master-of-the-nephew of Y.

* The most familiar relations are those which exist between one human being and another; of which the *relations* of consanguinity and affinity have almost usurped the name *relation* to themselves. But hardly a sentence can be written without expression or implication of other relations.

158. A combined relation may have a separate name, or it may not. Thus *brother of parent* has its own name, *uncle:* but *friend of parent* has no name which describes nothing else.

159. A combined relation may be of limited meaning, or it may not. Thus 'non-ancestor of a descendant of Z' has a limitation of meaning with reference to Z; he is certainly non-ancestor of Z. But 'ancestor of a descendant of Z' has no such limitation: any person may be the ancestor of a descendant of any other.

160. When a relation combined with itself reproduces itself, let it be called *transitive:* as *superior; superior* of *superior* is *superior*, the same sort of superiority being meant throughout. A transitive relation has a transitive converse: thus *inferior of inferior* is *inferior*.

161. Relations are conceivable both in extension and in intension (§ 131), both as aggregates and as compounds. Thus 'child of the same parents with' is aggregate of 'brother, sister, self': the relation of whole to part has among its components 'greater' and 'of same substance with'.

162. If two relations combine * into what is contained in a third relation, then the *converse* of either of the two combined with the *contrary* of the third, in the same order, is contained in

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the *contrary* of the other of the two. Thus the following three assertions are *identically the same*, superior and inferior being taken as contraries, that is, absolute equality not existing. Let the combination be 'master of parent' and the third relation 'superior'.

Every master of a parent is a superior
Every servant of an inferior is a non-parent
Every inferior of a child is a non-master.

From either of these the other two follow. This may be generally proved: at present it will be sufficient to deduce one of the assertions before us from another. Assume the second; from it follows that every parent is not any servant of an inferior, and therefore, if servant at all, only servant of superior, whence master of parent must be superior.

* This theorem ought to be called theorem K, being in fact the theorem on which depends the process (§ 108) indicated by the letter K in the old memorial verses.

163. The relation in which an object of thought stands *to itself*, is called *identity;* to every thing else, *difference.* Every thing is itself: nothing is anything but itself: and any two things being thought of, they are either the same or different, and can be nothing except one or the other. These principles enter into the distinction between truth and falsehood: but cannot distinguish one truth from another. They are antecedent* to all nomenclature, and to all decomposition.

* Many acute writers affirm that syllogism can be evolved from, and solely depends upon, three principles: 1. *Identity*, A is A; 2. *Difference*, A is not not-A; thirdly, *excluded middle*, Every thing either A or not-A. Now syllogism certainly demands the perception of *convertibility*, 'A is B gives B is A', and of *transitiveness*, 'A is B and B is C gives A is C'. Are the *two* principles deducible from the *three?* If so, either by syllogism or without. If by syllogism, then syllogism, *before* establishment upon the three principles, is made to establish itself, which of course is not valid. Consequently, we must take the writers of whom I speak to hold that convertibility and transitiveness follow from the principles of identity, difference, and excluded middle, without *petitio principii.* When any one of them attempts to show *how*, I shall be able to judge of the process: as it is, I find that others do not go beyond the simple assertion, and that I myself can detect the *petitio principii* in every one of my own attempts. Until better taught, I must believe that the *two* principles of identity and transitiveness are not capable of reduction to consequences of the *three*, and must be assumed on the authority of consciousness.

Should I be wrong here: should any logician succeed, without assuming syllogism, in deducing the syllogism of the *identifying* copula 'is' from

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what may be called the three principles of *identification*, I shall then admit a completely established specific difference between the ordinary syllogism and others in which the copula, though convertible and transitive, is not the substantive verb. I should expect, in such an event, to deduce the transitiveness and convertibility of 'equals' from 'A equals A', 'A does not equal not-A' and 'every thing either equals A or not-A', where A is magnitude only.

164. Identity is agreement in every thing and difference in nothing. Complex objects of thought usually agree in some things and differ in others. They get the same names in right of those points in which they agree, and different names in right of those points in which they differ. And thus, all resemblances or agreements giving an agreement of names, and all differences giving a difference of names, all the forms of inference are capable of being evolved out of those forms in which nothing but concomitance or non-concomitance of names is considered (§ 5 to § 73).

165. Relations which have immediate reference to, or are directly evolved from, the application of names and the mode of thinking about names in connexion with objects named, or with other names, may be called *onymatic* relations*.

* The logician has hitherto denied entrance to every relation which is not onymatic; declaring all others to be *material*, not *formal*. When the distinction of matter and form is so clearly defined that it can be seen why and how no connexions are of the form of thought except those which I have called onymatic, it will be time enough to attempt a defence of the introduction of other relations. In the meantime, looking at all that is commonly said upon the distinction of form and matter, I am strongly inclined to suspect that there is nothing but a mere confusion of terms; that is, that when the logician speaks of the distinction of form and matter, he *means* the distinction of onymatic and non-onymatic. Dr. Thomson, in his *Outlines, &c.* (§ 15, note) observes that the philosophic value of the terms matter and form is greatly reduced by the confusion which seems invariably to follow their extensive use. The truth is that the mathematician, as yet, is the only consistent handler of the distinction, about which nevertheless he thinks very little. The distinction of form and matter is more in the theory of the logician than in his practice: more in the practice of the mathematician than in his theory.

166. The only relation in which a *name*, as a name, can stand to an *object*, is that of *applicable* or *inapplicable*.

167. Names may have many grammatical and etymological relations to one another, but the only relations which are of any logical import are the relations in which they stand to one another arising out of the relations in which they stand to objects. Ac-

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cordingly we consider two names as having objects to which both apply, or as both applying to nothing whatsoever.

168. When X, Y, Z, are individual names, and we say 'X is Y, Y is Z, therefore X is Z', we can but mean that in speaking of X and Y we are speaking of one object of thought, and the same of Y and Z, so that in speaking of X and Z we are speaking of one object. The law of thought which acts in this inference is the *transitiveness* (§ 160) *of the notion of concomitancy*: if X go with Y, and Y go with Z, then X goes with Z.

169. When names denote *classes*, the primary relation between them is that of containing and contained, in the sense of aggregate and aggregant (§ 124): other relations spring out of this, as will be seen. This relation is *mathematical* in its character: a class is made up of classes, just as an area is made up of areas. It is physically possible to connect the two aggregations: we can imagine all *men* on one area, and all *brutes* on another: the aggregate of the areas contains the class *animal*, the aggregate of *man* and *brute*.

170. When names denote *attributes*, the primary relation is that of containing and contained, in the sense of compound* and component (§ 124). This relation is *metaphysical* in its character: the mode of junction of components is not mathematical, but is a subject for metaphysical discussion, though how that discussion may terminate is of no importance for logical purposes. The manner in which the sources of the notion *rational* are combined with those of the notion *animal* in the object which is called *man* has nothing to do with the laws of thought under which the compound and the components are and must be treated.

