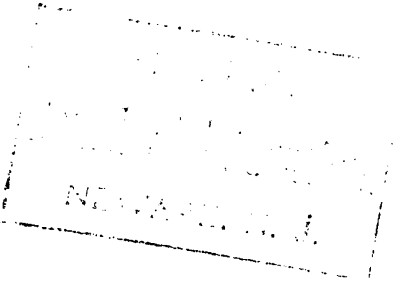

THE MODEL MENTAL ARITHMETIC;

CONTAINING

ORAL EXERCISES

FOR

Graded and Grammar Schools.

BY

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THE

MODEL SERIES OF ARITHMETICS

COMPRISES

I. The Model Primary Arithmetic.

II. The Model Elementary Arithmetic.

III. The Model Common School Arithmetic.

IV. The Model Mental Arithmetic.

These text-books on Arithmetic are published in the belief that they are based upon correct principles, and that a carefully matured plan is consistently adhered to throughout the series.

Progress from the easy to the difficult is gradual, but sure and effective; inductive oral exercises render plain the principles underlying every operation in written Arithmetic; and these principles the pupil is required to apply, as soon as they are learned, in the solution of problems varied and practical, and adapted to his understanding and his stage of advancement.

J. FAGAN & SON,
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PREFACE.

THE present work is designed to review the principles of the fundamental rules presented in the author's Primary Arithmetic, and to extend, by further illustrations and analyses, those included in the Elementary and the Common School Arithmetic. It is, however, complete in itself, and may be used independently or in connection with any series of Arithmetics.

The only essential difference between oral and written arithmetic is, that oral or mental arithmetic implies the performance of computations without the aid of written operations or solutions; hence, the arrangement of topics presented in the Written Arithmetics has been observed in this work. The same definitions, so far as they are necessary to a complete understanding of the subject, are given; as are, also, the same formal statements of principles. It is designed that pupils may perceive and understand that mental arithmetic is not a separate and distinct branch of the science of numbers; that it is not a substitute for written arithmetic; but that each is the complement of the other, and that both are but portions of one harmonious whole, which, in its entirety, requires a full and uniform development of all its subordinate parts.

The importance of mental arithmetic as a means of intellectual training can scarcely be overestimated. It ensures ability to catch principles from oral statement; to understand them; to carry them through a course of reasoning, and to apply them practically in the solution of problems. Its object, then, is twofold: First. To develop and strengthen *the thinking* and reasoning faculties. Second. To en-

sure accuracy and rapidity in combining numbers, and promptness in solving mentally such problems as are likely to occur in the every-day business affairs of life.

The plan of this work is intended to aid in the accomplishment of these important results. It unfolds in a natural order the principles underlying every topic presented, and leads by easy gradations and logical analyses to deductions more complex and abstract. The applications of principles are illustrated by numerous and varied examples arranged progressively; the problems and exercises are carefully and systematically classified; reviews combining new principles and methods of solution with those previously learned are placed at the close of almost every section and chapter.

Models of analysis are given, brief and logical in form and correct in expression. While they are offered as the result of long experience in the school-room, it is not expected that they shall be rigidly adhered to; but they will aid pupils to give additional analyses in their own words. The greatest care should always be taken to secure the strictest accuracy in the statement of the various steps by which fundamental truths are evolved; equal care should be taken to create and stimulate in pupils habits of self-dependence; to train them to rely upon principles and reason, rather than upon mere forms; and to think, compare, and investigate for themselves; for knowledge of this subject becomes useful and available only when they can perceive the conditions of any given problem, understand the relations existing between those conditions, and know how to apply principles in solutions necessary to obtain required results.

Answers to the more difficult problems, together with additional models of analysis and forms of solution are given in the Key; in it are offered also some suggestions as to methods of teaching this important branch of Arithmetic, which, it is hoped, will be found useful to the teacher.

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THE

MODEL MENTAL ARITHMETIC.

CHAPTER I.

INTEGERS.

Section I.

Definitions.

1. A ***Unit*** is one, or a single thing.

Thus, one *boy*, one *book*, or *one* is a *unit*.

2. A ***Number*** is a unit, or a collection of units.

Thus, *one chair*, *ten years*, *two*, or *twelve* is a *number*.

3. The ***Unit of a Number*** is *one* of the collection forming that number.

Thus, *one man* is the unit of *six men*; *one* is the unit of *six*.

4. An ***Integer*** is a number whose units are whole or undivided.

Thus, *one*, *three*, *seven days*, *twenty miles* are *integers*.

Integers are also called *Whole Numbers*, or *Integral Numbers*.

5. A ***Concrete Number*** is a number whose units are named.

Thus, *one yard*, *nine pens*, *twelve eggs* are *concrete numbers*.

6. An ***Abstract Number*** is a number whose units are not named.

Thus, *one, nine, twelve, twenty* are *abstract numbers.*

7. ***Similar Numbers*** are numbers whose units are alike.

Thus, *five feet* and *eight feet* are *similar numbers;* also, three ones and six ones; four tens and five tens; six and nine.

8. ***Dissimilar Numbers*** are numbers whose units are not alike.

Thus, *five feet* and *eight pints* are *dissimilar numbers;* also, three ones and six tens; four tens and eight hundreds.

9. ***Arithmetic*** is the science of numbers and the art of computing by them.

Note. — As a *science*, arithmetic treats of the principles, properties, and relations of numbers. As an *art*, it teaches how to apply the principles of numbers to practical purposes, or to business affairs.

10. ***Oral*** or ***Mental Arithmetic*** is the process of solving problems without the aid of written characters.

11. ***Written Arithmetic*** is the process of solving problems with the aid of written characters or figures.

12. A ***Problem*** is a question which requires a solution.

13. A ***Principle*** is a general truth upon which a process of solution is based.

14. An ***Analysis*** is a statement of the different steps in the solution of a problem.

Exercises.

1. How many units in one? In three? In nine? In one day? In two feet? In six pounds?

2. Two is a collection of how many units? Four? Six months? Five dollars? Seven days? Nine quarts?

3. What is the unit of two cents? Of four miles? Of three? Of five? Of eight inches? Of twelve? Of nine tons?

4. Is three yards a concrete, or an abstract number? Why? Three? Five hours? Two? Seven gallons? Ninety?

5. Are two dozen and four dozen similar, or dissimilar numbers? Why? Fourteen and sixty? Five minutes and seven bushels? Six tens and eight ones? Eight tens and eighty?

6. Name three units. Three numbers. Three concrete numbers. Three abstract numbers. Three similar numbers. Three dissimilar numbers. Three numbers that are concrete and similar. Three abstract and similar. Three concrete and dissimilar.

Section II.

NOTATION AND NUMERATION.

1. How many single things are one and one? Two and one? Three and one? Six and one? Eight and one?

2. How many single things, or units, are in a group of ten? Of ten and one? Ten and two? Ten and three? Ten and five? Ten and seven? Ten and nine?

3. How many single things, or units, are in two groups of ten single things each? In three groups of ten each? In four groups? In six? Eight? Nine?

4. How many single things, or units, are in ten groups of ten single things each? In ten groups and one group of ten each? In ten groups and five groups?

15. In naming numbers, objects are considered as being collected into *groups of ten*, the groups increasing in size or value tenfold, and receiving different names.

16. In naming numbers, a *single thing* is called ***one.***

A collection of *ten ones* considered as a single group is called ***one ten.***

A collection of *ten tens* considered as a single group is called ***one hundred.***

Ten hundreds taken together are called ***one thousand;*** *ten thousands* are called ***one ten-thousand;*** *ten ten-thousands* are called ***one hundred-thousand;*** etc.

17. A group of a *thousand ones* is called a ***thousand;*** a group of a *thousand thousands* is called a ***million;*** a group of a *thousand millions* is called a ***billion;*** etc.

18. Orders of Units are kinds or classes of units formed by grouping together ten lower or smaller units.

Thus, *ones* are units of the *first order; tens*, of the *second order; hundreds*, of the *third order; thousands*, of the *fourth order;* etc.

19. A ***Period*** is a group of three orders of units, beginning with ones.

20. The ***First,*** or ***Ones' Period,*** is formed of the first, the second, and the third orders of units, and comprises *ones, tens,* and *hundreds.*

21. The ***Second,*** or ***Thousands' Period,*** is formed of the fourth, the fifth, and the sixth orders, and comprises *thousands, ten-thousands,* and *hundred-thousands.*

22. The ***Third,*** or ***Millions' Period,*** is formed of the seventh, the eighth, and the ninth orders, and comprises *millions, ten-millions,* and *hundred-millions.*

23. In writing numbers, or expressing numbers by written characters, three methods are used: —

1. By ***words;*** as one, five, ten, etc.
2. By ***letters;*** as I, V, X, etc.; called the *Roman Method.*
3. By ***figures;*** called the *Arabic Method.*

FIGURES.— *0, 1, 2, 3, 4, 5, 6, 7, 8, 9.*

NAMES.—*naught, one, two, three, four, five, six, seven, eight, nine.*

24. The name of a figure denotes the number of units which it represents.

Thus, *0*, named *naught*, or *cipher*, denotes *nothing*, or the absence of number; *2* is called *two*, and denotes *two units; 3* is called *three*, and denotes *three;* etc.

25. Units of the first order, or ***ones,*** are expressed by writing the figures having the same name as the number itself.

Thus, *one* is expressed by *1; two*, by *2; six*, by *6;* etc.

26. A ***unit of the second order,*** or ***one ten,*** is expressed by writing the figure 1 with a cipher after it; thus, *10.*

In the same manner are expressed

Two tens, or twenty,	*20.*	*Six tens, or sixty,*	*60.*
Three tens, or thirty,	*30.*	*Seven tens, or seventy,*	*70.*
Four tens, or forty,	*40.*	*Eight tens, or eighty,*	*80.*
Five tens, or fifty,	*50.*	*Nine tens, or ninety,*	*90.*

27. The numbers between ten and twenty are expressed by writing 1 with the number of ones after it; thus,

Eleven,	*11.*	*Fourteen,*	*14.*	*Seventeen,*	*17.*
Twelve,	*12.*	*Fifteen,*	*15.*	*Eighteen,*	*18.*
Thirteen,	*13.*	*Sixteen,*	*16.*	*Nineteen,*	*19.*

The numbers between twenty and thirty are expressed by writing 2 with the number of ones after it.

Thus, *twenty-one, 21; twenty-five, 25; twenty-nine, 29;* etc.

28. The numbers between ten and one hundred are expressed by writing the figure denoting the number of tens in the left-hand or second place, and the figure denoting the number of ones in the right-hand or first place.

29. When a number is expressed by two figures written side by side, the right-hand figure denotes *ones*, and the left-hand figure denotes *tens.*

30. A ***unit of the third order,*** or ***one hundred,*** is expressed by writing the figure 1 with two ciphers after it; thus, *100.*

In the same manner are expressed

Two hundred,	*200.*	*Six hundred,*	*600.*
Three hundred,	*300.*	*Seven hundred,*	*700.*
Four hundred,	*400.*	*Eight hundred,*	*800.*
Five hundred,	*500.*	*Nine hundred,*	*900.*

31. The numbers between one hundred and two hundred are expressed by writing 1 with the number of tens and ones after it; thus,

One hundred and five	is written	*105.*
One hundred and twenty-three	"	*123.*
One hundred and seventy-nine	"	*179.*

32. The numbers between one hundred and one thousand are expressed by writing the figure denoting the hundreds in the third place, the tens in the second place, and the ones in the first place.

33. When a number is expressed by three figures written side by side, the right-hand figure denotes *ones*, the middle figure denotes *tens*, and the left-hand figure *hundreds*.

34. A ***unit of the fourth order,*** or ***one thousand,*** is expressed by writing 1 with three ciphers after it; thus, *1000.*

In the same manner are expressed

Two thousand,	*2000.*	*Six thousand,*	*6000.*
Three thousand,	*3000.*	*Seven thousand,*	*7000.*
Four thousand,	*4000.*	*Eight thousand,*	*8000.*
Five thousand,	*5000.*	*Nine thousand,*	*9000.*

35. The numbers between one thousand and ten thousand are expressed by writing the figure denoting the number of thousands in the fourth place, the hundreds in the third place, the tens in the second, and the ones in the first; thus,

One thousand two hundred and five	is written	*1205.*
Five thousand four hundred and sixty	"	*5460.*
Nine thousand eight hundred and seventy-six	"	*9876.*

36. A unit of the fifth order, or *one ten-thousand*, is expressed *10000; one hundred-thousand, 100000;* one million, *1000000;* etc.

37. The orders of units are denoted by the ***places*** in which the figures used to express a number stand.

38. A figure in the *first place* denotes ones, or *units of the first order;* in the *second place* tens, or *units of the second order;* in the *third place* hundreds, or *units of the third order;* etc.

Thus, in the number *987*, 7 denotes *7 ones*, or units of the first order; 8 denotes *8 tens*, or units of the second order; and 9 denotes *9 hundreds*, or units of the third order. The number is read *nine hundred and eighty-seven.*

Definitions.

39. ***Notation*** is the method of expressing numbers by figures.

Thus, the expressing of the number *three hundred and fifty-seven* by the figures *357* is the *notation* of three hundred and fifty-seven.

40. ***Numeration*** is the method of reading numbers that are expressed by figures.

Thus, the naming or reading of *468* as *four hundred and sixty-eight* is the *numeration* of 468.

41. ***Significant Figures*** are the figures which signify or denote a number of units.

The significant figures are 1, 2, 3, 4, 5, 6, 7, 8, 9.

42. The ***Value*** of a figure is the number of units which it represents.

Thus, the value of *3* is *three* ones; of *9* is *nine* ones; etc.

43. Significant figures have two values: a *simple value* and a *local value.*

44. The ***Simple Value*** of a figure is the value which it expresses when taken alone.

The ***Local Value*** of a figure is the value which it expresses when combined with other figures.

45. The *simple value* of a figure depends only upon the *number* of units that it expresses, while the *local value* depends upon both the *number* and the *order* of units that it expresses.

46. The simple value of a figure is always the same; but the local value varies according to the place in which it stands.

Thus, the simple value of *2* is *two ones* or *two;* in *20*, the simple value of *2* is two units, but its local value is *two tens*, or *twenty;* and in *200*, the local value of *2* is *two hundreds*, or *two hundred.*

47. The method or system of grouping into tens to express numbers is called the ***Decimal System,*** from the Latin word *decem*, which signifies *ten.*

48. A ***Scale*** in arithmetic is a succession of units increasing and decreasing in value according to a fixed law.

49. A ***Decimal Scale*** is a scale in which the orders of units increase uniformly by the multiplier 10, and decrease uniformly by the divisor 10.

50. The Arabic method of expressing numbers is based upon the decimal scale, and is called the ***Arabic*** or ***Decimal System of Notation.***

Orders and Periods.

NAMES OF PERIODS.	Millions' Period.			Thousands' Period.			Ones' Period.		
NAMES OF ORDERS.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Ones.
	9	8	7	6	5	4	3	2	1
PERIODS.	Third Period.			Second Period.			First Period.		

The fourth period is billions, and the fifth is *trillions.*

51. Principles.

I. *Ten units of any order are one unit of the next higher order.*

II. *The successive orders of units increase in value tenfold from right to left.*

Exercises.

1. How many ones are one ten? How many tens are one hundred? How many hundreds are one thousand? One thousand is how many hundreds? One hundred is how many tens? One ten is how many ones?

2. Name the units of the first order; the units of the second order; of the third order; of the fourth; the fifth. What order are ones? Tens? Thousands? Hundreds?

3. How many figures are needed to express ones? Hundreds? Ten-thousands? What does a number expressed by one figure denote? By three figures? By two? By four?

4. In what place in a number expressed by figures are ones written? Tens? Thousands? Hundreds? What order of units does a figure written in the first place denote? In the third place? In the second? In the fourth?

5. How is any number of ones expressed? How is ten expressed? A hundred? A thousand? Any number between ten and twenty? Between ten and one hundred? Between one hundred and one thousand? When a number is expressed by one figure, what does it denote? By three? By two? By four?

6. Tell what order of units each figure in the following numbers denotes; tell how many of each order; read each number, and write it in words: —

10	*11*	*100*	*908*	*1000*	*8190*	*4000*
20	*24*	*200*	*706*	*2000*	*7802*	*5100*
30	*36*	*310*	*514*	*5900*	*6307*	*6290*
50	*88*	*420*	*327*	*7800*	*5645*	*7803*
70	*97*	*540*	*139*	*9710*	*4938*	*8172*

7. Express by figures the numbers: —

Thirty; fifty; seventy; forty-two; sixty-four; eighty-seven; two hundred; four hundred; six hundred; one hundred and ninety; nine hundred and twenty; eight hundred and thirty; three hundred and nine; seven hundred and five; six hundred and three.

One thousand; nine thousand; two thousand eight hundred; eight thousand three hundred; three thousand seven hundred and ten; seven thousand two hundred and ninety; five thousand seven hundred and twenty; six thousand eight hundred and five; eight thousand five hundred and three; seven thousand and fifty-three; nine thousand and seventy; four thousand and six.

8. Name the different numbers that can be expressed by combining *3*, *4*, and *5*.

52. Model. — The three figures *3*, *4*, and *5*, may express the numbers *345*, *354*, *435*, *453*, *534*, and *543*.

Name the different numbers that can be expressed by

1 and 9	*2 and 8*	*2, 9, and 1*	*1, 3, and 0*	*9, 0, and 2*
3 and 7	*9 and 2*	*4, 0, and 9*	*2, 8, and 6*	*7, 3, and 4*
5 and 3	*3 and 6*	*6, 8, and 3*	*9, 5, and 0*	*6, 0, and 7*
7 and 2	*7 and 4*	*8, 0, and 7*	*8, 6, and 7*	*8, 5, and 5*
9 and 5	*4 and 8*	*3, 7, and 5*	*7, 1, and 0*	*5, 1, and 3*

Section III.

ADDITION.

1. How many boys are 3 boys and 2 boys? 4 boys and 5 boys are how many boys?

2. What number contains as many ones as 4 ones and 6 ones? As many as 6 ones and 5 ones?

3. What number is produced by uniting the ones in 3 ones and 7 ones? In 7 ones and 4 ones?

4. If George is 8 years old, and Charles is 9 years old, what is the sum of their ages?

5. What sum of money equals the number of dollars in 4 dollars and 8 dollars united?

6. What is the unit of 10 pints? Of 9 pints? Are the *units* of 10 pints and 9 pints like, or unlike?

7. Can 10 pints be added to 9 pints? What is the sum of 10 pints and 9 pints? The unit of their sum?

8. What is the unit of 6 yards? Of 12 pounds? Are their units like, or unlike? Can they be united in one sum? Why?

In each of the preceding questions, two or more numbers are given to find one number which shall contain as many units as are in all the numbers taken together.