* It is not uncommon among logical writers to declare that an attribute is the sum of the attributes which it comprehends; that, for example, *man*, completely described by the notions *animal* and *rational* conjoined, is the *sum* of those notions. This is quite a mistake: let any one try to sum up *animal* and *rational* into *man*, in the obvious sense and manner in which he sums up *man* and *brute* into *animal*. The distinction of aggregation and composition, very little noticed by logicians, if at all, runs through all cases of thought. In mathematics, it is seen in the distinction of addition and multiplication; in chemistry, in the distinction of mechanical mixture and chemical combination; in an act of parliament, in the distinction between 'And be it further enacted' and 'Provided always'; and so on.

Hartley has more nearly than any other writer produced the notion of composition as distinguished from aggregation. His compound idea has a force and meaning of its own, which prevents our seeing the components in it, just as, to use his own illustration, the smell of the compound medicine

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overpowers the smells of the ingredients. But even Hartley represents the compound of A and B by A + B.

171. When the class X is *contained in* the class Y, as an aggregant, the attribute Y is *contained in* the attribute X, as a component. Thus the class man is contained in the class animal: the attribute animal is contained in the attribute man. These two apparent contradictions are both true in their different senses: say he is man, you say he is in animal; say he is man, you say animal is in him. Class *man* is in class *animal*, as aggregant in aggregate; attribute *animal* is in attribute *man*, as component in compound.

172. In all things which do not depend on ourselves, we learn to think of that which *always* happens as *necessarily* happening, of that which *always* accompanies as being *essential*,* part of the *essence*, part of the *being*. This *metaphysical* notion is always in thought, in one form or other, whenever undeviating concomitance of one notion with another is established or supposed.

* Upon this word may be said, once for all, what is to be said concerning the use of metaphysical terms in logic. We have nothing to do with the way in which the mind comes *to* them; our affair is with the way in which the mind works *from* them. Thus it is absolutely essential to the fitness of three straight lines to be the sides of a triangle that any two should be together greater than the third; contradiction is inconceivable. It is naturally essential to an apple to be round; contradiction is unknown in nature. It is commercially and conveniently essential to a tea-pot to have a handle; any contradiction would be unsaleable and unusable. In all these cases, and whatever may be the force of the word *essential*, the mode of inference is the same: for the logical consequences of Y being an essential of X are but those of Y being always found whenever X is found. Why then do we not confine ourselves to this last notion, leaving the character of the conjunction, be it a necessity of thought, a result of uncontradicted observation, or a conventional arrangement, &c. entirely out of view? Simply because, by so doing, we fail to make logic an analysis of the way in which men *actually do think*. If men *will* be metaphysicians—and metaphysicians they will be—it must be advisable to treat the metaphysical views of the most common relations, the onymatic, in a system of logic. The metaphysical notion is a natural growth of thought, and children and uneducated persons are more strongly addicted to it than educated adults.

173. Out of these onymatic relations arise five different modes of enunciating the same proposition. One of these, the *arithmetical*, already treated, merely states, or sums up, an enumeration of concomitances, or non-concomitances: as in 'Every man is an animal'; or as in 'No man is a vegetable'.

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174. The four subjective modes of speaking which the notions of relation develope, are,—

1. *Mathematical.* Here both subject and predicate are notions of class: the *class* man contained in the *class* animal.

2. *Physical.* The subject a *class*, the predicate an *attribute.* As in 'man is mortal': the class *man* has the attribute *subject to death.*

3. *Metaphysical.* Both subject and predicate are notions of *attributes.* As in 'humanity is fallible': *fallibility* a component of the notion *humanity.*

4. *Contraphysical.* The subject an attribute, the predicate a class. As* in 'All mortality in the class man', or 'none but men are mortal': that is, we must attribute mortality only in the class man; or, all of which mortality is the attribute is in the class man.

* I take a falsehood for once, to remind the reader that with truth or falsehood of matter we have nothing to do.

175. All these modes of reading are concomitant: each one of the five gives all the rest. If all the men in the *universe** be so many animals, then the class man is in the class animal, and has the attribute animal as one of its class marks; also, the attribute animal is an essential of the attribute humanity, and the attribute humanity is to be sought only in the class animal.

* According to the universe understood, so is the mode of taking the meaning of the onymatic terms. For example, if the universe be the universe of *objective reality*, then, all existing men being ascertained to be animals, it is of the nature of man, as actually created, to be animal: the attributes of animal are naturally essential to man. If the universe be the universe of all possible thought, then, if all men conceivable be animals, if, for whatever reason, it be impossible to think of man without thinking of animal, then the attribute animal is an essential of the attribute humanity. And now arises a question of words, with which logic has nothing to do. Those creatures of thought which occur in the fables, dogs and oxen, &c. which are rational as well as animal, are they *men?* Certainly not, according to the notion which the word represents. Consequently, the phrase *rational animal* is a larger term than man, when all the possibilities of thought are in question. But this is not a question of logic. The logician, as such, does not know what man is, nor what animal is: but he knows how to combine 'every man is animal' with other propositions, so soon as he knows that he is permitted to use that proposition.

176. We have now to render the proposition and the syllogism into the four readings, mathematical, metaphysical, or mixed, in-

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venting appropriate terms for all the relations which occur. It will be sufficient, however, to treat the first and third system, the wholly mathematical, and the wholly metaphysical.

177. When Every X is Y, X))Y or Y((X, let the class X be called a *species* of the class Y, and Y a *genus** of X. In the contrary case, when some Xs are not Ys, X(·(Y or Y)·)X, let X be an *exient* of Y, and Y a *deficient* of X.

* In the common use of these words, the *species* is a part only of the *genus*. As here used the species may be the whole genus. This is, to my mind, the greatest liberty I have taken with the ordinary terms of logic.

178. When No X is Y, X)·(Y or Y)·(X, let each class be called an *external* of the other, or let the two be called *coexternals*. In the contrary case, when some Xs are Ys, X()Y or Y()X, let each be called a *partient* of the other, or let the two be called *copartients*.

179. When every thing is either X or Y, X(·)Y or Y(·)X, let each class be called a *complement* of the other. In the contrary case, when some things are neither Xs nor Ys, X)(Y or Y)(X, let each class be called* a *co-inadequate* of the other.

* Punsters are respectfully informed that the reading *coin-adequate*, and all jokes legitimately deducible therefrom, are already appropriated, and the right of translation reserved.

180. The spicular symbols may be made to stand for the relations themselves. Thus)) means *species* or *genus*, according as it is read forwards or backwards; ((, genus or species: and so on.

181. *Genus* and *species* are *converse* relations; as also *exient* and *deficient*: of *external, partient, complement, coinadequate*, each is its own converse. *Genus* and *deficient* are contrary relations; as are *species* and *exient, external* and *partient, complement* and *coinadequate*.

182. *Genus* is both *partient* and *coinadequate;* as also is *species*. *External* is both *exient* and *deficient*, and so is *complement*.

183. These are exercises in the meanings of the terms, and should be thought of until their truth is familiar; as also the following:—

The *genus* has the utmost *partience*, and may have the utmost *coinadequacy*. The *species* has the utmost *coinadequacy*, and may have the utmost *partience*. The *external* has the utmost *deficiency*,

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and may have the utmost *exience.* The *complement* has the utmost *exience,* and may have the utmost *deficiency.*

184. These relations have *terminal ambiguity,* founded on the notion of *contained* having two cases, filling the whole, or filling only a part. Thus

Genus is either *species* or *exient*
Species is either *genus* or *deficient*
External is either *complement* or *coinadequate*
Complement is either *external* or *partient.*

185. Read the identities in § 25 into this language, as in, Species is external of contrary, Contrary of species is complement, Contraries of species and genus are genus and species, &c.

186. The following are the combinations* of mathematical relation which take place in syllogisms. Each triad in the first list contains a universal and two particular syllogisms, the three being opponents (§ 47), connected also by the theorem in § 162. The second list (§ 187) contains the strengthened syllogisms.

))	))	Species of species is species
((	(·(	Genus of exient is exient
(·(	((	Exient of genus is exient
((	((	Genus of genus is genus
))	)·)	Species of deficient is deficient
)·)	))	Deficient of species is deficient
)·(	(·)	External of complement is species
)·(	(·(	External of exient is coinadequate
(·(	(·)	Exient of complement is partient
(·)	)·(	Complement of external is genus
(·)	)·)	Complement of deficient is partient
)·)	)·(	Deficient of external is coinadequate
))	)·(	Species of external is external
((	()	Genus of partient is partient
()	)·(	Partient of external is exient
((	(·)	Genus of complement is complement
))	)(	Species of coinadequate is coinadequate
)(	(·)	Coinadequate of complement is deficient
)·(	((	External of genus is external
)·(	()	External of partient is deficient
()	))	Partient of species is partient

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(·))) Complement of species is complement
(·))(Complement of coinadequate is exient
)(((Coinadequate of genus is coinadequate.

* Note that when, and only when, one of the combining words is either *genus* or *species*, the other two words are the same; and this throughout the fundamental or unstrengthened syllogisms. What law of thought does this represent? And except when one of these words so occurs, the three words of relation are all different.

187. (( )) Genus of species is partient
)) ((Species of genus is coinadequate
(·) (·) Complement of complement is partient
)·( )·(External of external is coinadequate
(( )·(Genus of external is exient
)) (·) Species of complement is deficient
(·) ((Complement of genus is exient
)·( )) External of species is deficient.

188. When we give what may be called, comparatively, *terminal precision*, as in § 57, we may use the following nomenclature,—

)o) A deficient species may be called a subidentical
|| A species and genus is an identical
(o(An exient genus may be called a superidentical

)o(A coinadequate external may be called a subcontrary
|·| An external complement is a contrary
(o) A partient complement may be called a supercontrary.

189. The complex syllogisms (§ 61) may be read as follows,—

)o))o) A subidentical of a subidentical is a subidentical
(o((o(A superidentical of a superidentical is a superidentical
)o((o) A subcontrary of a supercontrary is a subidentical
(o))o(A supercontrary of a subcontrary is a superidentical
)o))o(A subidentical of a subcontrary is a subcontrary
(o((o) A superidentical of a supercontrary is a supercontrary
)o((o(A subcontrary of a superidentical is a subcontrary
(o))o) A supercontrary of a subidentical is a supercontrary.

The following modes of connecting the symbols, as applied to the same two terms, may be useful,—

)o))), Species;)·), but not the greatest possible.
(o(((, Genus; (·(, but not the least possible.
)o( )·(, External;)(, but not the greatest possible.
(o) (·), Complement; (), but not the least possible.

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190. I now proceed to the metaphysical relations * between *attribute* and *attribute*.

* The terms of metaphysical relation are picked up without difficulty in our common language: but those of mathematical relation had in several instances to be forged. This means that the world at large has more of the metaphysical than of the mathematical notion in its usual form of thought. But though the *unconnected* words *essential*, *dependent*, *repugnant*, *alternative*, are constantly on the tongues of educated people, the combinations of these relations are not made with any security, and when thought of at all, enter under a cloud of words: while the analysis by which precision of speech and habit of security might be gained is treated with contempt, as being *logic*. A whole drawing-room of educated men may be without a single person who can expose the falsehood of the assertion that the essential of an incompatible must be incompatible; a proposition which I have heard maintained, though not in those words, by persons of more than respectable acquirements; sometimes by actual error, sometimes by confusion between the essential of an incompatible, and that to which an incompatible is essential. But even of the persons who are not thus taken in, very few indeed, when told that the answer to 'the essential of an incompatible is incompatible' is 'not so much; *only* independent', will be puzzled by the juxtaposition of incompatibility and independence as viewed in a relation of degree. In making these remarks, it will be remembered that I am not speaking of any words of my own, nor of any meanings of my own. The words are common, and I take them in their common meanings; but it is not generally seen that these common words, used in their common senses, are sufficient, in conjunction with their contraries, to express all the relations which occur in a completely quantified system of onymatic enunciation.

191. When X))Y let the attribute Y be called an *essential* of the attribute X, and X a *dependent* of Y. In the contrary case, X(·(Y, let Y be called an *inessential* of X, and X an *independent* of Y. Remember that *dependent on* does not mean *dependent wholly on*, or *dependent only on*.

192. When X)·(Y, let each attribute be called a *repugnant* of the other. When X()Y, let each be an *irrepugnant* of the other.

193. When X(·)Y let each attribute be called an *alternative* of the other. When X)(Y let each be called an *inalternative* of the other.

194. When difference of symbols is desired, the square bracket may be used instead of the parenthesis: thus]] may denote *dependent* when read forwards, and *essential* when read backwards, &c.

195. *Essential* and *dependent* are converse relations; as are also *inessential* and *independent*. Of *repugnant*, *irrepugnant*, *alter-*

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native, *inalternative*, each is its own converse. Essential and inessential are contrary relations; as are dependent and independent, repugnant and irrepugnant, alternative and inalternative. Compare § 181.

196. The *essential* is both *irrepugnant* and *inalternative:* as also is the *dependent*. The *repugnant* is both *independent* and *inessential:* as also is the *alternative*. Compare § 182.