Definitions.

53. ***Addition*** is the process of uniting two or more numbers to find their sum.

54. The ***Sum,*** or ***Amount,*** is the number obtained by adding two or more numbers together.

55. The numbers that are to be united to form the sum are called ***Parts.***

Thus, *5 days*, *7 days*, and *9 days* are the *parts* whose *sum* is *21 days.*

The sum always contains as many units as are in all the parts taken together.

56. The ***Sign of Addition*** is a short upright cross, +. It is named *plus*, and means *more.*

When the sign of addition is placed between two numbers, it shows that the number placed after it is to be added to the number placed before it.

Thus, $9 + 8$ is read *9 plus 8*, and means that 8 is *to be added to* 9.

57. The ***Sign of Equality*** is two short horizontal lines, =. It is read *equals*, or *equal.*

Thus, 18 pounds + 9 pounds = 27 pounds is read 18 pounds *plus* 9 pounds *equal* 27 pounds, and means that the sum of 18 pounds and 9 pounds is 27 pounds.

58. Principles.

I. *Only similar numbers can be added.*

II. *Only like orders of units can be added.*

III. *The sum is a number similar to the parts added.*

1. When every Part is less than 10.

1. Frank spent 3 cents for a pear and 4 cents for an orange. What did he spend for both?

59. ***Analysis.*** — If Frank spent 3 cents for a pear and 4 cents for an orange, for both he spent the sum of 3 cents and 4 cents, which is 7 cents. Hence, etc.

2. If I have 5 dollars and earn 3 dollars more, how much money will I then have?

3. Mary gave 4 cents for some paper and 5 cents for a lead-pencil. What did she give for both?

4. A grocer sold to Emma 6 pounds of white sugar and 3 pounds of brown sugar. How many pounds did she buy?

5. A boy found 4 eggs in one nest and 6 eggs in another. How many eggs did he find in both nests?

6. Harry walked 6 miles one morning and 5 miles in the afternoon. How far did he walk that day?

7. If a ball costs 7 cents and a top costs 5 cents, how much do both cost? If the top costs 4 cents?

8. After I had spent 6 dollars for a coat, I had 6 dollars left. How much money had I at first?

9. How much must be paid for a barrel of flour worth 7 dollars, and a ton of coal worth 6 dollars?

10. A tinsmith made 7 cans for one milkman and 8 cans for another. How many cans did he make for both?

How many are

11. 5 cents and 7 cents?	*16.* 6 hours and 7 hours?
12. 5 boys and 5 boys?	*17.* 7 sheep and 7 sheep?
13. 4 birds and 7 birds?	*18.* 8 slates and 9 slates?
14. 8 books and 4 books?	*19.* 9 yards and 8 yards?
15. 5 pears and 8 pears?	*20.* 9 horses and 9 horses?

21. James has 3 cents, Harry has 4 cents, and Frank has *5 cents. How* many cents have they all?

60. Analysis. — Since James has 3 cents, Harry has 4 cents, and Frank has 5 cents, they all have the sum of 3 cents and 4 cents and 5 cents.

3 cents and 4 cents are 7 cents, and 5 cents are 12 cents. Hence, if James has 3 cents, etc.

22. A hat cost 4 dollars, a vest 3 dollars, and a coat 6 dollars. What did they all cost?

23. If 6 blackbirds alighted on a tree, 4 on the fence, and 5 on the ground, how many were in the flock?

24. How far does a man go who drives 6 miles one hour, 5 miles the next hour, and 7 miles the next hour?

25. What must be paid for a locket worth 8 dollars, a chain worth 5 dollars, and a bracelet worth 6 dollars?

26. How many bushels of apples are 7 bushels of pippins, 6 bushels of greenings, and 6 of russets?

27. How many quarts of milk are needed to fill a 6-quart can, an 8-quart can, and a 7-quart can?

28. A farmer cut 8 tons of hay from one field, 7 tons from another, and 8 tons from another. How many tons from all?

29. Anna is 9 years old, Mary is 8 years old, and Fanny is 7 years old. What is the sum of their ages?

30. A newsboy bought some daily papers for 8 cents, and some weekly papers for 9 cents. For what must he sell them to clear 9 cents?

Exercises.

1. Add 1 to 1, 2, 3, 4, 5, 6, 7, 8, and 9, and back.

61. Model. — 1 and 1 are 2; 2 and 1 are 3; 3 and 1 are 4; 4 and 1 are 5; 5 and 1 are 6; 6 and 1 are 7; 7 and 1 are 8; 8 and 1 are 9; 9 and 1 are 10; 8 and 1 are 9; etc.

2. Add 2 to 1, 2, 3, 4, 5, 6, 7, 8, and 9, and back.

3. Add 3 to 1, 2, 3, 4, 5, 6, 7, 8, and 9, and reverse.

4. Add 4 to each number from 1 to 9 inclusive, and reverse.

5. Add 5 to each number from 1 to 9 inclusive, and reverse. Add 6 to each number, and reverse. Add 7 to each. Add 8, add 9.

6. Add rapidly 3 and 5; 6 and 4; 5 and 6; 7 and 4; 5 and 7; 7 and 6; 6 and 8; 8 and 7; 7 and 9; 8 and 8; 9 and 8; 9 and 9.

7. Tell promptly the sum of

2 and 3 and 4.	*3 and 4 and 7.*	*5 and 6 and 6.*	*7 and 6 and 9.*
4 and 4 and 4.	*4 and 7 and 6.*	*7 and 6 and 5.*	*8 and 7 and 8.*
4 and 3 and 4.	*7 and 6 and 5.*	*6 and 7 and 6.*	*7 and 9 and 8.*
4 and 6 and 4.	*6 and 5 and 4.*	*8 and 6 and 7.*	*8 and 8 and 9.*
5 and 4 and 6.	*9 and 8 and 3.*	*7 and 8 and 7.*	*9 and 8 and 9.*

2. When one or more of the Parts are greater than 9.

1. If John pays 12 cents for a knife, and sells it at a gain of 6 cents, what does he get for it?

62. Analysis. — If John pays 12 cents for a knife, and sells it at a gain of 6 cents, he gets for it the sum of 12 cents and 6 cents, which is 18 cents. Hence, etc.

2. If I pay 15 dollars for a coat and 7 dollars for a hat, what do both cost me?

3. A dairyman had 18 cows and bought 6 cows more. How many cows had he then?

4. A butcher paid 23 dollars for a cow and 7 dollars for her calf. What did he pay for both?

5. In a shop were 31 men and 8 boys at work. How many persons were at work in the shop?

6. Harry's father is 46 years old. What will be his age in 7 years? In 8 years? In 9 years?

7. Ellen's mother was 56 years old 8 years ago. How old is she now? What will be her age in 7 years?

8. A laborer had 62 dollars in a saving-bank, and put in 8 dollars more. How much had he in then?

9. A farmer paid 76 dollars for a mowing-machine, and 8 dollars for a plow. What did both cost?

10. A farmer had 85 acres of cleared land and 9 acres of *woodland.* How many acres in all had he?

How many are

11. 34 yards and 5 yards?
12. 38 cows and 5 cows?
13. 43 men and 7 men?
14. 50 cents and 8 cents?
15. 58 pounds and 7 pounds?
16. 62 chairs and 9 chairs?
17. 66 birds and 7 birds?
18. 79 miles and 8 miles?
19. 87 acres and 9 acres?
20. 95 acres and 9 acres?

21. A lady bought 15 yards of dress goods, 5 yards of linen, and 8 yards of muslin. How many yards in all?

22. What is the amount of a bill of $34 for parlor carpeting, $7 for oilcloth, and $8 for stair carpet?

23. How much does a merchant receive for a silk dress worth $43, a shawl worth $10, and some cloth worth $7?

24. At what price must an agent sell a sewing-machine that cost 50 dollars, so as to clear 20 dollars?

25. How many pounds of tea are in three chests weighing 50 pounds, 40 pounds, and 30 pounds?

26. How many tons in the sum of 40 tons of egg coal, 35 tons of stove coal, and 9 tons of chestnut coal?

27. A farmer owned 35 cows, and bought 27 cows more. How many cows had he then?

63. Analysis. — Since a drover owned 35 cows, and bought 27 cows more, he then had the sum of 35 cows and 27 cows.

27 cows are 20 cows and 7 cows. 35 cows and 20 cows are 55 cows, and 7 cows are 62 cows. Hence, etc.

28. How many pupils are in a school composed of 36 boys and 29 girls? Of 38 boys and 25 girls?

29. What is the cost of a fence for which I pay 46 dollars for work and 35 dollars for material?

30. How far does a man travel if he goes 48 miles by railroad and 43 miles by stage?

31. How many acres are in three fields containing 30 acres, 15 acres, and 18 acres?

32. What is the sum of 35 gallons of water, 20 gallons, and 17 gallons? Of 22 gallons, 34 gallons, and 48 gallons?

33. Henry has 45 dollars, and James has 37 dollars. If each earn 15 dollars more, how much will each then have?

34. What is the sum of 7, 8, 9? Of the tens in 30 and 44? In 9, 40, and 56? In 15, 35, and 50?

35. How many tens and ones in 31, 40, and 50? In 20, 35, and 46? In 32, 44, and 53? In 42, 54, and 66?

Exercises.

1. Add 1 to 10, 20, 30, 40, 50, 60, 70, 80, and 90, and back. Add 2 to each, and reverse. Add 3. Add 4. Add 5; 6; 7; 8; 9.

2. Add 1 to 11, 21, 31, 41, 51, 61, 71, 81, and 91, and reverse. Add 2 to each, and reverse. Add 3. Add 4. Add 5; 6; 7; 8; 9.

3. Add 1 to 12, 22, 32, 42, 52, 62, 72, 82, and 92, and reverse. Add 2 to each, and reverse. Add 3. Add 4. Add 5; 6; 7; 8; 9.

4. Add 1 to 13, 23, 33, 43, 53, 63, 73, 83, and 93, and reverse. Add 2 to each, and reverse. Add 3. Add 4. Add 5; 6; 7; 8; 9.

5. Add 1 to 14, 24, 34, 44, 54, 64, 74, 84, and 94, and reverse. Add 2 to each, and reverse. Add 3. Add 4. Add 5; 6; 7; 8; 9.

6. Add 1 to each tenth number from 15 to 95 inclusive, and reverse. From 16 to 96 inclusive; 17 to 97; 18 to 98; 19 to 99. Add 2 to each, and reverse. Add 3; 4; 5; 6; 7; 8; 9.

7. Count by 2's from 0 to 10. From 10 to 30. From 15 to 39. From 35 to 63. From 52 to 100. Count by 3's from 0 to 30. From 20 to 50. From 31 to 64. From 45 to 81. From 52 to 100.

8. Count by 4's from 10 to 30. From 20 to 52. From 31 to 71. From 41 to 77. From 52 to 100. By 5's from 0 to 30. From 20 to 55. From 32 to 77. From 43 to 93. From 45 to 100. By 6's from 0 to 30. From 14 to 50. From 23 to 71. From 34 to 100.

9. Add by 7's from 0 to 35. From 21 to 63. From 33 to 89. From 44 to 100. Add by 8's from 10 to 42. From 19 to 67. From 28 to 84. From 20 to 100. Add by 9's from 10 to 46. From 17 to 62. From 25 to 88. From 10 to 100.

10. Add, rapidly, 12 and 10; 20 and 15; 35 and 20; 25 and 17; 32 and 19; 24 and 37; 43 and 28; 36 and 34; 58 and 35; 47 and 45;

63 and 48; 54 and 76; 78 and 55; 68 and 84; 89 and 69; 75 and 87; 94 and 88.

11. Tell, promptly, the amount of

10 and 5 and 6.	*11 and 14 and 13.*	*50 and 26 and 22.*
17 and 7 and 5.	*23 and 15 and 14.*	*62 and 31 and 24.*
25 and 6 and 7.	*35 and 17 and 15.*	*74 and 34 and 30.*
34 and 8 and 8.	*47 and 21 and 17.*	*86 and 40 and 35.*
47 and 9 and 8.	*59 and 25 and 20.*	*98 and 52 and 44.*

Section IV.

SUBTRACTION.

1. How many cents must be added to 4 cents to make 7 cents? To make 9 cents?

2. A vessel that can contain 11 quarts has in it 6 quarts. How many quarts must be added to fill it?

3. 12 pounds are how many more than 5 pounds? 8 pounds are how many less than 13 pounds?

4. What number added to 8 ones makes 14 ones? What number taken from 15 ones leaves 7 ones?

5. How many pints remain after taking 8 pints from 15 pints? 7 pints from 16 pints?

6. If 8 books be taken from a pile of 17 books, how many books will remain? 9 books from 17 books?

7. If Charles is 17 years old, and Henry is but 9 years old, what is the difference of their ages?

8. What is the unit of 17 years? Of 8 years? Are the units of 17 years and 8 years like, or unlike?

9. Can 8 years be taken from 17 years? What is the difference between 17 years and 8 years?

10. What is the unit of 18 yards? Of 9 days? Are the units of 18 yards and 9 days like, or unlike? Can 9 days be taken from 18 yards? Why?

In each of the preceding questions there are given two numbers to find the difference between them, or to find the remainder of the larger after taking away the smaller.

Definitions.

64. Subtraction is the process of taking one of two numbers from the other.

65. The ***Difference,*** or ***Remainder,*** is the number obtained by subtracting one of two numbers from the other.

66. The ***Minuend*** is that one of two numbers from which the other is taken.

67. The ***Subtrahend*** is that one of two numbers which is taken from the other.

Thus, if 12 pounds are taken from 20 pounds, *20 pounds* is the *minuend, 12 pounds* is the *subtrahend,* and *8 pounds* is the *remainder,* or *difference.*

68. The ***Sign of Subtraction*** is a short horizontal line, —. It is named *minus,* and means *less.*

When the sign of subtraction is placed between two numbers, it shows that the number placed after it is to be taken from the number placed before it.

Thus, *20 pounds — 8 pounds* is read 20 pounds *minus* 8 pounds, and means that 8 pounds are *to be taken* or *subtracted from* 20 pounds.

69. Principles.

I. *Subtraction is the reverse operation of addition.*

II. *Only similar numbers, and only like orders of units, can be taken one from another.*

III. *The difference or remainder is a number similar to the minuend and the subtrahend.*

1. When the Subtrahend is less than 10.

1. A basket has in it 3 apples. How many apples must be added that it may contain 7 apples?

70. Analysis. — If a basket has in it 3 apples, that it may contain 7 apples, there must be added 4 apples, because 3 apples and 4 apples are 7 apples. Hence, etc.

2. Ada has 8 cents, and Emma has 5 cents. How many cents more has Ada than Emma?

3. Charles had 6 dollars, and earned enough to make his money equal 10 dollars. What did he earn?

4. James is eleven years old, and his brother five years old. What is the difference in their ages?

5. If a dressmaker earns 12 dollars a week, and spends 7 dollars, how much does she save?

71. Analysis. — If a dressmaker earns 12 dollars a week, and spends 7 dollars, she saves the difference between 12 dollars and 7 dollars, which is 5 dollars. Hence, etc.

6. From a lot of 13 pails, a tinsmith sold 6 pails. How many pails had he remaining?

7. Ellen is 14 years old, and Henry is 7 years younger. What is Henry's age?

8. From a pile of wood containing 15 cords, a farmer sold 6 cords. How many cords remained?

9. A newsboy bought some papers for 9 cents, and sold them for 16 cents. How many cents did he clear?

10. If a freight-train had in it 17 cars, and 8 of them were empty, how many were loaded?

How many are

11. 10 tons less 7 tons?

12. 11 boys less 6 boys?

13. 13 bushels less 5 bushels?

14. 15 years less 8 years?

15. 14 dollars less 7 dollars?

16. 15 cows less 9 cows?

17. 16 cords less 7 cords?

18. 16 baskets less 8 baskets?

19. 17 books less 9 books?

20. 18 melons less 9 melons?

21. Mary had 14 cents, and spent 3 cents for paper and 5 cents for a pencil. How much had she left?

72. Analysis. — Since Mary had 14 cents, and spent 3 cents for paper, she had left the difference between 14 cents and 3 cents, which is 11 cents; and if she then spent 5 cents more for a pencil, she had left the difference between 11 cents and 5 cents, which is 6 cents.

22. From a bin containing 16 bushels were sold 4 bushels for seed and 7 bushels to a miller. How many remained?

23. How much of 17 tons of coal remains after taking away 5 tons and 8 tons? 9 tons and 6 tons?

24. A farmer had 9 cows, bought 7 cows more, and then sold 8. How many had he then?

25. From the sum of 8 and 9 take 8. To the difference between 18 and 9 add 7.

Exercises.

1. Subtract 1 from 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and back.

73. Model. — 1 from 1 leaves 0; 1 from 2 leaves 1; 1 from 3 leaves 2; 1 from 4 leaves 3; 1 from 5 leaves 4; 1 from 6 leaves 5; 1 from 7 leaves 6; 1 from 8 leaves 7; 1 from 9 leaves 8; 1 from 10 leaves 9. 1 from 9 leaves 8; etc.

2. Subtract 2 from 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and reverse.

3. Subtract 3 from 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12, and reverse.

4. Subtract 4 from each number from 4 to 13 inclusive, and reverse.

5. Subtract 5 from each number from 5 to 14, and reverse. 6 from each number from 6 to 15, and reverse.

6. Subtract 7 from each number from 7 to 16, and reverse. 8 from each number from 8 to 17; 9 from 9 to 9 from 18.

7. Subtract, rapidly, 3 from 8; 5 from 9; 4 from 11; 5 from 12; 6 from 12; 6 from 15; 7 from 14; 7 from 16; 8 from 15; 8 from 17; 9 from 15; 9 from 16; 9 from 18.

8. Tell, promptly, the remainder of

11 less 3 less 2.	*13 less 4 less 8.*	*15 less 6 less 5.*
12 less 4 less 3.	*14 less 5 less 6.*	*16 less 8 less 3.*
13 less 2 less 6.	*15 less 4 less 8.*	*17 less 9 less 5.*
14 less 5 less 3.	*16 less 7 less 7.*	*18 less 7 less 6.*
15 less 4 less 5.	*17 less 3 less 9.*	19 less 8 less 5.