197. The essential has the utmost irrepugnance, and may have the utmost inalternativeness. The dependent has the utmost inalternativeness, and may have the utmost irrepugnance. The repugnant has the utmost inessentiality, and may have the utmost independence. The alternative has the utmost independence, and may have the utmost inessentiality. Compare § 183.

198. These relations also have terminal ambiguity (Compare § 184).

Essential is either dependent or independent
Dependent is either essential or inessential
Repugnant is either alternative or inalternative
Alternative is either repugnant or irrepugnant.

199. Read the identities in § 25 into this language, as in Dependent is repugnant of contrary, contrary of dependent is alternative, contraries of dependent and essential are essential and dependent, &c.

200. The following are the combinations* in syllogism, arranged as in § 186.

))	))	Dependent of dependent is dependent
((	(·(	Essential of independent is independent
(·(	((	Independent of essential is independent
((	((	Essential of essential is essential
))	)·)	Dependent of inessential is inessential
)·)	))	Inessential of dependent is inessential
)·(	(·)	Repugnant of alternative is dependent
)·(	(·(	Repugnant of independent is inalternative
(·(	(·)	Independent of alternative is irrepugnant
(·)	)·(	Alternative of repugnant is essential
(·)	)·)	Alternative of inessential is irrepugnant
)·)	)·(	Inessential of repugnant is inalternative

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))	)·(	Dependent of repugnant is repugnant
((	()	Essential of irrepugnant is irrepugnant
()	)·(	Irrepugnant of repugnant is independent
((	(·)	Essential of alternative is alternative
))	)(	Dependent of inalternative is inalternative
)(	(·)	Inalternative of alternative is inessential
)·(	((	Repugnant of essential is repugnant
)·(	()	Repugnant of irrepugnant is inessential
()	))	Irrepugnant of dependent is irrepugnant
(·)	))	Alternative of dependent is alternative
(·)	)(	Alternative of inalternative is independent
)(	((	Inalternative of essential is inalternative.

* Note that when, and only when, one of the combining words is either *essential* or *dependent*, the other two words are the same; and this throughout the fundamental or unstrengthened syllogisms. What law of thought does this represent? And except when one of these words so occurs, the three words of relation are all different.

201.	((	))	Essential of dependent is irrepugnant
	))	((	Dependent of essential is inalternative
	(·)	(·)	Alternative of alternative is irrepugnant
	)·(	)·(	Repugnant of repugnant is inalternative
	((	)·(	Essential of repugnant is independent
	))	(·)	Dependent of alternative is inessential
	(·)	((	Alternative of essential is independent
	)·(	))	Repugnant of dependent is inessential.

202. I now proceed to form metaphysical* terms expressing relations of terminal precision (compare § 188). Let an *inherent* be an attribute asserted; let an *excludent* be an attribute denied; let an *accident*, which is also *non-accident*, be an attribute affirmed of part and denied of the rest. Thus of man, *life* is an inherent, *vegetation* an excludent, *wisdom* an accident and a non-accident.

* This new formation cannot be overlooked, since it is the extension of the Aristotelian system of *predicables*, *genus* and *species* (used in the old sense) and *accident*, to the system in which contrary terms are permitted. Otherwise, the relations of terminal ambiguity, compounded, might serve the purpose.

203. Each of these relations may be either *generic* or *specific*. Either is *generic* when it applies in as large or a larger degree to a larger genus: *specific*, when it does not so apply to any larger

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genus. This being premised, the following relations will be found correctly stated,—

)o)	Inessential dependent	is	*Specific accident* *Generic non-accident*
||	Dependent essential	is	*Specific inherent*
(o(	Independent essential	is	*Generic inherent*
)o(	Inalternative repugnant	is	*Generic excludent*
|·|	Repugnant alternative	is	*Specific excludent*
(o)	Irrepugnant alternative	is	*Generic accident* *Specific non-accident.*

204. The following are examples of each of these terms, the universe being *terrestrial animal*,—

Specific accident; generic non-accident. *Lawyer* is in this relation to man: *accident* and *non-accident*, because an attribute of some men, and not of others; *specific* accident, because not found in the additional extent of any genus larger than man; *generic* non-accident, for the same reason.

Specific inherent. *Rational* is in this relation to man: *inherent*, because an attribute of all; *specific*, because no attribute of the additional extent in a larger genus.

Generic inherent. *Biped* is in this relation to man; *inherent*, because an attribute of all; *generic*, because an attribute of the additional extent of a larger genus.

Generic excludent. *Oviparous* is in this relation to *man; excludent*, because an attribute to be denied of man; *generic*, because to be also denied of the additional extent of some larger genera.

Specific excludent. *Dumb* (wanting articulate language with meaning) is in this relation to man; *excludent*, because to be denied of man; *specific*, because not to be denied of the additional extent of any larger genus.

Generic accident; specific non-accident. *Naked* (not artificially clothed) is in this relation to man; *accident* and *non-accident*, because some are and some are not; *generic* accident, because an accident of the additional extent of larger genera; *specific* non-accident, because not non-accident of any such additional extent.

205. When either of the relations belongs equally to a term and its contrary, it may be called *universal*. Thus an attribute of both term and contrary is a *universal inherent;* an accident and non-accident of both term and contrary is a *universal accident and non-accident;* an excludent of both term and contrary is a *universal excludent.* But the first and third of these terms are chiefly

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of use in defining the universe: the second is that relation which we suppose until some contradiction is affirmed.

206. With the arithmetical reading in *extension* may be joined that in *intension*, § 115, 131. In extension, the unit of enumeration is one of the objects all of which aggregate into the class: in intension, the unit of enumeration is one of the qualities all of which compose the object. The following is the system of arithmetical reading in intension: naturally connected (§ 129) with the metaphysical mode of viewing objects of thought. The inversion of the quantities, presently further described, will be easily seen; namely, that)X and X(now indicate that X is taken completely, in all its qualities; while X) and (X indicate that X is taken incompletely, in some (some or all, not known which) of its qualities. The term *any* (§ 22) is here introduced when grammatically desirable.

Symbol	*Metaphysical reading*	*Arithmetical reading in intension*
X))Y ,	X dependent of Y	All qualities of Y are some qualities of X
X(·(Y ,	X independent of Y	Some qualities of Y not any qualities of X
X)·(Y	X repugnant of Y	All things want either some qualities of X, or some qualities of Y
X()Y	X irrepugnant of Y	Some things want neither any quality of X, nor any quality of Y
X(·)Y	X alternative of Y	All things have either all the qualities of X or all the qualities of Y
X)(Y	X inalternative of Y	Some things want either some of the qualities of X, or some of the qualities of Y
X((Y	X essential of Y	Some qualities of Y are all the qualities of X
X)·)Y	X inessential of Y	Any qualities of Y are not some qualities of X.

207. I now proceed to further consideration of the subject of quantity. No new results can appear, but it will be necessary both to adapt the old results to the more subjective view of logical process, and also to consider the distinctions of quantity from new points of view.