2. *When the Subtrahend is more than 9.*

1. By selling a cow for 23 dollars, a farmer cleared 5 dollars. What did the cow cost him?

74. Analysis. — Since a farmer sold a cow for 23 dollars, and cleared 5 dollars, the cow cost him the difference between 23 dollars and 5 dollars, which is 18 dollars. Hence, etc.

2. A man bought a watch for 31 dollars, and sold it at a loss of 5 dollars. What did he get for it?

3. From a cask containing 42 gallons, 7 gallons leaked out. How many gallons were left in it?

4. From a 50-cent note, John received 6 cents in change. How much money had he spent?

5. Alice is 7 years old, and her father is 61 years old. What is the difference in their ages?

6. John's grandfather is now 65 years old. How old was he 8 years ago? How old 6 years ago?

7. The sum of two numbers is 73, and the less number is 7. What is the greater number? If the less is 9?

8. If the minuend is 75 dollars, and the subtrahend is 8 dollars, what is the remainder?

9. If the minuend is 82 miles, and the remainder is 8 miles, what is the subtrahend? If the minuend is 91 miles?

10. What is the remainder of the minuend 93 pounds after taking from it 9 pounds? After taking 8 pounds?

How many are

11. 25 roses less 6 roses?
12. 33 pinks less 5 pinks?
13. 42 lilies less 4 lilies?
14. 54 lilacs less 6 lilacs?
15. 65 shrubs less 7 shrubs?
16. 53 nails less 6 nails?
17. 64 knobs less 8 knobs?
18. 77 doors less 8 doors?
19. 86 panes less 7 panes?
20. 98 locks less 9 locks?

21. A merchant sold some goods that cost him $30 at a loss of $10. What did he receive for them?

22. If a coal-dealer buys 40 tons of coal, and sells 20 tons the same day, how many tons has he left?

23. From a flock of 58 sheep, a drover sold 30 sheep. How many then remained in the flock?

24. Henry's father is 64 years old, and Henry is 40 years younger. How old is Henry?

25. A clerk was paid $35, and he spent $19 for some clothing. What had he remaining?

75. Analysis. — Since the clerk was paid $35, and he spent $19 for some clothing, he had remaining the difference between $35 and $19.

$19 equal $10 and $9; $10 from $35 leave $25; and $9 from $25 leave $16. Hence, etc.

26. A man bought a watch for $42, and sold it for $14 less than the cost. How much did he get for it?

27. If I live 17 more years, I shall be 51 years of age. How old am I now? How old was I 15 years ago?

28. Of 63 passengers on a ferry-boat, 24 were women. How many of the passengers were men?

29. A fence was put up at a cost of $70. If the material cost $35, what was paid for the labor?

30. I exchanged a watch worth $48 for a horse worth $85, and paid the difference in cash. How much did I pay?

31. If a dollar-note is offered in payment for a bushel of potatoes worth 75 cents, what change must be given?

32. What is the remainder of the minuend 83 barrels after taking from it 49 barrels? After taking 54 barrels?

33. What is the excess of the minuend 94 acres over the subtrahend 55 acres? Over the remainder 66 acres?

34. What is the difference between the tens in 30 and the tens in 60? Between the tens and ones in 40 and 75?

35. How many tens and ones in 35 less 8? In 46 less 20? *In 60* less 45? In 75 less 57? In 82 less 63? 91 less 74?

Exercises.

1. Subtract 1 from 10, 20, 30, 40, 50, 60, 70, 80, and 90, and back. Subtract 2 from each, and back. 3, 4, 5, 6, 7, 8, 9.

2. Subtract 1 from 11, 21, 31, 41, 51, 61, 71, 81, 91, and reverse. Subtract 2 from each, and reverse. 3, 4, 5, 6, 7, 8, 9.

3. Subtract 1 from 12, 22, 32, 42, 52, 62, 72, 82, 92, and reverse. Subtract 2 from each, and reverse. 3, 4, 5, 6, 7, 8, 9.

4. Subtract 1 from 13, 23, 33, 43, 53, 63, 73, 83, 93, and reverse. Subtract 2 from each, and reverse. 3, 4, 5, 6, 7, 8, 9.

5. Subtract 1 from 14, 24, 34, 44, 54, 64, 74, 84, 94, and reverse. Subtract 2 from each, and reverse. 3, 4, 5, 6, 7, 8, 9.

6. Count back by 2's from 100 to 90. From 80 to 60. From 95 to 73. From 69 to 45. From 48 to 0. Count by 3's from 100 to 70. From 80 to 50. From 70 to 34. From 67 to 28. From 60 to 0.

7. Count by 4's from 100 to 80. From 90 to 60. From 79 to 47. From 52 to 0. By 5's from 100 to 80. From 80 to 50. From 78 to 38. From 65 to 0. By 6's from 100 to 76. From 90 to 60. From 85 to 43. From 72 to 0.

8. Count by 7's from 100 to 79. From 90 to 55. From 84 to 42. From 77 to 0. By 8's from 100 to 76. From 90 to 50. From 80 to 32. From 72 to 0. By 9's from 100 to 73. From 90 to 54. From 81 to 27. From 72 to 0.

9. Subtract, rapidly, 10 from 30; 20 from 35; 25 from 100; 27 from 50; 17 from 32; 21 from 40; 27 from 52; 35 from 61; 46 from 72; 57 from 75; 68 from 80; 75 from 93; 79 from 90.

10. Tell, promptly, the remainder of

15 less 4 less 5.	*35 less 13 less 15.*	*58 less 21 less 29.*
22 less 5 less 8.	*43 less 15 less 19.*	*64 less 39 less 26.*
31 less 7 less 6.	*57 less 17 less 16.*	*76 less 29 less 38.*
43 less 5 less 8.	*63 less 19 less 17.*	*84 less 35 less 45.*
54 less 6 less 9.	*75 less 18 less 19.*	*91 less 46 less 39.*

Review Problems.

1. The sum of two numbers is 25, and one of them is 18. What is the other number?

2. The difference between two numbers is 17, and the greater number is 25. What is the less number?

3. The difference between two numbers is 27, and the less number is 6. What is the greater number?

4. The remainder is 35 yards, and the subtrahend is 36 yards. What is the minuend?

5. If from $42 I spend $15 for a picture and $8 for a frame, how many dollars have I left?

76. Analysis. — If I spend $15 for a picture and $8 for a frame, I spend for both the sum of $15 and $8, which is $23.

Since I had $42, and spent $23, I have the difference between $42 and $23, which is $19. Hence, etc.

6. A bookseller sold 13 books from a lot of 42 books, and then bought 15 more. How many had he then?

7. A drover had 45 sheep, and sold 18 of them. How many must he buy to increase his flock to 52 sheep?

8. Henry had $35, and spent $16, and then earned enough to have $56. How much did he earn?

9. A dairyman sold 18 cows, then bought 16 cows, and then had 45 cows. How many had he at first?

10. Helen was 17 years of age 9 years ago, and her mother is now 52 years old. What is the difference of their ages?

11. From the sum of 23 yards and 28 yards take their difference. To their difference add their sum.

12. How much less than 80 feet is the sum of 28 feet and 36 feet? How much more than 47 feet?

13. What number must be added to 29 dollars to make an amount of 27 dollars less than 70 dollars?

14. What number must be taken from 70 dollars to make a difference of 27 dollars more than 29 dollars?

15. If two parts of 72 years are 39 years and 18 years, *what is* the other part? If two are 34 years and 29 years?

16. If the sum of three numbers is 83 feet, and two of the numbers are 37 feet and 28 feet, what is the third?

17. A merchant gave $68 for a box of goods, and paid $15 freight. For what must he sell them to clear $18?

18. If a boy sleeps 9 hours of the 24, studies 6 hours, and works 4, how many remain for recreation?

19. Charles had a 25-cent note and a 15-cent note; from them he paid 18 cents for paper and 15 cents for a book. How many remained? How many less than 33 cents?

20. A lady bought a bonnet for $12, a scarf for $5, and some silk for $18, and handed the clerk a fifty-dollar note. How much change did he return to her?

21. I had in bank $90, and gave one check for $25, another for $18, and a third for $39. What remained on deposit?

22. A man bought a horse for $55, received $18 for hiring him out, and paid $24 for his keeping. What is gained or lost by selling him for $58? By selling for $63?

23. A jeweler bought a watch for $48, a chain for $18, and a key for $6, and sold them all for $80. How much did he gain? How much by selling all for $85?

24. A wood dealer had 45 cords of oak wood, and 39 cords of pine. After selling 17 cords of each kind, how many cords remained? After selling 26 cords of each?

25. John had $25, and earned $28 more. James had $85, and spent $28. Which had then the greater amount of money, and how much?

Exercises.

1. Add 18, 3, 7, and 6; 23, 9, 4, and 7; 31, 12, 6, and 8; 41, 19, 8, and 7; 58, 23, 9, and 8; 62, 29, 18, and 8; 73, 38, 25, and 9; 84, 46, 35, and 12.

2. Add, alternately, 4 and 5 three times; 5 and 6 four times; 6 and 7 four times; 7 and 8 five times; 8 and 9 five times; 9 and 10 six times; 12 and 11 four times; 15 and 12 five times; 9, 8, and 7 four times.

3. From 100 subtract 7, 9, 4, and 10; 9, 11, 5, and 8; 12, 6, 9, and 7; *22, 8, 9, and 15; 34, 9,* 16, and 10; 41, 15, 12, and 11; 52, 25, 13, and 10.

4. From 100 subtract, alternately, 4 and 5 three times; 5 and 6 four times; 13 and 9 four times; 15 and 11 three times; 16 and 17 three times.

5. To 15 add 18; subtract 9; add 16; subtract 19; add 11; subtract 13; add 35; subtract 19; add 28; subtract 23; add 62.

6. From 100 take 16; add 9; take 15; add 17; take 26; add 14; take 27; add 25; take 32; add 12; take 24; add 15; take 52.

7. How many are

17 and 15 less 12?	*52 and 29 less 26?*	*42 less 14 and 12?*
19 and 17 less 14?	*63 and 38 less 27?*	*58 less 19 and 15?*
23 and 19 less 16?	*74 and 49 less 34?*	*64 less 27 and 18?*
38 and 22 less 21?	*89 and 51 less 48?*	*71 less 32 and 27?*
47 and 26 less 24?	*98 and 62 less 54?*	*92 less 43 and 34?*

Section V.

MULTIPLICATION.

1. If you spend 5 cents one time and 5 cents another time, how many times do you spend 5 cents?

2. When 5 is taken as many times as there are ones in 3, how many times is it taken?

3. What is the sum of 5 cents + 5 cents + 5 cents? How many cents are 5 cents and 5 cents and 5 cents? 3 times 5 cents?

4. If you take 6 apples 3 times from a fruit dish, how many apples will you take in all?

5. How many peaches are 6 peaches taken or repeated 4 times? 4 apples taken or repeated 6 times?

6. If 7 be placed 5 times in a column and then added, what will be the sum? How many are 5 times 7?

7. How many ones in 5? How many pounds are 8 pounds taken as many times as there are ones in 5?

8. Frank bought 6 oranges for 8 cents each. How many *times 8 cents did they* cost? How many cents?

9. How many dollars are 7 times 8 dollars? 8 times 7 dollars are how many dollars?

10. If you spend 8 dollars 7 times, do you spend more or less than if you spend 7 dollars 8 times?

11. If you buy 8 melons at 10 cents each, to find the cost do you take 10 cents 8 times, or take 8 10 times?

12. What is the unit of 10 feet? Of 7? Of 70 feet, the number produced by taking 10 feet 7 times?

In each of the preceding questions it has been required to find the result of taking one of two numbers as many times as there are ones in the other number.

Definitions.

77. ***Multiplication*** is the process of repeating one of two numbers as many times as there are ones in the other.

78. The ***Product*** is the number obtained by multiplying one of two numbers by the other.

79. The ***Multiplicand*** is that one of two numbers which is repeated or multiplied.

80. The ***Multiplier*** is that one of two numbers by which the other is multiplied.

The multiplier shows *how many times* the multiplicand is repeated or multiplied.

Thus, in the expression 7 times 8 dollars are 56 dollars, *8 dollars* is the *multiplicand*, *7* is the *multiplier*, and *56 dollars* is the *product*.

81. The multiplicand and the multiplier are called the ***Factors*** of the product.

Thus, *8 dollars* and *7* are the *factors* of *56 dollars*; 6 and 9, of 54.

82. The ***Sign of Multiplication*** is a short inclined cross, ×. It is read *multiplied by*.

When the sign of multiplication is placed between two numbers, it shows that the number placed before it is to be multiplied by the number placed after it.

Thus, *10 cents* $\times$ *8* is read 10 cents *multiplied by* 8, and means that 10 cents are *to be taken* or *repeated* 8 times.

83. Principles.

I. *Multiplication is a short method of performing addition.*

II. *The multiplier is always regarded as an abstract number.*

III. *The product is always a number similar to the multiplicand.*

1. When the Factors are less than 10.

1. If in one week there are 7 days, how many days are in 4 weeks?

84. Analysis. — If in one week there are 7 days, in 4 weeks there are 4 times 7 days, which are 28 days. Hence, etc.

2. A miller sold 5 barrels of corn-meal at $4 a barrel. What did he receive for it? How much for 6 barrels?

3. If 6 knives or forks make a set, how many knives are in 5 sets? How many in 6 sets? In 7 sets?

4. If the fare between two cities is $7, how much will the tickets for 4 passengers cost? For 6 passengers?

5. If a horse trots at the rate of 8 miles an hour, how far will he trot in 5 hours? In 6 hours? 7 hours?

6. At 5 cents a pound, how much will 8 pounds of brown soap cost? 7 pounds? 9 pounds?

7. If a dressmaker uses 7 yards of cloth to make a cloak, how many yards are needed to make 6 cloaks?

8. If the fare on a street-car is 5 cents, how much does a conductor collect from 9 passengers?

9. If a ton of coal is worth $8, how much are 9 tons worth? 5 tons? 7 tons? 6 tons? 8 tons?

10. A drayman takes 6 barrels of flour at a load. How many barrels does he take in 9 loads?

11. At $7 a barrel, how much must be paid for 8 barrels of flour? For 7 barrels? 9 barrels?

12. If 7 men can do a piece of work in 9 days, how many days will it take one man to do the work?

13. If 8 men do a piece of work in 8 days, how many men would it take to do the work in 1 day?

14. If 9 horses eat a quantity of oats in 8 days, how many horses would it take to eat it in 1 day?

15. If 8 pipes fill a cistern in 9 hours, how long will it take one pipe to fill it? If 9 pipes fill it in 9 hours?

How many are

16. 4 times 6 days?	*21.* 4 times 9 yards?	*26.* 8 times 6 coats?
17. 7 times 4 hours?	*22.* 6 times 6 trees?	*27.* 6 times 9 vests?
18. 5 times 5 miles?	*23.* 7 times 6 doors?	*28.* 9 times 7 boots?
19. 9 times 3 tons?	*24.* 9 times 5 cents?	*29.* 8 times 8 hats?
20. 5 times 7 feet?	*25.* 7 times 7 girls?	*30.* 9 times 9 pins?

Exercises.

1. Multiply 1 by 1, 2, 3, 4, 5, 6, 7, 8, and 9, and reverse.

85. Model.—1 time 1 is 1; 2 times 1 are 2; 3 times 1 are 3; 4 times 1 are 4; 5 times 1 are 5; 6 times 1 are 6; 7 times 1 are 7; 8 times 1 are 8; 9 times 1 are 9; etc.

2. Multiply 2 by 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, and reverse.

3. Multiply 3 by 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, and reverse.

4. Multiply 4 by 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, and reverse.

5. Multiply 5 by 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, and reverse. Multiply 6 by each, and reverse. Multiply 7 by each; 8; 9.

6. Multiply, rapidly, 4 by 5; by 7; by 9. 6 by 4; by 6; by 8. 5 by 3; by 5; by 7. 7 by 8; by 6; by 4. 9 by 5; by 7; by 9. 8 by 8.

7. Tell, promptly, the result of

5 times 4, less 4.	*7 times 8, and 6.*	*6 times 9, less 8.*
4 times 5, and 5.	*8 times 5, less 6.*	*9 times 7, and 9.*
3 times 6, less 6.	*7 times 7, and 5.*	*8 times 9, less 7.*
6 times 5, and 7.	*8 times 7, less 7.*	8 times 8, and 9.
7 times 4, less 6.	*8 times 8, and 9.*	9 times 9, less 9.

2. When one or more of the Factors are greater than 9.

1. At 10 dollars a barrel, how much will 6 barrels of flour cost? 7 barrels?

2. How many men will be needed to do as much in one day as 10 men do in 9 days?

3. If a dressmaker buys 5 patterns at 20 dollars each, how much does she pay for all? For 6 patterns?

4. How many yards are in 6 pieces of muslin, each containing 30 yards? In 7 pieces? In 8 pieces?

5. What sum of money must be paid for 40 silk hats at 6 dollars each? At 5 dollars? At 7 dollars?

6. An express train ran 50 miles an hour for 7 hours. How far did it go? In 6 hours? 8 hours?

7. At 23 cents a pound, how much must be paid for 5 pounds of butter?

86. Analysis. — Since 23 cents are paid for one pound of butter, for 5 pounds there must be paid 5 times 23 cents.

23 cents equal 20 cents and 3 cents. 5 times 20 cents are 100 cents, and 5 times 3 cents are 15 cents; 100 cents and 15 cents are 115 cents. Hence, etc.

8. If in a roll of carpet there are 36 yards, how many yards are in 4 such rolls? In 5 rolls?

9. At 33 dollars a ton, how much will 5 tons of hay cost? 6 tons? 7 tons? 8 tons? 9 tons?

10. At 36 cents a pound, how much must be paid for 6 pounds of Java coffee? 5 pounds? 7 pounds?

11. If 5 cows average $41, how much must be paid for them all? For 6 cows? 7 cows? 8 cows?

12. How much does a boy earn in 6 days if he earns 45 cents a day? If 43 cents? 46 cents? 48 cents?

13. How much will 48 pair of boots cost at $6 a pair?

87. Analysis. — Since one pair of boots cost $6, 48 pair will cost 48 times $6.

48 equals 40 and 8. 40 times $6 are $240, and 8 times $6 are $48; $240 and $40 are $280, and $8 are $288. Hence, etc.

Note. — In the solution of problems, either factor may, for convenience, be regarded as the *multiplier*, although the product must always be a number *similar* to the *true multiplicand*.

14. At $52 each, how much does an agent receive for 6 sewing-machines? At $53 each? At $55?

15. I paid $56 an acre for a field containing 7 acres. How much did the field cost me? How much at $59?

16. If a goldsmith sells 6 gold watches at an average price of $63, how much does he get for them? At $65?

17. A dairyman bought 7 cows at an average price of $65. What did he pay for them? For 8 cows? For 9 cows?

18. How much must 72 passengers pay for their railroad tickets when the fare is $8 each? When $7 each? $9 each?

19. From a carpet factory were taken 8 rolls of carpeting, each containing 84 yards. How many yards were in all?

20. If an iron foundry uses 92 tons of iron a month, how much will it use in 9 months? How much in 8 months?

What is the result of

21. 4 times 34 bushels?
22. 5 times 37 quarts?
23. 6 times 43 gallons?
24. 5 times 49 pints?
25. 6 times 54 barrels?
26. 7 times 58 rods?
27. 8 times 63 miles?
28. 7 times 74 feet?
29. 8 times 82 yards?
30. 9 times 96 inches?

Exercises.

1. Add by 2's from 0 to 8. How many times have you added 2? From 0 to 18. How many times have you added 2? Add by 3's from 0 to 15. From 10 to 28. From 0 to 30. How many times have you added 3 in each?

Multiply from 1 time 2 to 4 times 2. From 3 times 2 to 8 times 2. From 1 time 2 to 10 times 2, and reverse.

Multiply from 1 time 3 to 5 times 3. From 4 times 3 to 8 times 3. From 1 time 3 to 10 times 3, and reverse.

2. Add by 4's from 0 to 16. From 12 to 32. From 0 to 40. By 5's from 0 to 20. From 10 to 35. From 0 to 50. By 6's from 0 to 18. From 18 to 48. From 0 to 60.

Multiply from 1 time 4 to 5 times 4. From 3 times 4 to 7 times 4. From 1 time 4 to 10 times 4, and reverse.

Multiply from 1 time 5 to 4 times 5. From 4 times 5 to 8 times 5. From 1 time 5 to 10 times 5, and reverse.

Multiply from 1 time 6 to 4 times 6. From 4 times 6 to 8 times 6. From 1 time 6 to 10 times 6, and reverse.

3. Add by 7's from 0 to 21. From 4 to 39. From 0 to 70. By 8's from 0 to 32. From 16 to 56. From 0 to 80. By 9's from 0 to 36. From 9 to 54. From 0 to 90.

Multiply from 1 time 7 to 4 times 7. From 3 times 7 to 8 times 7. From 1 time 7 to 10 times 7, and reverse.

Multiply from 1 time 8 to 4 times 8. From 2 times 8 to 7 times 8. From 1 time 8 to 10 times 8, and reverse.

Multiply from 0 times 9 to 4 times 9. From 3 times 9 to 8 times 9. From 1 time 9 to 10 times 9, and reverse.

4. Multiply 10, 20, 30, 40, 50, 60, 70, 80, and 90 by 1, and reverse. By 2, and reverse. By 3; 4; 5; 6; 7; 8; 9.

Multiply 11, 21, 31, 41, 51, 61, 71, 81, and 91 by 1, and reverse. By 2, and reverse. By 3; 4; 5; 6; 7; 8; 9.

Multiply 12, 22, 32, 42, 52, 62, 72, 82, and 92 by 1, and reverse. By 2, and reverse. By 3; 4; 5; 6; 7; 8; 9.

Multiply 13, 23, 33, 43, 53, 63, 73, 83, and 93 by 1, and reverse. By 2, and reverse. By 3; 4; 5; 6; 7; 8; 9.

Multiply 14, 24, 34, 44, 54, 64, 74, 84, and 94 by 1, and reverse. By 2, and reverse. By 3; 4; 5; 6; 7; 8; 9.

Multiply every tenth number from 15 to 95 by 1, and reverse. By 2; 3; 4; 5; 6; 7; 8; 9. From 16 to 96; 17 to 97; 18 to 98; 19 to 99.

5. Multiply, rapidly, 20 by 5; 4 by 25; 30 by 6; 7 by 35; 37 by 4; 43 by 7; 52 by 6; 8 by 54; 63 by 7; 7 by 68; 69 by 8; 8 by 77; 83 by 8; 9 by 86; 97 by 8; 9 by 98.

3 and 2 and 4; 3, 4, and 5; 4, 4, and 5; 5, 4, and 6; 7, 5, and 6; 6, 5, and 8; 7, 6, and 6; 5, 7, and 8; 7, 6, and 8; 7, 7, and 8; 8, 7, *and 8; 7, 8, and 9;* 9, 8, and 8; 9, 9, and 9.

6. Tell, promptly, the result of

4 times 5, and 8.	*7 times 8, less 23.*	*12 times 5, and 25.*
5 times 7, less 6.	*8 times 6, and 35.*	*23 times 6, less 39.*
4 times 9, and 7.	*7 times 9, less 44.*	*45 times 7, and 43.*
6 times 6, less 8.	*8 times 8, and 57.*	*56 times 8, less 59.*
5 times 9, and 9.	*9 times 8, less 65.*	*85 times 9, and 68.*

Review Problems.

1. The sum of two numbers is 33, and the larger number is 17. What is 5 times the smaller number?

2. If the difference between two numbers is 17, and the smaller is 16, what is 5 times the larger number?

3. If from 41 yards you take 28 yards, what will 5 times the remainder be? 6 times the remainder?

4. What is the difference between 6 times 14 and 7 times 13? The sum of 5 times 15 and 6 times 13?

5. What is the difference between 9 times 8 less 15, and 5 times 9 plus 17? Find their sum. Twice their sum.

6. How much more will a dozen lemons cost at 6 cents each, than a dozen oranges at 4 cents each?

7. If A drives 8 miles an hour, and B drives 6 miles, how much further will A drive in 7 hours than B?

8. If I buy 6 pounds of sugar at 13 cents a pound, what change should I receive if I hand the clerk a dollar?

9. Charles earned 24 dollars, and Henry earned 3 times as many. How many dollars had both?

10. A clerk, who earned $90 a month, spent $65. How much had he saved at the end of 6 months?

11. Edward is 14 years old, and his father is 6 times as old, lacking 37 years. What is the sum of their ages?

12. A merchant cut 15 yards of cloth from a piece containing 43 yards. What was the remainder worth at $5 a yard?

13. How much less than $61 will 6 tons of coal cost at $7 a ton? How much more than $37?

14. A farmer bought 4 plows at $18 each, and offered in payment 8 ten-dollar bills. What change was due him?

15. A grocer bought 16 barrels of flour at $7 a barrel, and sold them at a profit of $18. Find the selling price.

16. How much must be paid for 17 cords of wood at $5 a cord, and 15 tons of coal at $6 a ton?

17. Arthur bought 6 pounds of sugar at 14 cents a pound, and had 21 cents left. How much had he at first?

18. If I buy a book for 50 cents, some paper for 38 cents, and envelopes for 22 cents, what change should I get from a two-dollar note? How much from five 25-cent notes?

19. When cherries are 12 cents a quart, and strawberries are 16 cents, how much more will 7 quarts of strawberries cost than 7 quarts of cherries?

20. John had $24, Louis had 4 times as much less $48, and Frank had twice as much as both together plus $18. How much had each? Find the sum of their amounts.

21. A merchant sold cloth at $6 a yard. A tailor bought at one time 15 yards of it, and 20 yards at another time. What was the amount of his bill?

22. A farmer having 75 acres of land, gave 25 acres to each of his two sons. What was the remainder worth at $50 an acre?

23. Two men started from the same place and walked in the same direction, one going 28 miles a day, and the other 24 miles a day. How far apart were they at the end of 5 days?

24. Two men started from the same place and walked in opposite directions, one going 28 miles a day, and the other 24 miles. How far apart were they at the end of 5 days?

25. Two men were 400 miles apart, and walked toward each other, one going 28 miles a day, and the other 24 miles. *How far apart* were they at the end of 5 days?

Exercises.

1. Add, alternately, 12 and 10 three times; 11 and 13 three times; 13 and 12 four times; 12 and 14 four times; 15 and 12 five times; 13 and 15 five times; 16 and 14 six times; 17 and 16 six times.

2. From 200 subtract, alternately, 10 and 9 three times; 9 and 11 three times; 11 and 10 four times; 12 and 11 four times; 11 and 13 five times; 13 and 12 five times; 12 and 15 six times; 16 and 14 six times.

3. To 12 add 18, subtract 16, multiply by 4, add 18, subtract 59, multiply by 5, subtract 46, add 21, multiply by 6.

4. From 25 take 17, multiply by 4, add 18, subtract 40, add 9, multiply by 4, subtract 38, add 26, multiply by 5, subtract 85, add 34.

5. Multiply 15 by 6, subtract 74, add 15, multiply by 3, subtract 78, add 18, multiply by 4, subtract 97, add 25, multiply by 5, add 75.

6. How many are

4 times 16, and 18?	*5 times 13, less 17?*	*12 times 5, less 24?*
5 times 18, and 24?	*6 times 15, less 28?*	*70 less 5 times 13?*
6 times 22, and 32?	*7 times 23, less 39?*	*15 times 7, and 36?*
7 times 24, and 40?	*8 times 25, less 58?*	*80 and 7 times 16?*
8 times 30, and 55?	*9 times 30, less 61?*	*14 times 6, and 70?*

Section VI.

DIVISION.

1. How many cents are 6 cents less 6 cents? 6 cents less 3 cents less 3 cents? 6 cents less 2 cents less 2 cents less 2 cents?

2. How many times can 6 cents be taken from 6 cents? 3 cents from 6 cents? 2 cents from 6 cents?

3. A can holds 8 pints. How often can 4 pints be taken from it? 2 pints can be taken from it how often?

4. If 5 is taken from 15, and 5 from the remainder, and 5 from the remainder, how often is 5 taken from 15?

5. How many are 3 times 5? 15 is how many times 5? 15 is how many times 3?

6. John has 18 cents. How many times 6 cents has he? How many times 3 cents?

7. How many times 5 pears make 20 pears? 20 pears are how many times 4 pears?

8. How many are 4 times 6 quarts? 24 quarts are how many times 6 quarts? How many times 4 quarts?

9. How many times can 5 pints be taken from 30 pints? How many times 5 make 30? How many 5's in 30?

10. What is the product of 7 times 5 gallons? 35 gallons contain 7 gallons how often? 5 gallons how often?

11. 7 times what number make 42? 42 is 7 times what number? How many 6's in 42? How many 7's?

12. How many times can 8 apples be taken from a pile of 35 apples? How many will remain?

13. How many times can 7 dollars be spent from 58 dollars? How many dollars remain? 8 dollars, how often?

14. When you find how many times 7 dollars are contained in 56 dollars, is the result an abstract, or a concrete number?

15. How many times can 9 feet be taken from 63 feet? How many times 9 feet make 63 feet? 9 times how many feet are 63 feet? How many times do 63 feet contain 9 feet?

In each of the preceding questions it has been required to find how many times one of two numbers can be taken from the other, or to find how many times one of two numbers is contained in the other.

Definitions.

88. ***Division*** is the process of finding how many times one of two numbers is contained in the other.

89. The ***Quotient*** is the number obtained by dividing one of two numbers by the other.

90. The ***Dividend*** is that one of two numbers which is separated or divided.

91. The ***Divisor*** is that one of two numbers by which the other is divided.

The quotient shows *how many times* the divisor is contained in the dividend.

Thus, in the expression 42 quarts divided by 7 quarts = 6, *42 quarts* is the *dividend*, *7 quarts* is the *divisor*, and *6* is the *quotient*.

92. The ***Remainder*** is the part of the dividend left after division.

Thus, if 58 dollars are divided among 8 men, each man receives 7 dollars, and there is a *remainder* of *2 dollars*.

93. The ***Sign of Division*** is a short horizontal line with a dot above and a dot below the middle of it, ÷. It is read *divided by*.

When the sign of division is placed between two numbers, it shows that the number placed before it is to be divided by the number placed after it.

Thus, *80 cents ÷ 8 cents = 10* is read 80 cents *divided by* 8 cents equals 10, and means that 8 cents are contained *10 times* in 80 cents.

94. The ***Object of Division*** is twofold: —

First. — To find how often one of two numbers is contained in the other.

Thus, if it is desired to know how many chairs at 6 dollars each can be bought for 30 dollars, the object is to find *how many times* 6 dollars are contained in 30 dollars; and

Since 6 dollars taken 5 times equal 30 dollars, 6 dollars are contained in 30 dollars 5 times. Hence, at 6 dollars each, 5 chairs can be bought for 30 dollars.

Second. — To separate one of two numbers into as many equal parts as there are ones in the other.

Thus, if it is desired to divide thirty dollars equally among 5 men, the object is to separate 30 dollars into 5 equal parts; and

Since 5 times \$6 are \$30, or \$6 and \$6 and \$6 and \$6 and \$6 are \$30, one of the 5 equal parts of \$30 is \$6. Hence, if \$30 are divided equally among 5 men, each man receives \$6.

95. The ***names of the equal parts*** of a thing or number vary according to the *number* of those parts.

Thus, one of *two* equal parts is named *one half;* one of *three* equal parts is named *one third;* one of *four* equal parts, *one fourth;* etc.

One half is expressed $\frac{1}{2}$; *one third*, $\frac{1}{3}$; *one fourth*, $\frac{1}{4}$; *one fifth*, $\frac{1}{5}$; etc.

96. The equal parts into which a thing or number is divided are called ***Fractions.***

97. In *multiplication*, *both factors* are given to find the *product;* in *division* the *product*, corresponding to the *dividend*, and *one factor*, are given, to find the other factor, corresponding to the *quotient.*

98. Principles of Division.

I. *Division is a short method of performing subtraction, and is the reverse of multiplication.*

II. *The dividend is the product of the divisor multiplied by the quotient.*

III. *The divisor and the dividend are similar numbers.*

IV. *The quotient is always an abstract number.*

1. When the Divisor and the Quotient are less than 10.

1. How many times can 5 pints be taken from 15 pints? From 25 pints? From 30 pints?

2. How many yards are 4 times 5 yards? What is the product of 6 times 4 yards? 5 times 7 yards?

3. How many times 6 quarts are 30 quarts? 35 gallons contain 7 gallons how many times? Why?

4. 28 inches are how many times 4 inches? Why?

99. Analysis. — 28 inches are 7 times 4 inches, because 4 inches can be taken from 28 inches 7 times; or, because 7 times 4 inches are 28 inches. Hence, etc.

5. 20 feet are how many times 5 feet? Why? 20 feet contain 5 feet how many times? Why?

6. A 24-quart can will fill a 4-quart can how often? 24 quarts contain 4 quarts how often? Why?

7. 6 quarts can be put in a 24-quart can how often? 6 quarts are contained in 24 quarts how often? Why?

8. If 35 blocks be separated into groups of 7 blocks each, how many groups will there be? Why?

9. 42 apples were divided among some boys so that each boy had 6 apples. How many boys were there?

10. How many times are 6 days contained in 48 days? 48 days contain 8 days how many times?

11. The product of two factors is 45, and one of the factors is 5. What is the other factor?

100. Analysis. — Since the product of two factors is 45, and one of the factors is 5, the other factor is the number of times 5 must be taken to make 45, which is 9 times. Hence, etc.

12. If the product of two numbers is 49 days, and one of the numbers is 7 days, what is the other number?

13. The product of two numbers is 54 hours, and one of the numbers is 9 hours. What is the other number?

14. If the dividend is 56 pounds, and the divisor is 7 pounds, what is the quotient? If the divisor is 8 pounds?

15. 9 men taken a certain number of times will equal 63 men. How many times must 9 men be taken?

16. If a table costs 9 dollars, how many tables can be bought for 36 dollars?

101. Analysis. — Since one table costs 9 dollars, for 36 dollars there can be bought as many tables as the number of times 9 dollars are contained in 36 dollars, which is 4 times. Hence, etc.

17. An agent received 40 dollars for some books at 5 dollars each. How many books did he sell?

18. If a farmer sells 5 pounds of butter for a dollar, how many dollars will he receive for 45 pounds?

19. A laborer paid 48 dollars for 6 months' rent of his house. How much did he pay per month?

20. If I pay 54 dollars for my winter's supply of coal, at 6 dollars a ton, how many tons do I lay in?

21. A farm of 56 acres was divided into fields of 7 acres each. How many fields were in the farm?

22. At 8 cents a paper, how many papers of carpet-tacks can be bought for 56 cents? For 64 cents? For 72 cents?

23. One man did a certain piece of ditching in 63 days. In how many days could 9 men have done it?

24. If a merchant paid 72 dollars for 9 pieces of goods, what price per piece did he pay? If 81 dollars?

25. If a gentleman pays 81 dollars for 9 weeks' board, how much per week does he pay? If 90 dollars?

How many times

26. 5 feet are 45 feet?	*31.* 7 is 56?	*36.* Is 6 contained in 54?
27. 6 pints are 48 pints?	*32.* 6 is 42?	*37.* Is 7 contained in 63?
28. 7 men are 49 men?	*33.* 7 is 42?	*38.* Is 8 contained in 48?
29. 8 yards are 64 yards?	*34.* 8 is 56?	*39.* Is 9 contained in 72?
30. 8 days are 72 days?	*35.* 9 is 63?	*40.* Is 9 contained in 81?

Exercises.

1. Divide by 2 from 2 in 2 to 2 in 18, and reverse.

102. Model.— 2 in 2 once; 2 in 4 twice; 2 in 6 3 times; 2 in 8 4 times; 2 in 10 5 times; 2 in 12 6 times; 2 in 14 7 times; 2 in 16 8 times; 2 in 18 9 times; 2 in 16 8 times; etc.

2. Divide by 3 from 3 in 3 to 3 in 27, and reverse.

3. Divide by 4 from 4 in 4 to 4 in 36, and reverse.

4. Divide by 5 from 5 in 5 to 5 in 45, and reverse.

5. Divide by 6 every sixth number from 6 to 54, and reverse. By 7 every seventh number from 7 to 63, and reverse.

6. Divide by 8 every eighth number from 8 to 72, and reverse. By 9 every ninth number from 9 to 81, and reverse.

7. Divide, rapidly, 16 by 2; by 4; by 8. 18 by 3; by 6; by 9. 21 by 3; by 7. 24 by 4; by 6; by 8. 36 by 4; by 6; by 9. 40 by 5; by 8. 48 by 6; by 8. 56 by 7; by 8. 63 by 7; by 9. 72 by 8; by 9.