208. The distinction of the two *tensions*, *extension* and *intension* (§ 131), or, for brevity, *extent* and *intent*, may for clearness (§ 129) be applied only to classes and attributes. The extent of a class embraces all the classes of which it is aggregated: the intent of an attribute embraces all the attributes of which it is compounded.

209. A class may be subdivided down to the distinct and non-

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interfering individual objects of thought of which it is composed: and here subdivision must stop. But it is not for human reason to say what are the simple attributes into which an attribute may be decomposed: the decomposition of the notion *rational*, for example, into distinct and non-interfering component notions, is the subject of an old controversy which will perhaps never be settled. But this difficulty is of no logical importance.

210. The relation of quantity as exhibited in the arithmetical view of the proposition (§ 13, 14), giving the distinction of *universal* and *particular* quantity, as it is commonly expressed, or of *total* and *partial* quantity, as I have expressed it, may be in this part of the subject most conveniently attached to other names. Let the terms *full extent** and *vague extent* be used to replace *total extension* and *partial extension:* and let *full intent* and *vague intent* replace *total intension* and *partial intension.*

* These terms are convenient from their brevity: *full extent* is shorter than *universal extension.* But they are still more useful as avoiding the ambiguity of the words *some*, *particular*, *partial*, which, as we have seen (§ 14, note †) misleads even the highest writers. The logical opposition of quantity is not *quantity universal* and *quantity not universal*, but *quantity asserted to be universal* and *quantity not asserted to be universal.* Two words cannot be found which express the opposition of undertaking to assert and not undertaking to assert universality. We may therefore be content with *full* and *vague*, which, if they do not *express* opposition, at least do not, like *universal* and *particular*, express the *wrong* opposition.

211. Additional extent can only be gained by a new aggregate containing extent which is not in the collective extent of the others: additional intent only by a new component which is not in the joint intent of the others. Thus the extent of the class *animal* is not augmented by the aggregation of the class *having volition*, if the universe be the *visible earth.* Again, the intent* of the notion *plane triangle* is not augmented by the junction of the notion *capable of inscription in a circle.* The distinction between these real and apparent augmentations is of the matter, not of the form: and is of no logical import except this, that when we say that a new aggregant increases extent, and a new component increases intent, we must be prepared, with the mathematicians, to reckon 0 among the cases of quantity.

* There is a remarkable difference between extent and intent, which, though logically nothing at all, is psychologically very striking. Say we discover extent hitherto unknown, without the necessity of reducing intent to include it within a class thought of. Columbus did this when he first was able to add the class *American* to the classes then known under *man.*

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Here is nothing beyond what was possible in previous thought, which could people the seas to any extent. But when we add intent without diminishing extent, which knowledge is doing every day, we cannot conceive beforehand what kind of additions we shall make. A beginner in geometry gradually adds to the intent of *triangle*, which at first is only rectilinear three-sided figure, the components—can be circumscribed by a circle—has bisectors of sides meeting in a point—has sum of angles equal to two right angles—and other properties, by the score. The distinction is that class aggregation joins *similars*—but that composition of attributes joins things perfectly distinct, of which no one can predicate anything merely by what he knows of another thing. When the old logicians threw the notion of intent out of logic into metaphysics, they were guided by the material differences *of* qualities, and did not apprehend their similarity of properties *as* qualities.

212. The distinction of extent and intent has found its way into common language, in the words *scope** and *force*, which I shall sometimes use. Thus in 'every man is animal' the term *man* is used in all its scope, but not in all its force; a person incognisant of some of the components of the notion *man*, that is, of the whole *force* of the term, might have the means of knowing this proposition. But *animal* is used in all its force, and not in all its scope. This answers to saying that in 'Every X is Y', the term X is of full extent and vague intent; the term Y is of full intent and vague extent.

* The logicians, until our own day, have considered the *extent* of a term as the only object of logic, under the name of the *logical whole*: the *intent* was called by them the *metaphysical whole*, and was excluded from logic. In our own time the English logical writers, and Sir William Hamilton among the foremost, have contended for the introduction of the distinction into logic, under the names of *extension* and *comprehension*: Hamilton uses *breadth* and *depth*. Now I say that in the perception of the distinction between scope and force, as well as in other things, the world, which always runs after quack preparations, has ventured for itself out of the logical pharmacopœia. This certainly in a rude and imperfect way: and without apprehension of any *theorems*. I have not found, though I have looked for it, any such amount of recognition that the greater the scope the less the force as I could present without suspicion of the *aut inveniam aut faciam* bias. But I think it likely enough that some of my readers may casually pick up passages which show a *feeling* of this theorem.

213. The quantity considered in the arithmetical view of logic (§ 5–111) was entirely quantity of extent. I now proceed to the comparison of extent and intent.

214. In every use of a term, one of the tensions* is full, and the other vague: the full extent and the full intent cannot be used at one and the same time; and the same of the vague extent and the vague intent. Thus X) and (X must stand for X used

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in full extent and vague intent: and)X and X(for vague extent and full intent.

The proof of this proposition is as follows. When a term is full in extent, we can abandon or dismiss any aggregant of that extent we please: the proposition, though reduced or crippled by the dismissal, is true of what is left: but we may not annex an aggregant at pleasure. When a term is vague in extent, we cannot dismiss any aggregant whatever: for we know not by what aggregant the proposition is made true: but we may annex any aggregant at pleasure: for we do not thereby throw out what makes the proposition true, even if we annex no additional truth; and we do not, when speaking vaguely, affirm or deny of any one selected aggregant. And as the extent must be full or vague, and we must be either competent or incompetent to dismiss an aggregant taken at pleasure, and must be either competent or incompetent to annex one, the converses follow (§ 151), namely, that when we are competent to dismiss, the extent is full, and when we are incompetent the extent is vague: and also that when we are competent to annex, the extent is vague, and when not, the extent is full. Precisely the same proposition may be established upon the intent of a term, and its components.

Now let a term be of *full* extent. In diminishing the extent, which we may do, we can so do it as to augment the intent: and if we be competent to augment the intent, that intent must be *vague*, as just proved. Similarly, if a term be of *vague* extent, we are competent to annex an aggregant, that is, to diminish the intent; whence the intent must be *full*. And the same may be proved in like manner when either kind of intent is first supposed instead of extent; though by use of § 151 this case may be seen to be contained in that already treated. And the learner may gather the whole from instances. Thus A, B))PQ gives A))PQ, and A))P; but not A,B,C))PQ, nor A,B))PQR. But PQ))A,B does not give P)))A, B, nor PQ))A, though it does give PQR))A,B and PQ))A,B,C; and so on. And further, from § 133, this proposition can be made good of all universals when it is known of one: and the same of all particulars.