8. Tell, promptly, the result of

4 times 6, divided by 3.	$(16 + 8) \div 6.$	*42 and 14, divided by 8.*
6 times 6, divided by 4.	$(24 + 8) \div 4.$	*85 less 22, divided by 7.*
8 times 3, divided by 6.	$(48 - 12) \div 9.$	*49 and 23, divided by 9.*
9 times 4, divided by 6.	$(54 \div 9) + 28.$	*91 less 19, divided by 8.*
8 times 6, divided by 8.	$(63 \div 7) + 45.$	*36 and 45, divided by 9.*

2. *To Find One of the Equal Parts of a Number.*

1. What is one of the two equal parts of 8?

103. Analysis. — One of the 2 equal parts of 8 is 4, since 4 taken 2 times is 8; or, because 2 is contained in 8 4 times.

2. Find one of the three equal parts of 12; of 18; of 24. One of the four equal parts of 20; of 28; of 36. One of the five equal parts of 35; of 40; of 45.

3. What is one of the 6 equal parts of 24? Of 30? Of 36? Of the 7 equal parts of 42? Of 49? Of the 8 equal parts of 56? Of 64? Of the 9 equal parts of 72? Of 81?

4. How is one of 2 equal parts of a number found? One of 3 equal parts? One of 4? Of 5? Of 6? Of 7? Of 8? Of 9?

5. If any thing or number is separated or divided into 2 equal parts, what is each part called?

104. Analysis.—If any thing or number is separated or divided into 2 equal parts, each part is called one half of that thing or number.

6. If any number is divided into 3 equal parts, what is each part called? If divided into 4 equal parts? If divided into 5 equal *parts,* what is each part called?

7. What is one of six equal parts of a number called? One of 7 equal parts? Of 8 equal parts? Of 9 equal parts?

8. How many cents is one half of 12 cents?

105. Analysis. — One half of 12 cents is as many cents as the number of times 2 is contained in 12, which is 6 times. Hence, one half of 12 cents is 6 cents.

9. What is one third of 15 yards? Of 18 feet? One fourth of 28 gallons? Of 32 quarts? One fifth of $40? Of 45 cents?

10. How much is one sixth of 30? Of 36? One seventh of 42? Of 49? One eighth of 56? Of 64? One ninth of 72? Of 81?

11. How is one half of any number found? One third? One fourth? One fifth? One sixth? One seventh? One eighth? One ninth?

12. If 6 apples be shared equally between 2 girls, what part of 6 apples will each girl have?

13. If 12 apples are divided into 3 equal groups, how many are in each group? If into 4 groups? Into 6 groups?

14. If 21 blocks are divided into 3 equal groups, what part of 21 blocks is in each group? How many are in each part?

15. If 6 tons of coal cost 42 dollars, what is the cost per ton?

106. Analysis. — If 6 tons of coal cost 42 dollars, the cost per ton is one sixth of 42 dollars, which is 7 dollars. Hence, etc.

16. In a large foundry, 48 tons of coal were used in 8 days. How many were used each day?

17. How many horseshoes can a blacksmith set with 56 nails, using 8 nails for each shoe? With 72 nails?

18. If you find how many times $9 are contained in $72, is the result a concrete, or an abstract number?

19. If you find one of 9 equal parts of $72, is the result a concrete, or an abstract number?

20. If you divide $72 among 9 men, to find how many *dollars* each man gets, do you find how often 9 men are

contained in 72 dollars, or do you find one of 9 equal parts of 72 dollars?

How much is

21. One third of 21 feet?	*26.* $\frac{1}{5}$ of 35?	*31.* One fourth of 36?
22. One fourth of 24 days?	*27.* $\frac{1}{6}$ of 48?	*32.* One sixth of 59?
23. One fifth of 35 hours?	*28.* $\frac{1}{7}$ of 49?	*33.* One eighth of 72?
24. One sixth of 48 pints?	*29.* $\frac{1}{8}$ of 64?	*34.* One seventh of 63?
25. One seventh of $49?	*30.* $\frac{1}{9}$ of 72?	*35.* One ninth of 81?

Exercises.

1. Find one half of 2, 4, 6, 8, 10, 12, 14, 16, and 18, and reverse.

107. Model. — One half of 2 is 1; one half of 4 is 2; one half of 6 is 3; one half of 8 is 4; one half of 10 is 5; one half of 12 is 6; one half of 14 is 7; one half of 16 is 8; one half of 18 is 9; one half of 16 is 8; etc.

2. Find one third of 3, 6, 9, 12, 15, 18, 21, 24, and 27, and reverse.

3. Find one fourth of 4, 8, 12, 16, 20, 24, 28, 32, and 36, and reverse.

4. Find one fifth of every fifth number from 5 to 45, and reverse.

5. Find one sixth of every sixth number from 6 to 54, and reverse. The seventh of every seventh number from 7 to 63, and reverse.

6. Find the eighth of every eighth number from 8 to 72, and reverse. The ninth of every ninth number from 9 to 81, and reverse.

7. Find, rapidly, $\frac{1}{3}$ of 21; of 27. $\frac{1}{4}$ of 24; of 36. $\frac{1}{5}$ of 35; of 45. $\frac{1}{6}$ of 42; of 54. $\frac{1}{7}$ of 35; of 56; of 63. $\frac{1}{8}$ of 40; of 64; of 72. $\frac{1}{9}$ of 54; of 63; of 81.

8. Tell, promptly, the result of

$\frac{1}{4}$ *of 24, and 24.*	$\frac{1}{5}$ *of (23 + 17).*	*One fourth of 17 and 19.*
$\frac{1}{5}$ *of 30, and 37.*	$\frac{1}{6}$ *of (29 + 25).*	*One sixth of 51 less 21.*
$\frac{1}{6}$ *of 42, and 45.*	$\frac{1}{7}$ *of (63 — 14).*	*One seventh of 29 and 27.*
$\frac{1}{7}$ *of 49, and 53.*	$\frac{1}{8}$ *of (82 — 18).*	*One eighth of 91 less 27.*
$\frac{1}{8}$ *of 72, and 72.*	$\frac{1}{9}$ *of (90 — 18).*	*One ninth of 58 and 32.*

3. When the Quotient is more than 9.

1. A farmer received 20 dollars for wheat at 2 dollars a bushel. How many bushels did he sell?

2. A clerk received a salary of $60 a month, and spent one third of it for clothes. What did they cost?

3. How many days must a man work to receive 60 dollars at 6 dollars a day? At 5 dollars a day?

4. At 8 dollars each, how many sheep can be bought for 80 dollars? At 5 dollars each?

5. At $6 each, how many tables can be bought for $84?

108. Analysis.—Since each table costs $6, for $84 there can be bought as many tables as the number of times $6 are contained in $84. 84 equals 60 and 24. 6 is contained in 60 10 times, and in 24 4 times; and 10 times and 4 times are 14 times. Hence, etc.

6. How many 4-quart cans are needed to hold 44 quarts of milk? To hold 64 quarts?

7. If a lady pays 88 cents for 4 yards of ribbon, what is the price per yard? For 8 yards?

8. At $5 each, how many sheep can be bought for $50? For $65? For $75? For $80? For $100? For $125?

9. A farmer sold a lot from his farm for $90, and spent one fifth for a plow. What did the plow cost?

10. How many sets of 6 chairs each can be made from 60 chairs? From 96 chairs? 132 chairs? 144 chairs?

11. At $7 a ton, how many tons of coal can be bought for $84? For $105? For $112? For $154?

12. Allowing 7 yards to a suit, how many suits can be made from 91 yards of cloth? From 105 yards?

13. If a painter receives $126 for painting wagons at $7 each, how many does he paint? If he receives $161?

14. If 8 persons earn $176 in a month, what are their average monthly wages? If they earn $224?

15. Henry earned $200, and paid out one eighth of it for board. How much did he pay for his board?

16. At a yearly rent of $5 an acre, how large a farm can a man rent for $175? For $200? For $225?

17. If I buy a lot for $216, and pay one sixth of the cost at each payment until clear, how much do I pay each time?

18. A carpet-weaver put 222 yards of carpet in 6 rolls. How many yards were in each roll?

19. If a farmer raises 231 bushels of wheat from 7 acres, what is the average yield per acre?

20. The fare between two cities being $7, how many passengers can be carried between them for $245?

21. A ferry-boat used 8 tons of coal a day. How long at that rate would 320 tons last? 344 tons?

22. If I pay $328 for 8 months' rent of my house, how much rent per month do I pay? If $360?

23. A dairyman paid $405 for 9 cows. What was the average price paid for them? If he paid $432?

24. A merchant's sales one day amounted to $423, one ninth of which he deposited in bank. How much was put in bank?

How much is

25. One fifth of 325 feet?
26. One sixth of 300 yards?
27. One seventh of 364 tons?
28. One eighth of 424 miles?
29. One ninth of 505 cords?
30. One ninth of 522 pages?
31. One eighth of 648 acres?
32. One seventh of 721 dozen?
33. One sixth of 822 rods?
34. One fifth of 945 bales?

109. In division, the quotient depends upon the relative values of the divisor and the dividend. Hence, any change in either must affect the quotient, as shown in the following illustrations: —

I. *If the dividend is multiplied by any number, the quotient is made greater.*

Thus, if the dividend in the expression $48 \div 8 = 6$ is multiplied by 2, the quotient is $(48 \times 2) \div 8 = 12$, which is 2 times as great as the first quotient, 6.

II. *If the dividend is divided by any number, the quotient is made less.*

Thus, if the dividend in the expression $48 \div 8 = 6$ is divided by 2, the quotient is $(48 \div 2) \div 8 = 3$, which is but half as great as the first quotient, 6.

III. *If the divisor is multiplied by any number, the quotient is made less.*

Thus, if the divisor in the expression $48 \div 8 = 6$ is multiplied by 2, the quotient is $48 \div (8 \times 2) = 3$, which is but half as great as the first quotient, 6.

IV. *If the divisor is divided by any number, the quotient is made greater.*

Thus, if the divisor in the expression $48 \div 8 = 6$ is divided by 2, the quotient is $48 \div (8 \div 2) = 12$, which is two times as great as the first quotient, 6. Also,

V. *If the divisor and the dividend are both multiplied by the same number, the quotient is not changed.*

Thus, if the divisor and the dividend in the expression $48 \div 8 = 6$ are both multiplied by 2, the quotient is $(48 \times 2) \div (8 \times 2) = 6$, which is the same as the first quotient.

VI. *If the divisor and the dividend are both divided by the same number, the quotient is not changed.*

Thus, if the divisor and the dividend in the expression $48 \div 8 = 6$ are both divided by 2, the quotient is $(48 \div 2) \div (8 \div 2) = 6$, which is the same as the first quotient.

From the preceding illustrations are deduced the following:

110. General Principles of Division.

I. The quotient is multiplied —

1. *By multiplying the dividend;* or,
2. *By dividing the divisor.*

II. The quotient is divided —

1. *By dividing the dividend;* or,
2. *By multiplying the divisor.*

III. The quotient is not changed —

1. *By multiplying the dividend and the divisor by the same number.*
2. *By dividing the dividend and the divisor by the same number.*

Exercises.

1. Subtract by 2's from 10 to 0. How many times have you subtracted 2? From 20 to 0. How many times have you subtracted 2? Subtract by 3's from 15 to 0. From 30 to 0. How often have you subtracted 3?

Subtract by 4's from 20 to 0. How many times have you subtracted 4? By 5's from 45 to 0. By 6's from 54 to 0. By 7's from 56 to 0. By 8's from 64 to 0. By 9's from 90 to 0. How often have you subtracted each?

2. Multiply by 2 from 2 times 0 to 2 times 5. From 2 times 0 to 2 times 10, and reverse. Divide by 2 from 2 in 2 to 2 in 10. From 2 in 2 to 2 in 20, and reverse.

Multiply by 3 from 3 times 2 to 3 times 7. From 3 times 0 to 3 times 10, and reverse. Divide by 3 from 3 in 3 to 3 in 15. From 3 in 3 to 3 in 30, and reverse.

3. Multiply by 4 from 4 times 2 to 4 times 6. From 4 times 1 to 4 times 10, and reverse. By 5 from 5 times 2 to 5 times 7. From 5 times 1 to 5 times 10, and reverse. By 6 from 6 times 3 to 6 times 8. From 6 times 1 to 6 times 10, and reverse.

Divide by 4 from 4 in 8 to 4 in 24. From 4 in 4 to 4 in 40, and reverse. By 5 from 5 in 15 to 5 in 35. By 5 from 5 in 5 to 5 in 50, and reverse. By 6 from 6 in 24 to 6 in 48. By 6 from 6 in 6 to 6 in 60, and reverse.

4. Multiply by 7 from 7 times 2 to 7 times 5. From 7 times 1 to 7 times 10, and reverse. By 8 from 8 times 3 to 8 times 7. From 8 times 1 to 8 times 10, and reverse. By 9 from 9 times 2 to 9 times 8. From 9 times 1 to 9 times 10, and reverse.

Divide by 7 from 7 in 14 to 7 in 35. From 7 in 7 to 7 in 70, and

reverse. By 8 from 8 in 24 to 8 in 56. From 8 in 8 to 8 in 80, and reverse. By 9 from 9 in 36 to 9 in 63. From 9 in 9 to 9 in 90, and reverse.

5. Divide, rapidly, 42 by 2; 63 by 3; 168 by 4; 265 by 5; 384 by 6; 525 by 7; 688 by 8; 873 by 9; 891 by 9; 696 by 8; 532 by 7; 390 by 6; 275 by 5; 402 by 6; 546 by 7; 704 by 8.

Find, rapidly, $\frac{1}{2}$ of 48; of 104. $\frac{1}{3}$ of 54; of 108. $\frac{1}{4}$ of 68; of 116. $\frac{1}{5}$ of 85; of 130. $\frac{1}{6}$ of 96; of 192. $\frac{1}{7}$ of 203; of 343. $\frac{1}{8}$ of 416; of 520. $\frac{1}{9}$ of 648; of 756; of 891.

6. Tell, promptly, the result of

$(18 + 12) \div 3.$	$(38 - 18) \div 5.$	$(18 \times 3) \div 6.$	$144 \div (4 \times 2).$
$(24 + 16) \div 5.$	$(55 - 19) \div 6.$	$(32 \times 4) \div 8.$	$216 \div (4 \times 3).$
$(32 + 22) \div 6.$	$(72 - 23) \div 7.$	$(54 \times 6) \div 9.$	$504 \div (6 \times 3).$
$(45 + 18) \div 7.$	$(81 - 25) \div 8.$	$(72 \times 9) \div 6.$	$735 \div (7 \times 5).$
$(52 + 28) \div 8.$	$(90 - 18) \div 9.$	$(96 \times 6) \div 8.$	$960 \div (6 \times 8).$

General Review.

1. I had \$25, spent \$9, and then earned enough to make my money amount to \$33. What did I earn?

2. If a merchant gave \$28 for a case of goods, and paid \$7 cartage, for what must he sell them to clear \$18?

3. A man deposited in bank \$36 and \$18, and gave checks for \$15, \$16, and \$17. What remained in bank?

4. Having six 10-dollar notes and a 5-dollar note, I paid \$28 for a coat, \$13 for a pair of pants, and \$9 for a pair of boots. How much had I left?

5. When lemons are worth 6 cents, and oranges 4 cents, what will a dozen of both cost?

6. Emma bought 7 yards of tape at 7 cents, and 4 yards of ribbon at 12 cents. What change from a dollar-note did she get?

7. What must a butcher pay for 9 sheep at \$3, 8 sheep at \$5, and 10 sheep at \$4?

8. How much is lost by selling 8 writing-desks, that cost *45 dollars, at 5* dollars each?

9. John lives 5 miles from a certain city, his brother 9 miles further, and a cousin 6 times as far as his brother. How far from the city is his cousin?

10. The product of three numbers is 90, and two of the numbers are 5 and 6. What is the other number?

11. The product of three numbers is 240 yards, and two of the numbers are 5 and 8. What is the third?

12. The divisor is 9, the quotient 8, and the remainder is 7. What is the dividend?

13. The divisor is 12, the dividend 116, and the quotient is 9. What is the remainder?

14. The quotient is 12, the dividend is 125, and the remainder is 5. What is the divisor?

15. Five times the sum of two numbers is 175. If one of the numbers is 9, what is one-half of the other number?

16. A man earns $12 a week, and his son $5. How much more will the father earn in 12 weeks than his son?

17. If a man earns $12 while his son earns $5, how much will the son earn while the father earns $120?

18. While a man earns $15, a boy earns $7. How much will both have earned when the man has $135?

19. If 14 yards of cloth are worth $70, and 25 yards of another kind are worth $100, what is the difference in the price per yard paid for the two kinds?

20. What is the cost of 12 tons of coal at the rate of 16 tons for $112? What is the cost of 18 tons?

21. If a man earns $180 in 12 weeks, how many dollars can he earn in 11 weeks at the same rate?

22. When 36 bushels of wheat pay for 9 barrels of flour at $8 per barrel, what is the price of the wheat per bushel?

23. If a man buys twelve acres of land for $240, how must he sell it per acre to double his money?

24. If a man earns $90 in 6 weeks, how much will he earn in one half of 24 weeks?

25. When 12 horses eat 20 bushels of oats in a certain time, how many bushels would 15 horses eat in the same time?

26. In how many days can 12 men earn as much as 15 men earn in 12 days?

27. A farmer bought 8 acres of land at $25 an acre, and 12 acres at $30. What was the average price?

28. A drover bought 12 cows at $30 each, and 18 at $25 each. What was the average price paid?

29. A farmer sold 24 sheep at $2 each, 18 sheep at $3, and 12 at $5. What average price did he get?

30. A boat ran up stream 8 hours at the rate of 12 miles an hour, but returned in 6 hours. How fast did it run down stream?

31. If you buy 60 apples at the rate of 4 apples for 3 cents, and sell them at a cent a piece, what do you clear?

32. William can walk 360 yards in 3 minutes, and Thomas in 4 minutes. How long will it take William to gain 150 yards on Thomas? How far will both have walked?

33. I owe $50 for one bill of goods, and $49 for another. How much of the debt can I pay with 10-dollar bills? How many one-dollar bills will pay the balance?

34. If 18 gentlemen and 24 ladies go on a picnic, and their expenses at $3 each are paid by the gentlemen, how much must each gentleman pay?

35. James has 50 cents, Charles has one-half as much money, and Henry has twice as much as both the others together. How many cents have they all?

36. The sum of two numbers is 55, and their difference is 17. What are the numbers?

111. Analysis. — Since the greater number is made up of the less number and the difference between them, the sum of the greater *and the less is twice* the less plus the difference. Therefore,

If from the sum of the two numbers, 55, the difference, 17, is taken, the remainder, 38, is twice the less number. Now if 38 is twice the less number, the less number is one-half of 38, which is 19; and the greater number is 55 minus 19, which is 36.

37. The sum of two numbers is 69, and their difference is 21. What are the numbers? The sum is 76, and the difference is 28. The sum is $102, and the difference $24.

38. If the sum of two numbers is 95 gallons, and the difference between them is 29 gallons, what are the two numbers?

39. John and James together have 85 cents, and James has 15 cents more than John. How many cents has each?

40. The sum of Henry's age and his father's age is 71 years, and the difference of their ages is 29 years. How old is each?

112. Solve the following problems, and illustrate each by an example: —

1. Given several numbers to find their sum.

2. Given the parts to find the whole.

3. Given two numbers to find their difference.

4. Given the greater of two numbers and their difference to find the less.

5. Given the less of two numbers and their difference to find the greater.

6. Given the minuend and the subtrahend to find the remainder.

7. Given the minuend and the remainder to find the subtrahend.

8. Given the subtrahend and the remainder to find the minuend.

9. Given the sum of two numbers and one of them to find the other number.

10. Given the sum of several numbers and all of them but one to find that one.

11. Given the whole and all the parts but one to find that one.

12. Given two numbers, or two factors, to find their product.

13. Given more than two numbers to find their continued product.

14. Given the multiplicand and the multiplier to find the product.

15. Given the product and the multiplicand to find the multiplier.

16. Given the product and the multiplier to find the multiplicand.

17. Given the product of two factors and one of them to find the other factor.

18. Given the product of several factors and all the factors but one to find that one.

19. Given two numbers to find the quotient.

20. Given the divisor and the dividend to find the quotient.

21. Given the divisor and the quotient to find the dividend.

22. Given the dividend and the quotient to find the divisor.

23. Given the divisor, the quotient, and the remainder to find the dividend.

24. Given the dividend, the quotient, and the remainder to find the divisor.

25. Show that the sum of two numbers plus their difference equals twice the greater. That the sum of two numbers less their difference equals twice the less.

26. Show that addition and subtraction are converse operations. That multiplication and division are converse operations.

27. Show that the *sum* in addition and the *minuend* in subtraction are corresponding terms. That the *parts* in addition and the *subtrahend* and the *remainder* in subtraction are corresponding terms.

28. Show that the *product* in multiplication and the *dividend* in division are corresponding terms. The *multiplicand* and the *divisor*. The *multiplier* and the *quotient*.

Exercises.

1. To 14 add 8; subtract 7; multiply by 5; divide by 3; add 25; multiply by 6; divide by 10.

2. From 32 take 17; add 25; multiply by 7; divide by 8; add 45; subtract 39; multiply by 7; divide by 14.

3. Multiply 24 by 6; divide by 12; add 69; subtract 17; multiply by 8; divide by 16; subtract 19; add 87.

4. Divide 91 by 7; multiply by 8; subtract 26; add 22; divide by 20; multiply by 23; subtract 38; add 31; divide by 12.

5. Divide 104 by 8; add 27; divide by 20; add 35; multiply by 9; subtract 33; divide by 6; add 32; subtract 28; multiply by 4; divide by 18; add 76; subtract 31; multiply by 9; divide by 27.

How many are

$\frac{1}{2}$ *of 4 times 17?*	*5 times* $\frac{1}{5}$ *of 65?*	$\frac{1}{4}$ *of 136, less 17?*
$\frac{1}{5}$ *of 6 times 50?*	*6 times* $\frac{1}{7}$ *of 105?*	*75 less* $\frac{1}{6}$ *of 216?*
$\frac{1}{4}$ *of 8 times 56?*	*7 times* $\frac{1}{8}$ *of 144?*	$\frac{1}{8}$ *of 12 times 48?*
$\frac{1}{6}$ *of 9 times 54?*	*8 times* $\frac{1}{6}$ *of 156?*	*84 and* $\frac{1}{7}$ *of 175?*
$\frac{1}{8}$ *of 7 times 56?*	*9 times* $\frac{1}{9}$ *of 171?*	*15 times* $\frac{1}{9}$ *of 225?*

CHAPTER II.

PROPERTIES OF NUMBERS.

1. Factors, or Divisors.

1. How many are 3 times 1? 5 times 1? 7 times 1?

2. How many ones in 3? In 5? In 7? In 11?

3. What two numbers multiplied together produce 5? 7?

4. What two numbers only can divide 5 without a remainder? What two only can exactly divide 7? 11?

5. What two numbers other than itself and 1 multiplied together produce 6? 10? 15? 21? 35?

6. What two numbers other than itself and 1 can divide 6 without a remainder? 10? 15? 21? 35?

7. Of what number are 2 and 3 the factors? 3 and 3? 2, 2, and 3? 2, 3, and 3? 3, 3, and 5? 3, 4, and 5?

8. Name all the numbers that can divide 12 without a remainder; 15; 18; 20; 24; 28; 30; 36.

9. Name five numbers each of which is produced only by multiplying together itself and 1.

10. Name five numbers each of which is produced by multiplying together other numbers than itself and 1.

Definitions.

113. The ***Factors*** of a number are those numbers which, multiplied together, produce that number.

Thus, *2* and *5* are *factors* of *10;* 2, 3, and 5, of 30.

114. A ***Prime Number*** is a number that has no other factor than 1 and itself.

Thus, *5* is a *prime number;* so, also, are 7, 11, 13, etc.

115. A ***Composite Number*** is a number that has other factors than 1 and itself.

Thus, *12* is a *composite number*, since $12 = 3 \times 4$, or $12 = 2 \times 6$.

116. A ***Prime Factor*** is a prime number used as a factor; and a ***Composite Factor*** is a composite number used as a factor.

Thus, in $3 \times 4 = 12$, *3* is a *prime factor* of *12*, and *4* is a *composite factor.*

117. An ***Exact Divisor*** is a number that divides ***any given number*** without a remainder.

Thus, *3* is an *exact divisor* of *15;* 5, of 20; 7, of 28; etc.

Note. — The terms *factor* and *divisor* differ only in use: *factor* suggesting the process of *multiplication*, and *divisor*, the process of *division*.

11. Name all the prime numbers from 0 to 30.

12. Name all the composite numbers from 0 to 30.

13. Name the prime factors of 12; 18; 20; 24; 28.

14. What are the composite factors of 18? 24? 32? 36?

15. What are the smallest numbers other than 1 that can exactly divide 15? 24? 27? 35?

16. What are the largest numbers other than the number itself that can exactly divide 24? 28? 36? 42? 48?

17. What is the smallest and what is the largest prime factor of 42? Name all the composite factors.

18. Name all the different sets of two factors of which 24 is the product. Of which 36 is the product.

19. Name all the different sets of three factors of which 36 is the product. Of which 48 is the product.

20. What numbers from 40 to 60 are the product of two or more prime factors? Of two or more composite factors? Of a prime and a composite factor?

21. Show that a factor of any number is also a factor of any number of times that number.

22. Name all the different values of articles that can be bought for 48 cents. For 54 cents.

118. Principles.

I. *Every composite number is the product of all its prime factors.*

II. *Every factor of a number is an exact divisor of that number.*

III. *Every number can be exactly divided only by its prime factors and by the product of any two or more of them.*

IV. *Any factor of a number is also a factor of any integral number of times that number.*

Exercises.

Name all the different divisors of 24.

119. Model. — The prime factors or divisors of 24 are 2, 2, 2, and 3. The composite divisors are 2 times 2, or *4;* 2 times 3, or *6;* 2 times 2 times 2, or *8;* and 2 times 2 times 3, or *12.* Hence, all the divisors of 24 are *2, 2, 2, 3, 4, 6, 8,* and *12.*

1. Tell which of the following numbers are prime, and give the reason: 5, 7, 9, 14, 17, 18, 21, 23, 27, 29, 31, 35, 39, 41, 45, 49.

2. Tell which of the following numbers are composite, and give the reason: 6, 8, 11, 15, 17, 19, 25, 33, 37, 42, 51, 53, 57, 59, 61, 69.

Name the prime, and the composite factors, and all the different divisors of

1. 15, 18, and 20.
2. 24, 27, and 32.
3. 36, 40, and 45.
4. 48, 54, and 56.
5. 64, 66, and 70.
6. 75, 80, and 81.
7. $84, 88 feet, and 90 yards.
8. $92, 96 tons, and 98 years.
9. $100, 108 days, and 112 cords.

2. Common Factors, or Divisors.

1. What are the factors of 15? Of 21? Of 33?

2. What are the exact divisors of 15? Of 21? Of 33?

3. What factors, or divisors, are common to 15 and 21?

4. Name a factor, or exact divisor, common to 18 and 24.

5. Name two numbers of which 7 is a common factor.

6. What prime factors have 18 and 27 in common?

7. What is the greatest factor that exactly divides 18 and 27? 24 and 36? 24, 32, and 40?

8. What factor, or exact divisor, is common to 7 and 3 times 7? To 9 and 5 times 9?

9. What factor, or divisor, is common to 6 and 9? To 6 and 9, and 54, their product?

10. What factor, or divisor, is common to 18 and 24, and to 42, their sum? To 18 and 24, and 6, their difference?

Definitions.

120. A ***Divisor,*** or ***Measure,*** is a number that divides any given number without a remainder.

Thus, *6* is a *divisor* of *24*, since 6 divides 24 without a remainder.

121. A ***Common Divisor*** is a number that divides each of two or more given numbers without a remainder.

Thus, *6* is a *common divisor* of *24* and *36*, since it divides each of them without a remainder.

122. The ***Greatest Common Divisor*** is the greatest number that divides each of two or more given numbers without a remainder.

Thus, *12* is the *greatest common divisor* of *24* and *36*, since it is the greatest number that divides each of them without a remainder.

123. Numbers are ***prime to each other*** when they have no common factors, or divisors.

Thus, *7* and *12* are *prime to each other;* also, 9 and 16; 9, 16, and 25.

11. What numbers are exact divisors of 12 feet? Of 16 feet?

12. Show that every common divisor of 12 and 16 is also a divisor of their sum, 28.

13. Show that a common divisor of any two numbers is also a divisor of their sum.

14. Show that every common divisor of 12 and 28 is also a divisor of their difference, 16.

15. Show that a common divisor of any two numbers is also a divisor of their difference.

16. Name four odd numbers of which 7 is the greatest common divisor. Four even numbers.

17. What divisors are common to 20 and 30? To their sum? To their difference? To their product?

18. What is the unit of 14 feet? Of 21 feet? Of the common divisor of 14 feet and 21 feet?

19. Name four numbers of which 3 and 9 are common divisors. Four of which 9 is the greatest common divisor.

20. Tell every number that is a common factor of 18 yards and 24 yards. Of 27 feet and 36 feet.

21. What is the length of the longest pole that will exactly measure 20 feet, 30 feet, and 40 feet?

22. What is the greatest number of men among whom \$24, \$32, and \$40 could be exactly divided, so that each should have a whole number of dollars?

124. Principles.

I. *A factor common to two or more numbers is a common divisor of those numbers.*

II. *The only common divisors of two or more numbers are their common prime factors, and the product of any two or more of them.*

III. *The greatest common divisor of two or more numbers is the product of all their common prime factors.*

IV. *The common divisor of two numbers is also a divisor of their sum and of their difference.*

Exercises.

Name the common prime divisors, the common composite divisors, and the greatest common divisor of 24 and 36.

125. Model. — The prime divisors of 24 are 2, 2, 2, and 3; the prime divisors of 36 are 2, 2, 3, and 3. The common prime divisors of 24 and 36 are *2*, *2*, and *3*. The common composite divisors of 24 and 36 are 2 times 2, or *4*; 2 times 3, or *6*; and 2 times 2 times 3, or *12*. The greatest common divisor of 24 and 36 is the product of all their common prime divisors, 2, 2, and 3, or *12*.

Name the common prime divisors, the common composite divisors, and the greatest common divisor of

1. 21 and 35.	*4. 32 and 48.*	*7. 24, 28, and 42.*	*10. 56 feet, 72 feet.*
2. 27 and 36.	*5. 36 and 60.*	*8. 36, 42, and 48.*	*11. 64 tons, 80 tons.*
3. 28 and 42.	*6. 42 and 63.*	*9. 45, 54, and 63.*	*12. \$54, \$72, \$90.*

3. Common Multiples, or Dividends.

1. What number is the product of the factors 3 and 5?

2. What number can be exactly divided by 3 and 5?

3. Name three products of which 3 is a factor; 5; 7.

4. What are the prime factors of 6 and 2 times 6? Of 8 and 3 times 8? Of 9 and 5 times 9?

5. Of what two numbers is 21 the product? By what two numbers can 21 be exactly divided?

6. Name some product of which 6 and 9 are common factors. Some dividend of which 6 and 9 are common divisors.

7. What is the least number of which 6 and 9 are common factors? The least number of which they can be exact divisors?

8. Name the prime factors of 10 and of 15. What is the product of the different prime factors of 10 and 15?

9. What are the prime factors of the least number that can be exactly divided by 10 and 15?

10. What is the least number of which 4, 6, and 8 are common factors? The least number of which they are common divisors?

Definitions.

126. A ***Multiple,*** or ***Dividend,*** is a number that can be divided by any given number without a remainder.

Thus, *15* is a *multiple*, or dividend, of *5*, since 5 is a factor, or divisor, of 15.

127. A ***Common Multiple*** is a number that can be divided by each of two or more numbers without a remainder.

Thus, *15* is a *common multiple* of *5* and *3*; so, also, is 30.

128. The ***Least Common Multiple*** is the least number that can be divided by each of two or more numbers without a remainder.

Thus, *12* is the *least common multiple* of *3* and *4*, or of *2* and *6*, since it is the least number that can be exactly divided by 3 and 4, or by 2 and 6.

Note. — The terms *multiple* and *dividend* differ only in use: *multiple* suggesting the process of *multiplication*, and *dividend*, of *division*.

11. Name three multiples of 9 feet. Of 12 feet.

12. Show that a number can have any number of multiples.

13. Name three common multiples of 9 feet and 12 feet.

14. Show that two or more numbers may have any number of common multiples.

15. Name the least common multiple of 9 feet and 12 feet.

16. Show that two or more numbers can have but one least common multiple.

17. What is the unit of 10 yards? Of 15 yards? Of their least common multiple?

18. Name two numbers of which 6 and 8 are factors. The least number of which they are common factors.

19. What number is a common multiple of 5 quarts, 6 quarts, and 8 quarts?

20. What is the smallest sum of money that can be made up of 2-cent, 3-cent, 5-cent, or 10-cent pieces?

21. What is the narrowest box in which ribbons 2 inches wide, 3 inches, or 4 inches could be packed?

22. What is the smallest sum of money for which you could buy a number of oranges at 5 cents, 8 cents, or 10 cents each?

129. Principles.

I. *A multiple of a number contains all the prime factors of that number.*

II. *A common multiple of two or more numbers contains all the prime factors of each of those numbers.*

III. *The least common multiple is the least number that contains all the prime factors of each of two or more numbers.*

Exercises.

Name two common multiples, and the least common multiple of 9 and 12.

130. Model. — Since the common factors of a number are also common divisors of that number, a common multiple of 9 and 12 is any number of which they are factors, and they are factors of their product, 9 times 12, or *108;* also, of twice 108, or *216;* etc.

The prime factors of 9 are 3 and 3; the prime factors of 12 are 2, 2, and 3. The least common multiple of 9 and 12 is the least number that is the product of the different prime factors of 9 and 12, or the product of 2, 2, 3, and 3, which is *36.*

Name two common multiples, and the least common multiple of

1. 12 and 15.
2. 15 and 20.
3. 16 and 24.
4. 18 and 30.
5. 25 and 40.
6. 36 and 60.
7. 4 tons, 5 tons, and 6 tons.
8. 6 feet, 8 feet, and 10 feet.
9. $9, $10, and $15.

CHAPTER III.

FRACTIONS.

1. Common Fractions.

1. If a cake is cut into two equal parts, what is each part called? What part of the cake is each part?

2. Into how many halves can an apple be cut? How many halves are in a unit, or in anything?

3. If an apple is divided into four equal parts, what name is given to one of the parts? How many fourths in one?

4. What is one of the six equal parts of a unit named? What are two of the parts named? How many sixths in a unit?

5. Into how many halves can a unit, or anything, be divided? Into how many thirds? Fifths? Sevenths?

6. How many fourths are there in a unit? How many sixths? How many eighths? Tenths?

7. What is meant by one half of a unit? By one fourth? One sixth? One fifth? One seventh? One ninth?

8. What is meant by two thirds of a unit? By three fourths? Three fifths? Five sixths?

9. Which is the greater, one half or one fourth of a unit? One third, or one sixth? One fourth, or one eighth?

10. Which are the smaller parts of a unit, halves or thirds? Fourths, or fifths? Sixths, or eighths? Ninths, or tenths?

Definitions.

131. A ***Fraction*** is one or more of the equal parts of a unit.

Thus, *1 fourth, 2 fifths, 3 eighths* are *fractions.*

132. Fractions are divided into two classes: *Common Fractions* and *Decimal Fractions.*

133. A ***Common Fraction*** is expressed in figures by two numbers written one above the other, with a line between them. Thus,

One half	is expressed	$\frac{1}{2}$.	*Three fourths*	is expressed	$\frac{3}{4}$.
One third	"	$\frac{1}{3}$.	*Five sixths*	"	$\frac{5}{6}$.
Two thirds	"	$\frac{2}{3}$.	*Seven eighths*	"	$\frac{7}{8}$.

134. The two numbers used to express a fraction are called the *Denominator* and the *Numerator.*

135. The ***Denominator*** or *namer* of a fraction is the number written below the line.

It shows into how many equal parts the unit is divided, and *denominates* or *gives name* to the parts.

Thus, *8* is the *denominator* of the fraction $\frac{7}{8}$; it shows that the unit *is divided into 8* equal parts, which are named *eighths.*

136. The ***Numerator*** or *numberer* of a fraction is the number written above the line.

It shows the number of the equal parts of the unit expressed by the fraction.

Thus, 7 is the *numerator* of the fraction $\frac{7}{8}$; it shows that 7 of eight equal parts of a unit *are expressed* by the fraction.

137. The ***Terms*** of a fraction are its numerator and its denominator.

Thus, 7 and 8 are the *terms* of the fraction $\frac{7}{8}$.