* The logicians who have *recently* introduced the distinction of extension and comprehension, have altogether missed this opposition of the quantities, and have imagined that the quantities remain the same. Thus, according to Sir W. Hamilton 'All X is some Y' is a proposition of comprehension, but 'Some Y is all X' is a proposition of extension. In this the logicians have abandoned both Aristotle and the laws of thought from which he drew the

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few clear words of his dictum: 'the genus is said to be part of the species; but in another point of view (ἄλλως) the species is part of the genus'. *All animal* is in *man*, notion in notion: *all man* is in *animal*, class in class. In the first, all the notion *animal* part of the notion *man:* in the second, all the class *man* part of the class *animal.* Here is the opposition of the quantities.

215. It appears then that the *elements* of a tension (aggregants of an extent, components of an intent) may be dismissed from the term used fully, but cannot be introduced; may be introduced into a term used vaguely, but cannot be dismissed. The dismissible is inadmissible: the indismissible is admissible.

216. Elements of either tension may, under the limitations of a rule to be shown, be transposed from one term of a proposition to the other, either directly, or by contraversion, *without either loss or gain of import to the proposition.* Thus AB)·(Y is the same proposition as A)·(BY, and X))A,B is the same as Xa))B. The demonstration of this may best be seen by observing that every universal is a declaration of *incompossibility,** and every particular a corresponding declaration of *compossibility.* Thus X)·(Y is an assertion that X and Y, as names of one object, are incompossible; and X()Y that they are compossible. Again, X))Y declares X and y to be incompossible; and so on.

Now it will be seen that AB)·(Y is merely a statement that the three names A, B, Y, are incompossible; and so is A)·(BY. Hence AB)·(Y and A)·(BY are identical. Similarly AB()Y and A()BY are identical, both declaring the compossibility of A, B, Y: or thus, if two propositions be identical, their contraries must be identical. Hence we learn that in Y))a, b we have YB))a &c. Carrying this through all transformations, we arrive at the following rules:—

1. In universal propositions, vague elements (the elements of terms of either vague tension) are transposible; directly in negatives, by contraversion in affirmatives. But full elements are intransposible.

2. In particular propositions, full elements (the elements of terms of either full tension) are transposible; directly in affirmatives, by contraversion in negatives. But vague elements are intransposible.

Thus in X))Y, Y is of vague extent; if it be (A, B), its aggregant A is transposible, the proposition being affirmative, by contraversion: that is X))A, B is identical with Xa))B. The rules are for comparison and generalisation, not for use. Nothing

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can be more evident than that if every X be either A or B, every X which is not A is B.

* These good words are Sir William Hamilton's (see § 14, note †), to whom, in matters of language, I am under what he would have called obligations general and obligations special. His occasional writing of the adjective after the substantive is a useful revival of an old practice, tending much to clearness. As to my obligations special, he, finding the word *parenthesis* not enough to erect his reader's hair, described my notation as "horrent with mysterious spiculæ". This was the very word I wanted, § 21: for *parenthesis* has come to mean, not the punctuating sign, but the matter which it includes: and *parenthetic* notation would have been ambiguous.

217. It has in effect been noticed that for every full term in a proposition a term of as much or less tension may be substituted; and for every vague term a term of as much or more tension. This is the whole principle of onymatic syllogism, or rather may be made so: for the varieties of principle upon which all onymatic inference may be systematically introduced are numerous. Thus in X()Y)·(Z, giving X(·(Z, all we do is to substitute for Y used vaguely in extent, the as extensive or more extensive term z. Or thus;—for Y of full intent, we substitute the as intensive or less intensive term z. For Y)·(Z or Y))z, shows that z, if anything, is of greater extent and less intent than Y.

218. There are processes which appear like transpositions, but are not so in reality. Thus X))PQ certainly gives XP))Q: here is a universal proposition, in which the element of a full tension is transposible. But not transposible within the description in § 216, in which it is affirmed that the proposition after transposition is identical with the proposition before transposition. This is not the case here; for though X))PQ gives XP))Q, yet XP))Q does not give X))PQ. Here, since X))PQ gives X))Q and X))P, the term XP is really X. And further, since X))PQ gives X))Q from whence XR))Q, be R what it may so long as XR has existence, the deduction of XP))Q from X))PQ is a case of something different from mere transposition: for P, in XP))Q, may be changed into anything else.

219. The dismissal of the elements of terms comes under what may be called the *decomposition of propositions.* When the elements of both terms are of the full tension, the proposition is a compound of $m \times n$ propositions, if m and n be the numbers of elements in the two terms. Thus A, B))CD gives and is given by the four propositions A))C, A))D, B))C, B))D.

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220. Species, external, deficient, coinadequate
Dependent, repugnant, inessential, inalternative

must carry the notion of full extent and vague intent. For example, the universe being *England*, *farmer* is a deficient of *landowner:* of all the class* *farmer,* no part is identical with a certain part of the class *landowner.* To know this by extent I must know the whole class *farmer:* but to know it by intent, I need not know all the attributes of the notion *farmer.* Let there be *but one* of these attributes which is not an essential of *landowner,* and the proposition is established.

Genus, complement, exient, partient
Essential, alternative, independent, irrepugnant

must carry the notion of vague extent and full intent. The symbols will here help the memory of those who have fully connected them with the words.

* The student must, in any one proposition, be on his guard against thinking inconsistently of *class* and of *attribute.* Either of these modes of thought may be chosen, but not both together, unless the attribute be made to distinguish the class, without exceptions. For a remarkable instance, take the word *gentleman:* what different things people usually mean, according as they are speaking by notion of class or of attribute; the common attribute excludes a percentage of the class, and admits many who are not of the class. The reader may be puzzled to make out the text, unless his *character* of landowner correspond to his *class.*

221. The rules of § 50–52 must be translated as follows. A vague term in the conclusion takes extent or intent (scope or force) as follows.

1. In *universal syllogisms,* if one term of conclusion be of vague scope or force, it has the scope or force of the other; if both, one has the scope or force of the whole middle term, the other of its whole contrary.

2. In *fundamental particular syllogisms,* the vague term or terms of the conclusion take scope or force from the vague premise.

3. In *strengthened particular syllogisms,* the vague term or terms of conclusion take scope or force from the whole middle term or from its whole contrary, according to which is of full scope or force in both premises.

222. For example, (actual) farming depends on occupation of land (see the caution in § 191, often wanted in reference to this

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very instance); and occupation of land is an essential of county respectability: therefore farming and such respectability are in-alternative. Here the terms of conclusion are both of vague intent or force, and the middle term of full intent: the force is precisely so much as is contained in the notion of *occupying land*. Any component either of actual farming or of county respectability which can be possessed by a non-occupier of land is of no import in the conclusion as from the above premises.