138. A ***Fractional Unit*** is one of the equal parts into which the unit is divided.

Thus, *1 fifth* is the *fractional unit* of *4 fifths*; $\frac{1}{5}$, of $\frac{4}{5}$; \$$\frac{1}{5}$, of \$$\frac{4}{5}$.

139. Principles.

The less the number of equal parts into which the unit is divided, the greater is the value of each of the parts. And

The greater the number of equal parts into which the unit is divided, the less is the value of each of the parts.

Exercises.

1. Express in figures, or write, the fraction *nine tenths*.

140. Model. — The fraction *nine tenths* is expressed in figures by writing *9* as the *numerator* and *10* as the *denominator*, giving $\frac{9}{10}$, the fraction required.

Notate, or write, the following fractions: —

1. Three fifths.	*6.* Eight tenths.	*11.* Twelve fiftieths.
2. Five sixths.	*7.* Three twelfths.	*12.* Fifteen sixtieths.
3. Six sevenths.	*8.* Six twentieths.	*13.* Twenty sixty-sixths.
4. Two eighths.	*9.* Four fortieths.	*14.* Thirty ninetieths.
5. Three ninths.	*10.* Eight seventieths.	*15.* Forty forty-fifths.

2. Express in words, or read, the fraction $\frac{11}{12}$.

141. Model. —The fraction $\frac{11}{12}$ is read by stating the *number* of equal parts shown by the *numerator*, *eleven*, and then the name of

the equal parts shown by the *denominator, twelfths*, giving *eleven twelfths*, the expression required.

Numerate, or read, the following fractions: —

1. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{2}{5}$; $\frac{5}{6}$.	5. $\frac{3}{10}$; $\frac{4}{13}$; $\frac{5}{11}$; $\frac{7}{12}$.	9. $\frac{3}{101}$; $\frac{4}{210}$; $\frac{75}{322}$.
2. $\frac{3}{5}$; $\frac{5}{6}$; $\frac{2}{7}$; $\frac{1}{8}$.	6. $\frac{5}{12}$; $\frac{9}{14}$; $\frac{8}{13}$; $\frac{11}{15}$.	10. $\frac{501}{498}$; $\frac{99}{505}$; $\frac{3}{654}$.
3. $\frac{2}{5}$; $\frac{2}{6}$; $\frac{4}{7}$; $\frac{5}{9}$.	7. $\frac{11}{20}$; $\frac{5}{32}$; $\frac{9}{40}$; $\frac{11}{51}$.	11. $\frac{75}{624}$; $\frac{9}{734}$; $\frac{97}{876}$.
4. $\frac{1}{5}$; $\frac{5}{8}$; $\frac{6}{7}$; $\frac{8}{9}$.	8. $\frac{13}{60}$; $\frac{32}{71}$; $\frac{29}{85}$; $\frac{45}{97}$.	12. $\frac{100}{763}$; $\frac{88}{807}$; $\frac{987}{998}$.

3. In each of the preceding exercises, what does the denominator of each fraction show and name, and what does each numerator show?

2. The Value of a Fraction.

1. If an apple is cut into fourths, how many fourths of an apple are there?

2. Is the value of $\$\frac{3}{4}$ greater, or less than \$1? Is $\frac{4}{5}$ greater, or less than 1? $\frac{6}{6}$? $\frac{7}{8}$?

3. Which has the greater value, $\$\frac{4}{4}$ or \$1? $\frac{3}{3}$ of a yard, or 1 yard? $\frac{8}{8}$ of a gallon, or 1 gallon? Why?

4. If 5 apples are divided into halves, how many half-apples are there in all? If 6 apples are divided into halves?

5. 5 half-apples are equal to how many whole apples? Why? 3 apples are what part of 6 apples?

6. Compare the value of each of the following fractions with a unit, or 1: $\frac{3}{4}$, $\frac{6}{6}$, $\frac{7}{8}$, $\frac{10}{10}$, $\frac{1}{20}$, $\frac{20}{12}$, $\frac{15}{7}$.

7. Indicate in the form of a fraction the division of 1 by 2; 4 by 1; 5 by 8; 10 by 10; 16 by 8; 20 by 6; 24 by 10.

8. As an expression of division, what is the value of $\frac{6}{3}$? Of $\frac{7}{8}$? Of $\frac{16}{8}$? Of $\frac{12}{12}$? Of $\frac{20}{20}$? Of $\frac{24}{8}$?

9. Show that the value of a fraction is less than 1 when the numerator is less than the denominator.

10. Show that the value of a fraction is 1 when the numerator and the denominator are alike.

11. Show that the value of a fraction is greater than 1 when the numerator is greater than the denominator.

12. Show that the value of a fraction depends upon the number of equal parts into which the unit is divided, and the number of equal parts expressed by the fraction.

Definitions.

142. According to *value*, fractions are divided into two classes: *Proper Fractions* and *Improper Fractions.*

143. A ***Proper Fraction*** is a fraction whose value is less than 1.

The numerator of a proper fraction is always less than its denominator.

Thus, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{15}{20}$ are *proper fractions.*

144. An ***Improper Fraction*** is a fraction whose value is equal to, or greater than 1.

The numerator of an improper fraction is always equal to, or greater than its denominator.

Thus, $\frac{3}{3}$, $\frac{8}{8}$, $\frac{12}{10}$, $\frac{25}{20}$ are *improper fractions.*

145. A ***Mixed Number*** is a number expressed by an integer and a fraction.

Thus, $5\frac{1}{4}$ is a *mixed number*, read 5 *and* $\frac{1}{4}$; so, also, is $7\frac{8}{9}$; etc.

146. Principle.

The value of a fraction is the quotient of the numerator divided by the denominator.

Exercises.

Analyze the fraction $\frac{7}{8}$.

147. Model. — $\frac{7}{8}$ is read *seven eighths;* its terms are 7 and 8; the *denominator is 8,* which shows that the unit is divided into 8 equal

parts named *eighths;* the *numerator* is 7, which shows that 7 of these equal parts are used, or are expressed by the fraction; it is a *proper fraction,* since the numerator is less than its denominator, and its value is less than 1.

1. Analyze each of the following fractions: —

1. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$.	4. $\frac{3}{10}$; $\frac{1}{11}$; $\frac{7}{12}$; $\frac{8}{13}$.	7. $\frac{50}{73}$; $\frac{13}{84}$; $\frac{60}{97}$.
2. $\frac{2}{5}$; $\frac{3}{7}$; $\frac{1}{6}$; $\frac{5}{8}$.	5. $\frac{3}{20}$; $\frac{19}{31}$; $\frac{7}{42}$; $\frac{11}{53}$.	8. $\frac{1}{100}$; $\frac{3}{205}$; $\frac{13}{350}$.
3. $\frac{4}{6}$; $\frac{5}{7}$; $\frac{3}{8}$; $\frac{7}{9}$.	6. $\frac{21}{25}$; $\frac{1}{45}$; $\frac{44}{56}$; $\frac{10}{63}$.	9. $\frac{75}{412}$; $\frac{100}{524}$; $\frac{234}{789}$.

2. What is meant by half of any number or thing? By 1 third? 1 fourth? 1 fifth? 1 sixth? 1 eighth? 1 tenth?

By 2 thirds of any number or thing? By 3 fourths? By 2 fifths? 6 sevenths? 7 ninths? 3 tenths? 11 twelfths?

By 2 halves of any number or thing? By 3 thirds? 4 fourths? 5 fifths? 6 sixths? By $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{3}$?

3. *Principles of Fractions.*

1. What is the fractional unit of $\frac{3}{4}$? How many fractional units in $\frac{3}{4}$? How many are four times three fourths?

2. What is the name or size of the fractional unit in $\frac{3}{4}$? In $\frac{6}{4}$? What is the number of fractional units in $\frac{3}{4}$? In $\frac{6}{4}$?

3. How many fifths are 2 times $\frac{2}{5}$? How many sevenths are 2 times $\frac{3}{7}$? 3 times $\frac{4}{9}$ are how many ninths?

4. Show that if the numerator of a fraction is multiplied by any number, the value of the fraction is made greater.

5. How many fractional units in $\frac{4}{5}$? How many in one half as many fifths? In one half of $\frac{6}{5}$?

6. If the number of fractional units in $\frac{6}{5}$ be divided by 2, how many fractional units of the same size will there be?

7. How many ninths in one half of $\frac{6}{9}$? One third of $\frac{6}{9}$? One fourth of $\frac{8}{9}$ equals how many ninths?

8. Show that if the numerator of a fraction is divided by *any number, the* value of the fraction is made less.

9. What is the fractional unit of $\frac{5}{3}$? Of $\frac{5}{6}$? Which expresses the larger fractional unit, $\frac{5}{3}$ or $\frac{5}{6}$? Which expresses the greater number of units?

10. What does $\frac{5}{3}$ become if its denominator is multiplied by 2? Is the fractional unit made larger, or smaller? Is the number of units changed?

11. If the denominator of $\frac{5}{7}$ be multiplied by 2, how will the value of the fraction be affected? Which has the greater value, $\frac{1}{7}$ or $\frac{1}{14}$? $\frac{1}{14}$ is what part of $\frac{1}{7}$? Why?

12. Show that if the denominator of a fraction is multiplied by any number, the value of the fraction is made less.

13. What is the fractional unit of $\frac{5}{8}$? Of $\frac{5}{4}$? Which are the greater, eighths or fourths? How much greater?

14. What number of fractional units in $\frac{5}{8}$? In $\frac{5}{4}$? Which expresses the larger fractional unit? Which the greater number of units? Which the greater value?

15. If the denominator of $\frac{2}{9}$ be divided by 3, how will the value of the fraction be affected? $\frac{2}{3}$ is how many times $\frac{2}{9}$? Why?

16. Show that if the denominator of a fraction is divided by any number, the value of the fraction is made greater.

17. What will be the result if 4 be multiplied by 3, and the product be then divided by 3?

18. What is the result if the numerator of $\frac{3}{4}$ is multiplied by 3, and the denominator of $\frac{9}{4}$ is multiplied by 3? Which is the greater, $\frac{3}{4}$ or $\frac{9}{12}$? Why?

19. How will the value of the fraction be affected if the numerator of $\frac{5}{8}$ be multiplied by 2, and the denominator of the product be multiplied by 2?

20. Show that if both terms of a fraction are multiplied by the same number, the value of the fraction is not changed.

21. What will be the result if both terms of $\frac{9}{12}$ be divided by 3? If both terms of $\frac{3}{4}$ be multiplied by 3?

22. Fourths are how many times as large as twelfths? How many times as many twelfths as fourths must be taken to give the same part of a unit?

23. How will the value of the fraction be affected if the numerator of $\frac{9}{12}$ be divided by 3, and the denominator of the quotient be divided by 3?

24. Show that if both terms of a fraction are divided by the same number, the value of the fraction is not changed.

148. General Principles of Fractions.

I. The value of a fraction is multiplied
   1. *By multiplying its numerator;* or,
   2. *By dividing its denominator.*

II. The value of a fraction is divided
   1. *By dividing its numerator;* or,
   2. *By multiplying its denominator.*

III. The value of a fraction is not changed
   1. *By multiplying both terms by the same number;* or,
   2. *By dividing both terms by the same number.*

Exercises.

Multiply the numerator of $\frac{3}{7}$ by 2, and tell how the value of the fraction is affected.

149. Model. — If the numerator of $\frac{3}{7}$ is multiplied by 2, the result is $\frac{6}{7}$, and the value of the fraction is made twice as great; because, while the *size* of each part remains the *same*, the *number* of the parts is made *twice as great.*

Multiply both terms of $\frac{2}{3}$ by 3, and tell how the value of the fraction is affected.

150. Model. — If both terms of $\frac{2}{3}$ are multiplied by 3, the result *is* $\frac{6}{9}$, *and the value of* the fraction is not changed, because, while the

size of each part is made *3 times as small*, the *number* of parts is made *3 times as great.*

1. Multiply each of the following fractions, and tell how the value of the fraction is affected: —

The numerator of	The denominator of
1. $\frac{2}{3}$ *by 2;* $\frac{3}{5}$ *by 3;* $\frac{4}{7}$ *by 4.*	*3.* $\frac{3}{4}$ *by 2;* $\frac{2}{5}$ *by 3;* $\frac{5}{6}$ *by 4.*
2. $\frac{3}{8}$ *by 5;* $\frac{4}{9}$ *by 4;* $\frac{3}{10}$ *by 6.*	*4.* $\frac{4}{7}$ *by 4;* $\frac{5}{8}$ *by 5;* $\frac{5}{9}$ *by 6.*

2. Divide each of the following fractions, and tell how the value of the fraction is affected: —

The numerator of	The denominator of
1. $\frac{6}{7}$ *by 3;* $\frac{8}{9}$ *by 3;* $\frac{12}{7}$ *by 4.*	*3.* $\frac{5}{8}$ *by 2;* $\frac{7}{9}$ *by 3;* $\frac{11}{12}$ *by 4.*
2. $\frac{10}{11}$ *by 5;* $\frac{12}{5}$ *by 6;* $\frac{14}{15}$ *by 7.*	*4.* $\frac{9}{10}$ *by 5;* $\frac{7}{12}$ *by 6;* $\frac{9}{14}$ *by 7.*

3. Tell, with your reasons, how the value of each of the following fractions is affected by

Multiplying both terms of	Dividing both terms of
1. $\frac{2}{5}$ *by 2;* $\frac{4}{7}$ *by 3;* $\frac{5}{9}$ *by 4.*	*3.* $\frac{6}{8}$ *by 2;* $\frac{6}{9}$ *by 3;* $\frac{8}{12}$ *by 4.*
2. $\frac{5}{6}$ *by 4;* $\frac{7}{8}$ *by 5;* $\frac{5}{9}$ *by 6.*	*4.* $\frac{12}{20}$ *by 4;* $\frac{15}{20}$ *by 5;* $\frac{18}{30}$ *by 6.*

Section II.

REDUCTION OF FRACTIONS.

151. Reduction of Fractions is the process of changing their form without changing their value.

1. To Change a Fraction to Lower Terms.

1. How many thirds of an inch in 1 inch? In 6 sixths of an inch? In 2 sixths of an inch? In 4 sixths?

2. How many fourths in 6 eighths?

152. Analysis. — Since one fourth equals 2 eighths, 6 eighths

equal as many fourths as the number of times 2 eighths are contained in 6 eighths, which is 3 times. Hence, in 6 eighths there are 3 fourths.

3. How many fifths in 2 tenths? In 4 tenths? How many fifths in $\frac{6}{10}$? In $\frac{8}{10}$? In $\frac{12}{20}$? In $\frac{20}{25}$?

4. Six ninths of a yard are how many thirds of a yard? Express $\frac{6}{9}$ in terms one third as large.

5. Express the value of $\frac{12}{24}$ as halves; fourths; eighths.

6. What factor is common to the terms of $\frac{10}{18}$? Divide both terms by their common factor.

7. What common divisor have the terms of the fraction $\frac{14}{21}$? What does the fraction become if both terms are divided by 7?

8. What fraction is produced by dividing both terms of the fraction $\frac{16}{24}$ by their greatest common divisor? Is the value of the fraction changed? Why?

153. A fraction is changed to ***lower terms*** when it is changed to a fraction of equal value having a smaller numerator and denominator.

154. A fraction is changed to its ***lowest terms*** when its numerator and denominator are made prime to each other.

Thus, $\frac{18}{24}$ expressed in lower terms equals $\frac{9}{12}$; but changed to its lowest terms equals $\frac{3}{4}$, since 3 and 4 are prime to each other.

9. Express the value of $\frac{24}{36}$ in terms $\frac{1}{2}$ as great; $\frac{1}{3}$ as great; $\frac{1}{4}$ as great; $\frac{1}{6}$; $\frac{1}{12}$.

10. Is $\frac{10}{15}$ in its lowest terms? Why not? What is $\frac{10}{15}$ expressed in its lowest terms? Why is $\frac{2}{3}$ equal to $\frac{10}{15}$?

11. By what number must both numerator and denominator of $\frac{12}{20}$ be divided to change them to fifths? To tenths?

12. Express the value of $\frac{14}{21}$ in its lowest terms. By what common factor can both terms be divided?

13. Frank has $\frac{6}{8}$ of a dollar, and Harry has $\frac{75}{100}$ of a dollar. *Which has the* greater sum of money?

14. From one tree a gardener took $\frac{9}{12}$ of a bushel of pears, from another $\frac{2}{16}$, and from another $\frac{24}{32}$. Which yielded most?

15. Divide both terms of $\frac{27}{36}$ by 9, their greatest common divisor, and show that the value of the fraction is not changed.

16. Show that the reduction of a fraction to its lowest terms does not change the value of the fraction. Tell by what process a fraction is reduced to its lowest terms.

155. Principle.

Dividing both terms of a fraction by the same number does not change the value of the fraction.

Exercises.

Change $\frac{18}{30}$ to its lowest terms.

156. Model. — Since dividing both terms of a fraction by the same number does not change the value of the fraction, $\frac{18}{30}$ is changed to its lowest terms by dividing the numerator and the denominator by 6, their greatest common divisor, giving $\frac{3}{5}$, the fraction required.

Change, or reduce, to their lowest terms

1. $\frac{6}{15}$; $\frac{9}{21}$; $\frac{12}{28}$.
2. $\frac{15}{25}$; $\frac{20}{35}$; $\frac{35}{49}$.
3. $\frac{18}{45}$; $\frac{21}{56}$; $\frac{24}{60}$.
4. $\frac{26}{39}$; $\frac{32}{48}$; $\frac{44}{55}$.
5. $\frac{36}{54}$; $\frac{42}{56}$; $\frac{45}{75}$.
6. $\frac{46}{69}$; $\frac{52}{78}$; $\frac{60}{84}$.
7. $\frac{48}{60}$; $\frac{54}{72}$; $\frac{57}{78}$; $\frac{60}{75}$.
8. $\frac{65}{78}$; $\frac{68}{85}$; $\frac{76}{95}$; $\frac{84}{96}$.
9. $\frac{72}{84}$; $\frac{78}{91}$; $\frac{84}{98}$; $\frac{96}{108}$.

2. To Change a Fraction to Higher Terms or to a Given Denominator.

1. How many eighths of a yard in 1 yard? In 1 fourth of a yard? In 2 fourths? 3 fourths?

2. In 2 thirds of a yard how many sixths?

157. Analysis. — Since 1 is equal to 6 sixths, 1 third is equal to one third of 6 sixths, which is 2 sixths, and 2 thirds are 2 times 2 sixths, or 4 sixths. Hence, etc.