Take the mathematical form of the above:—Farmers are a species of occupiers of land; the county respectables are a species of occupiers; whence the farmer is a coinadequate of the county respectable, or both together do not make up the whole universe (that is, as implied, the population of the county). Here the contrary of the middle term, the class of *non-occupiers* of land, forms the extent of coinadequacy of the terms of conclusion implied in the premises.

223. The admission of complex terms, and of copular relations more general than the word of identification *is*, enable us to include in common syllogism all the cases known as hypothetical syllogisms, conditional syllogisms, disjunctive syllogisms, dilemmas, &c. I shall merely take a few cases of these.

If P be true, Q is true; but P *is* true, therefore Q is true. This is an *hypothetical* syllogism, so called. To reduce it to a common syllogism between 'Q', 'true', and a middle term, we have 'Q' is 'a proposition true when P is true'; 'A proposition true when P is true' is 'true' (because P is true); therefore Q is true. Many other ways might be given. In truth, though the reduction is possible, the law of thought connecting hypothesis with necessary consequence is of a character which may claim to stand before syllogism, and to be employed in it, rather than the converse. But the discussion of this subject is not for a syllabus: see § 226. In a similar way may be treated 'If P be true, Q is true; Q is not true: therefore P is not true'.

224. Say that P is either A, or B, or C; A is not X, B is not X, C is not X; then P is not X. This is the syllogism

P))A,B,C)·(X giving P)·(X

a common syllogism with the middle term an aggregate.

225. Either P is true, or Q; if P be true, X is true; if Q be true, Y is true; therefore either X or Y is true. The truth is the alternative of the truth of P or Q; which is the alternative

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of the truth of X or Y; therefore the truth is the truth of the alternative of X or Y.

Various other instances will be found in my *Formal Logic*, pp. 115–125.

226. In all syllogisms the *existence* of the middle term is a *datum*. If the conclusion be false, the syllogism being logically valid, and the premises true if the terms exist, then the non-existence of one of the terms is the error. And if the terms which remain in the conclusion be existent, the nonexistence of the middle term may be inferred. When the syllogism is subjective in character, the transition into the objective syllogism frequently hinges on this point. Suppose success in a certain undertaking, such success being *conceivable*, depends both upon X and upon Z: then X and Z are not subjectively repugnant. Suppose that in objective reality they are repugnant: their coexistence being a thing wholly unknown and incredible. It follows then that success is objectively unattainable; impossible as things are, people say. The metaphysical premises X((Y))Z, X, Y, Z, being conceivable, give X()Z: and if X and Z have objective existence, and X)·(Z, it follows that Y does not exist; for if it did, the premises X((Y))Z would give X()Z. Suppose a qualification which depends both upon natural talent and early training; and suppose the talent to be one which cannot be developed early, *as things go;* then, as things go, the qualification is unattainable.

227. The remaining logical whole of which we have to consider the parts is *belief*. This feeling is one the magnitude of which ranges between two extremes; *certainty for*, such as we have as to the proposition 'Two and two make four'; and *certainty against*, such as we have as to the proposition 'Two and two make five'. The first has the *whole belief*, or *no unbelief;* the second has *no belief*, and the *whole unbelief*. These extremes are represented by 1 and 0, on the scale of belief: and would be represented by 0 and 1, if we chose (which is not necessary) to have a scale of *unbelief*.

228. That which *may be* or *may not be* claims* a portion of belief and a portion of unbelief: that is, we partly believe in the "*may*" and partly in the "*may not*." Thus if an urn contain 13 white balls and 7 black balls, and nothing else, and I am going to draw a ball without knowing which, and without more belief in

one ball than in another; then my belief in the drawing of white is to my belief in the drawing of black as 13 to 7, that is, 1 representing certainty, I have $\frac{13}{20}$ of belief in a white ball, and $\frac{7}{20}$ in a black ball. This is usually expressed by saying that the *odds* in favour of a white ball are 13 to 7, and the *chances*, or *probabilities*, of the drawing of white or black ball, are $\frac{13}{20}$ and $\frac{7}{20}$. I shall call 13:7 the *ratio of belief* in a white ball, or of unbelief in a black ball; and 7:13 the ratio of belief in a black ball, or of unbelief in a white ball; and 13 and 7 the favourable and unfavourable terms for the white ball.

* Here, as in all other things, there are portions which are too small to be of perceptible effect. Cæsar *may* not have died in the manner stated: he *may*, if there were such a person, which *may* not be true, have been captured by the Britons, and detained in captivity for the rest of his life. But the received history absorbs so much of our belief that we have but a mere atom to divide among all the different ways in which that story may be wrong. There are two opposite fallacious methods of thinking: first, the confusion of high moral certainty with absolute knowledge in right of the nearness of the *quantities* of belief in the two; secondly, the confusion of high moral certainty with matters of practical uncertainty, in right of the want of absolute knowledge in both.

229. Referring to my *Formal Logic* for full explanation on the subject, I shall here only digest a few rules relative to the measures of belief and unbelief, in questions especially relating to logic.

230. Any alteration of our minds with respect to belief or unbelief of a proposition is derived from two sources,—

1. *Testimony*, assertion for or against by those of whose knowledge we have some opinion. This, when absolutely unimpeachable, is *authority;* though this word is used loosely for testimony of high value. Testimony speaks to the thing asserted, to its truth or falsehood; it turns out good if the proposition be true, and bad if the proposition be false.

2. *Argument*, reasoning for or against, addressed to the mind on its own force. This, when absolutely unimpeachable, is *demonstration;* though this word is used loosely for reasoning of great force. Reasoning speaks, not simply to the truth or falsehood, but to the truth as proved in one particular way. If an argument be invalid, it does not follow that the proposition is false, but only that it cannot be established in that one way.

231. When the proposer of an argument believes in its con-

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clusion, he is one of the testimonies in favour of the conclusion, independently of his argument.

232. Among the testimonies to a conclusion must be counted the receiver himself, whose initial state of mind enters as the testimony* of a witness into the mathematical formulæ, though a thing of a very different kind. Suppose that, all circumstances duly considered so far as he is able, the receiver begins with an impression that the proposition in hand has 7 to 3 against it, or (3 : 7) is his ratio of belief at the outset. Upon this belief the future testimonies and arguments are to act: and the mathematical effect is the same as if the first witness bore testimony (7 : 3) against the proposition.

* This is the point on which the mathematical study of this theory throws most light. Simple as the thing may appear, there is not one writer in a thousand who seems to know that the *legitimate* result of argument and testimony depends upon the initial state of the receiver's mind. They request him to begin without any bias; to make himself something which he is not by an act of his own will. Judges request juries to dismiss all that they know about the case beforehand: and this when the juries *know*, and the judges know that they know it, that the mere fact of the prisoner's appearance at the bar is itself three or four to one in favour of his guilt. Now the jury do not dismiss this presumption, because they cannot: and they need not, because the sound remedy against the presumption lurks in their own minds, and is ready to act. It would not be advisable to discuss in a short note the method in which common honesty manages to hit the truth, in spite of prepossession. But I may state my conviction that if the juryman were consciously to aim at being somebody else, that is, a person without any preconceived notion, he would give a wrong verdict far more often than he does. I should recommend him not to think about himself at all, but to forget himself altogether, or at least not to be active in bringing himself before himself; and to listen to the evidence. And further, to remember that the inquiry does not terminate in the jury-box; that the trial of the evidence commences when the jury retire; that the evidence of eleven other men to the character of the evidence is itself part of the evidence; and that the demand for unanimity on the part of the jury is the expression of the determination of the law that the juryman shall be *forced*, if needful, to take other opinions into account. I trust this necessity for unanimity will never be done away with.

233. In assigning numerical value to degrees of belief, we are supposing cases which are nearly as unusual in human affairs as numerically definite propositions (§ 13). But by the study of accurate data, supposed attainable, we analyse the sources of error to which our minds are subject in the rough processes which our state of knowledge obliges us to use.

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234. The method of compounding testimonies is by multiplying together all the favourable numbers for a favourable number, and all the unfavourable numbers for an unfavourable number. For instance, a person thinks it 10 to 3 against an assertion. Two witnesses affirm it, for whose accuracy it is in his mind 7 to 4 and 8 to 3: two witnesses deny it, for whose accuracy it is in his mind 11 to 5 and 3 to 1. What ought to be his state of belief after the testimony?

The several ratios *for* the assertion are

$$3:10,\ 7:4,\ 8:3,\ 5:11,\ 1:3$$

And $3 \times 7 \times 8 \times 5 \times 1 : 10 \times 4 \times 3 \times 11 \times 3$, or $7:33$ is the ratio of belief as it should be after the whole testimony is taken into account: or 33 to 7 against the assertion.

235. When several arguments are advanced on one side of a question, of which the several chances of validity are given, the chance that the side taken is proved, that is, that one or more of the arguments are valid, is as follows. Take the product of the unfavourable numbers for the unfavourable number, and subtract it from the product of the several totals for the favourable number. Thus if three arguments be advanced on one side, the ratios of belief in which are $(4:3)$, $(2:1)$, $(3:7)$, the unfavourable number is $3 \times 1 \times 7$, which subtracted from the product of $4+3$, $2+1$, $3+7$, gives the favourable number. Hence $(189:21)$ or $(9:1)$ is the chance of the side being established by one or more of the arguments.

236. Every argument, however weak, lends some force to its conclusion: for it may be valid, and if invalid does not disprove the conclusion. But it must be remembered that this conclusion is modified by the argument on the other side which arises from the production of weak arguments, or none but weak arguments. Weak arguments from a strong person themselves furnish an argument. If an assertion be true, it is next to certain that very strong arguments exist for it; if such arguments exist, it is highly probable that such and such a person could find them: but he cannot find them; whence there is strong presumption that the arguments do not exist, and from thence that the assertion is not true. This kind of reasoning really prevails, and leads to a rational conclusion that the production of none but weak arguments is a strong presumption against the truth of their conclusion. But when weak arguments are mixed with strong ones,

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they may rather tend to reinforce the conclusion, though the general impression is that they only weaken their stronger companions.

237. If ever an argument be of such nature that according as it is *valid* or *invalid* the conclusion is *true* or *false*, that argument is of the nature of a *testimony*, and must be combined with the rest as in § 234.

238. When testimony and arguments on both sides are to be combined, the result is obtained as follows. Combine all the testimony into one result, as in § 234, all the arguments for as in § 235, and all the arguments against in the same way. Then form the favourable and unfavourable numbers in the ratio of belief required, as follows:—

Favourable number.	*Unfavourable number.*
Multiply together	Multiply together
The favourable number of the testimony	The unfavourable number of the testimony
The unfavourable number of the argument against	The unfavourable number of the argument for
The total of the argument for	The total of the argument against

For instance, testimony giving (7 : 3), argument for, (5 : 2) and argument against, (8 : 1), the ratio of belief for the truth of the assertion should be ($7 \times 1 \times 7 : 3 \times 2 \times 9$) or (49 : 54), that is, it is 54 : 49 against the assertion being true.

239. When testimony is evenly balanced, (1 : 1), it may be altogether omitted. When the arguments for and against are evenly balanced, the arguments may be omitted. When the arguments on both sides are very strong, even though not evenly balanced, the mind may be presumed unable to compare the two very small quantities which they want of certain validity, and the arguments may be treated as evenly balanced.

240. When no argument is offered for, let (0 : 1) represent the ratio of belief which is to be used in the above rule: and the same when no argument is offered against.

241. When testimony is evenly balanced, and argument for is ($m : n$), there being no argument against, we have ($1 \times 1 \times m + n : 1 \times n \times 1$), or $m + n : n$ for the truth of the assertion. Thus, on a matter on which our minds have no bias, an argument which has only an even chance of validity gives 2 to 1 for the truth of the conclusion.

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242. Any one may wisely try a few cases, setting down in each, to the best of his judgment, or rather feeling, his ratios of belief as to testimony, argument for, argument against, and final conclusion. If the last do not agree with the calculation made from the first three, he does not agree with himself. This he may very easily fail to do, for, in such matters of appreciation, one element may have more than justice done to it at the expense of the rest, on the principle laid down in the Gospel of St. Matthew, xxv. 29.

243. The distinction of aggregation and composition occurs in the two leading rules of application of the theory of probabilities. When events are mutually exclusive, that is, when only one of them can happen, the chance that one or other shall happen is found from the separate chances of happening by a rule of aggregation, namely, by addition. But when events are entirely independent, so that any two or more of them may happen together, the chance of all happening is found by applying to the separate chances a rule of composition, namely, multiplication. The connexion of the formulæ of probability with those of logic in general has been most strikingly illustrated by Professor Boole, in his *Mathematical Analysis of Logic*, Cambridge, 1847, 8vo., and subsequently in his *Investigation of the Laws of Thought*, London, 1854, 8vo. In these works the author has made it manifest that the symbolic language of algebra, framed wholly on notions of number and quantity, is adequate, by what is certainly not an accident, to the representation of all the laws of thought.

244. I end with a word on the new symbols which I have employed. Most writers on logic strongly object to all symbols except the venerable *Barbara*, *Celarent*, &c. in § 109. I should advise the reader not to make up his mind on this point until he has well weighed two facts which nobody disputes, both separately and in connexion. First, logic is the only science which has made no progress since the revival of letters: secondly, logic is the only science which has produced no growth of symbols.

Erratum. *Page* 55, *line* 20, *for* will be puzzled *read* will not be puzzled.

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