3. How many twelfths in 1 half? In 2 thirds? How many in $\frac{3}{4}$? In $\frac{5}{6}$?

4. Express the value of $\frac{3}{5}$ as tenths; as fifteenths. In terms 4 times as great. In terms 5 times as great.

5. Name the first three multiples of 4, the denominator of $\frac{3}{4}$. Express $\frac{3}{4}$ as eighths; as twelfths; sixteenths.

6. The number of twelfths in a unit is how many times the number of thirds? How many times the fourths? Sixths?

7. Express $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{6}$, $\frac{9}{8}$, and $\frac{7}{12}$ as 24ths and as 48ths.

8. What fraction is produced by multiplying both of the terms of the fraction $\frac{3}{7}$ by 3? Is the value of the fraction changed? Why?

9. Express the value of $\frac{7}{8}$ in terms 2 times as large; 3 times as large; 4 times; 5 times; 6 times.

10. What is $\frac{4}{5}$ expressed in terms 4 times as large? How are fifths changed to 20ths? Why is $\frac{4}{5}$ equal to $\frac{16}{20}$?

11. By what number must both numerator and denominator of $\frac{3}{7}$ be multiplied to change it to 14ths? To 21sts?

12. Express the value of $\frac{7}{8}$ in sixteenths. By what factor must both terms be multiplied?

13. Change $\frac{9}{12}$ of a foot and $\frac{15}{18}$ of a yard to fractions having terms 3 times as great; 4 times; and 5 times.

14. Edward has $\frac{3}{4}$ of a dollar, and William has $\frac{17}{20}$ of a dollar. Which has the greater sum of money?

15. In one lot are $\frac{3}{4}$ of an acre, and in another $\frac{4}{10}$ of an acre. Which is the larger lot? Prove it.

16. How is a fraction changed to higher terms? Show that the reduction does not change the value of the fraction.

158. A fraction is changed to ***higher terms*** when it is changed to a fraction of equal value having a larger numerator and denominator.

Thus, $\$\frac{1}{4}$ *may be expressed as* $\$\frac{2}{8}$, $\$\frac{5}{12}$, $\$\frac{5}{20}$, etc.

159. Principle.

Multiplying both terms of a fraction by the same number does not change the value of the fraction.

Exercises.

Change $\frac{3}{8}$ to twenty-fourths.

160. Model. — Since multiplying both terms of a fraction by the same number does not change the value of the fraction, $\frac{3}{8}$ may be changed to 24ths by multiplying the denominator by 3, and the numerator by 3, giving $\frac{9}{24}$, the fraction required.

Change, or reduce

1. $\frac{2}{3}$ and $\frac{4}{5}$ to tenths.
2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{6}$ to twelfths.
3. $\frac{4}{5}$ and $\frac{9}{10}$ to twentieths.
4. $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$ to 24ths.
5. $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{10}$ to 30ths.
6. $\frac{7}{8}$, $\frac{11}{12}$, $\frac{15}{16}$ to 48ths.
7. $\frac{5}{9}$, $\frac{7}{12}$, $\frac{11}{18}$ to 36ths.
8. $\frac{5}{6}$, $\frac{7}{9}$, $\frac{13}{18}$ to 54ths.
9. $\frac{3}{9}$, $\frac{7}{24}$, $\frac{11}{36}$ to 72nds.

3. To Change an Integer or Mixed Number to an Improper Fraction.

1. In \$3 how many fourths of a dollar?

161. Analysis. — Since in \$1 there are 4 fourths of a dollar, in \$3 there are 3 times 4 fourths, or 12 fourths of a dollar. Hence, etc.

2. In 1 yard are how many thirds of a yard? In 9 yards? How many thirds in 3 yards? In 5 yards? In 9 yards?

3. In 4 gallons are how many fourths of a gallon? How many eighths? How many twelfths? Why?

4. How may an integer be changed to thirds? To fifths? To ninths? To sixths? To eighths? Why?

5. How many sixths in $4\frac{5}{6}$?

162. Analysis. — Since in 1 there are 6 sixths, in 4 there are 4 times 6 sixths, or 24 sixths; 24 sixths and 5 sixths are 29 sixths.

6. How many eighths of a dollar in \$5? In $\$5\frac{3}{8}$? How many tenths in \$6? In $\$6\frac{9}{10}$? In $\$9\frac{7}{10}$? Why?

7. In $4\frac{2}{5}$ how many fifths? In $8\frac{5}{7}$ how many sevenths? In $7\frac{8}{9}$ how many ninths? Why?

8. Among how many boys can $4\frac{3}{4}$ quarts of chestnuts be divided so that each boy shall have $\frac{1}{4}$ of a quart?

9. Into how many lots, each $\frac{1}{8}$ of an acre, can a piece of ground containing $5\frac{7}{8}$ acres be divided?

10. If $\frac{3}{4}$ of a pound of gold are worth \$12, what is $\frac{1}{4}$ of a pound worth? What are $4\frac{1}{4}$ pounds worth? $5\frac{3}{4}$?

11. How many half-pound packages can be filled by $7\frac{1}{2}$ pounds of tea? Quarter-pound packages?

12. Show how an integer or a mixed number can be changed to an improper fraction.

Exercises.

Change $5\frac{4}{5}$ to an improper fraction.

163. Model. — Since in 1 there are 6 sixths, in 5 there are 5 times 6 sixths, or 30 sixths; 30 sixths and 5 sixths are 35 sixths.

Change, or reduce

1. *8 and 10 to fourths.*
2. *9 and 12 to sevenths.*
3. *10 and* $10\frac{5}{6}$ *to sixths.*
4. *12 and* $12\frac{5}{9}$ *to ninths.*
5. $9\frac{3}{5}$ *and* $8\frac{3}{8}$ *to improper fractions.*
6. $7\frac{5}{10}$ *and* $9\frac{3}{10}$ *to improper fractions.*
7. $10\frac{3}{12}$ *yards to twelfths of a yard.*
8. \$20 *and* $\$30\frac{6}{8}$ *to quarter-dollars.*

4. To Change an Improper Fraction to an Integer or a Mixed Number.

1. How many dollars are in 4 fourths of a dollar? How many in 8 fourths of a dollar?

2. How many fifths make a unit, or 1? Then how many units are in 10 fifths? How many in $\frac{15}{5}$? In $\frac{20}{5}$? Why?

3. A tailor used 12 thirds of a yard of cloth to make a coat. How many yards did he use?

4. How is any number of fourths changed to ones, or to an integer? Any number of sixths? Eighths? Why?

5. How many dollars are $\frac{12}{5}$ of a dollar?

164. Analysis. — Since in 1 dollar there are 5 fifths of a dollar, in 12 fifths of a dollar there are as many dollars as the number of times 5 fifths are contained in 12 fifths, which is $2\frac{2}{5}$ times. Hence, etc.

6. How many days are $\frac{27}{4}$ of a day? Weeks in $\frac{21}{7}$ of a week? Years in $\frac{39}{12}$ of a year? Why?

7. If a man can walk 25 eighths of a mile in an hour, how many miles can he walk in 4 hours?

8. James received $\frac{25}{10}$ of a dollar as his week's wages. How many dollars did he receive?

9. How many bushels of peaches in 55 baskets, each containing $\frac{1}{3}$ of a bushel? Each $\frac{2}{5}$ of a bushel?

10. How much will 45 oranges cost at $\frac{1}{10}$ of a dollar each? At $\frac{1}{20}$ of a dollar? At $\frac{1}{50}$ of a dollar?

11. At a concert 40 dollars, 50 half-dollars, and 55 quarter-dollars were taken in. How much money was received?

12. Explain how an improper fraction can be changed to an integer or a mixed number.

Exercises.

Change $\frac{25}{6}$ to a mixed number.

165. Model. — Since in 1 there are 6 sixths, in 25 sixths there are as many ones as the number of times 6 sixths are contained in 25 sixths, which is $4\frac{1}{6}$ times. Hence, etc.

Change, or reduce

1. $\frac{19}{7}$ *of a week to weeks.*
2. $\frac{31}{8}$ *of a bushel to bushels.*
3. $\frac{47}{10}$ *and* $\frac{47}{20}$ *of a ton to tons.*
4. $\frac{18}{9}$, $\frac{40}{10}$, $\frac{48}{12}$ *to integers.*

Change to mixed numbers

5. $\frac{25}{9}$ *of a yard and* $\frac{60}{12}$ *of a dollar.*
6. $\frac{35}{6}$ *of a quart and* $\frac{20}{16}$ *of a bushel.*
7. $\frac{37}{12}$ *of a pound and* $\frac{66}{20}$ *of a ton.*
8. $\frac{35}{8}$, $\frac{57}{10}$, $\frac{62}{11}$, $\frac{73}{12}$, $\frac{75}{13}$, and $\frac{82}{15}$.

5. *To Change Dissimilar Fractions to Similar Fractions.*

1. Frank has $\frac{1}{2}$ of a dollar, and William has $\frac{6}{8}$ of a dollar. How many fourths of a dollar has each?

166. ***Analysis.*** — $\frac{1}{2}$ of a dollar changed to fourths equals $\frac{2}{4}$ of a dollar; and $\frac{6}{8}$ of a dollar changed to fourths equal $\frac{3}{4}$ of a dollar. Hence, etc.

2. If the denominator of $\frac{3}{4}$ be multiplied by 2, by what must be the numerator be multiplied so as not to change the value of the fraction?

3. If both terms of the fraction $\frac{6}{9}$ be divided by 3, is the value of the fraction changed? Give the reason.

4. How can $\frac{3}{4}$ and $\frac{5}{6}$ each be changed to twelfths? By what number must both terms of each fraction be multiplied?

5. Change $\frac{5}{6}$ and $\frac{18}{24}$ to 12ths. Is the value of the fractions changed? Why? Change $\frac{7}{8}$ and $\frac{11}{12}$ to 24ths, and to 48ths.

6. What is the common multiple of 5 and 6? Of 6 and 7, the denominators of $\frac{5}{6}$ and $\frac{6}{7}$?

7. Change $\frac{2}{3}$ and $\frac{3}{4}$ to equal fractions having the same denominator; $\frac{4}{5}$ and $\frac{5}{8}$. Are the values changed? Why?

8. What is the least common multiple of 4, 5, and 6? Of the denominators of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$?

9. What is the least number of which the denominators of $\frac{4}{5}$, $\frac{7}{8}$, and $\frac{9}{10}$ are common factors?

10. What is the least common multiple of the denominators of $\frac{5}{6}$, $\frac{8}{9}$, and $\frac{11}{12}$? Change them to fractions of equal value having the least common denominator.

167. ***Similar Fractions*** are fractions whose denominators are alike.

Thus, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$ are *similar fractions;* also $\frac{5}{3}$, $\frac{8}{3}$, and $\frac{9}{3}$.

168. ***Dissimilar Fractions*** are fractions whose *denominators* are not alike.

Thus, $\frac{2}{7}$, $\frac{5}{4}$, and $\frac{7}{10}$ are *dissimilar fractions*; also $\frac{2}{5}$, $\frac{5}{6}$, and $\frac{10}{9}$.

169. A *Common Denominator* is a denominator common to two or more fractions, and

The *Least Common Denominator* is the least or smallest denominator common to two or more fractions.

11. What is the least common denominator of the fractions equal to $\frac{5}{8}$, $\frac{7}{9}$, and $\frac{11}{12}$? What is each new numerator?

12. To what similar fractions may the dissimilar fractions $\frac{3}{4}$, $\frac{5}{12}$, $\frac{7}{8}$, and $\frac{10}{9}$ be changed?

13. Change to equal fractions having the least common denominator $\frac{7}{8}$ of an acre, $\frac{9}{10}$ of an acre, and $\frac{11}{16}$ of an acre.

14. Change $\$2\frac{1}{5}$, $\$3\frac{1}{8}$, and $\$5\frac{1}{10}$ to similar fractions having the least common denominator.

15. Explain how dissimilar fractions may be changed to similar fractions, and show that the value is not changed.

170. Principles.

I. *The least common denominator of two or more fractions is the least common multiple of their denominators.*

II. *Multiplying and dividing both terms of a fraction by the same number does not change the value of the fraction.*

Exercises.

Change $\frac{5}{6}$ and $\frac{7}{8}$ to similar fractions having the least common denominator.

171. Model. — The least common multiple of the denominators 6 and 8, or the least common denominator, is 24.

$\frac{5}{6}$ is changed to 24ths by multiplying the denominator and the numerator by 4, giving $\frac{20}{24}$; and $\frac{7}{8}$ is changed to 24ths by multiplying the denominator and the numerator by 3, giving $\frac{21}{24}$. Hence, $\frac{5}{6}$ and $\frac{7}{8}$ are equal to $\frac{20}{24}$ and $\frac{21}{24}$.

Change to similar fractions having the least common denominator

1. $\frac{5}{6}$ and $\frac{7}{9}$.
2. $\frac{7}{8}$ and $\frac{9}{10}$.
3. $\frac{8}{9}$ and $\frac{7}{12}$.
4. $\frac{5}{10}$ and $\frac{11}{12}$.
5. $\frac{4}{5}$, $\frac{7}{8}$, and $\frac{9}{10}$.
6. $\frac{5}{6}$, $\frac{5}{8}$, and $\frac{5}{12}$.
7. $\frac{1}{6}$, $\frac{5}{9}$, and $\frac{2}{12}$.
8. $\frac{9}{8}$, $\frac{9}{10}$, and $\frac{9}{20}$.
9. $\frac{9}{10}$, $\frac{11}{12}$, and $\frac{11}{15}$.
10. $\frac{7}{12}$, $\frac{11}{15}$, and $\frac{13}{20}$.
11. $\frac{11}{12}$, $\frac{11}{16}$, and $\frac{11}{24}$.
12. $\frac{14}{15}$, $\frac{19}{20}$, and $\frac{29}{30}$.

Section III.

ADDITION OF FRACTIONS.

1. What is the unit of 2 fourths? Of 3 fourths? What is the sum of 2 fourths and 3 fourths?

2. How many sevenths are 2 sevenths and 3 sevenths? How many are $\frac{3}{7}$ and $\frac{5}{7}$? How many are $\frac{2}{7}$ and $\frac{4}{7}$?

3. What is the sum of $\frac{3}{8}$ and $\frac{7}{8}$? Of $\frac{2}{9}$ and $\frac{4}{9}$ and $\frac{5}{9}$?

4. What is the numerator of the sum of $\frac{3}{10}$, $\frac{7}{10}$, and $\frac{9}{10}$? What is the denominator of the sum?

5. How do you add similar fractions, or fractions having a common denominator?

6. What is the fractional unit of $\frac{2}{3}$? Of $\frac{3}{4}$? Can $\frac{2}{3}$ and $\frac{3}{4}$ be united in one sum? Why?

7. How many twelfths in $\frac{2}{3}$? In $\frac{3}{4}$? Change $\frac{2}{3}$ and $\frac{3}{4}$ to 12ths. How many 12ths in their sum?

8. To what kind of fraction must $\frac{5}{8}$ and $\frac{7}{8}$ be changed before they can be added? What do they equal? Find their sum.

9. William paid $\$\frac{5}{8}$ for a knife and $\$\frac{7}{8}$ for a pair of skates. What did he pay for both?

172. ***Analysis.*** — Since William paid $\$\frac{5}{8}$ for a knife and $\$\frac{7}{8}$ for a pair of skates, he paid for both the sum of $\$\frac{5}{8}$ and $\$\frac{7}{8}$.

The least common multiple of the denominators is 24. $\frac{5}{8}$ changed to 24ths is $\frac{15}{24}$; and $\frac{7}{8}$ changed to 24ths is $\frac{21}{24}$. Now $\$\frac{15}{24}$ and $\$\frac{21}{24}$ are $\$\frac{36}{24}$, or $\$1\frac{12}{24}$. Hence, etc.

10. How do you add dissimilar fractions, or fractions having different denominators?

11. If a farmer sells $\frac{1}{4}$ of his wheat to one man, and $\frac{1}{6}$ of it to another, what part of his wheat does he sell to both?

12. If John does $\frac{1}{8}$ of a piece of work one day, and $\frac{1}{8}$ of it the next, what part does he perform in both days?

13. A man having $\frac{3}{4}$ of a ton of coal, bought $\frac{4}{5}$ of a ton more. How much coal had he then?

14. I used $\frac{4}{7}$ of a ton of coal one week, and $\frac{4}{5}$ of a ton the next. How much did I use in the two weeks?

15. A gardener has $\frac{3}{4}$ of an acre of raspberries, and $\frac{4}{5}$ of an acre of strawberries. How many acres of berries has he?

16. In a village lot are $\frac{5}{8}$ of an acre, and in another are $\frac{7}{10}$ of an acre. How much land is in both lots?

17. A owns $\frac{2}{5}$ of a vessel, and B owns $\frac{3}{8}$ of it. What part of the vessel do both of them own?

18. What fraction is that from which, if $\frac{5}{8}$ be taken, the remainder will be $\frac{2}{9}$?

19. William saved $\$\frac{4}{5}$ one week, $\$\frac{7}{8}$ the next, and $\$\frac{7}{10}$ the next. How much did he save in the three weeks?

Find the sum of

20. $\frac{7}{8}$ of a mile and $\frac{9}{10}$ of a mile.

21. $\frac{9}{10}$ of an acre and $\frac{1}{12}$ of an acre.

22. $\frac{11}{12}$ of a yard and $\frac{15}{16}$ of a yard.

23. $\frac{1}{20}$ of a ton and $\frac{11}{25}$ of a ton.

24. $\frac{13}{24}$ of a cord and $\frac{13}{32}$ of a cord.

25. $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$ of a bushel.

26. $\frac{5}{8}$, $\frac{6}{7}$, and $\frac{7}{8}$ of a gallon.

27. $\frac{5}{6}$, $\frac{8}{9}$, and $1\frac{1}{12}$ of a foot.

28. $\frac{9}{10}$, $\frac{11}{12}$, and $\frac{14}{15}$ of a rod.

29. $\frac{11}{12}$, $\frac{13}{15}$, and $\frac{19}{20}$ of a pint.

30. A farmer sold $2\frac{3}{4}$ tons of hay to one man, and $3\frac{4}{5}$ tons to another. How much did he sell to both?

173. Analysis. — Since the farmer sold $2\frac{3}{4}$ tons to one man, and $3\frac{4}{5}$ tons to another, he sold to both the sum of $2\frac{3}{4}$ tons and $3\frac{4}{5}$ tons.

2 tons and 3 tons are 5 tons. $\frac{3}{4}$ changed to 20ths is $\frac{15}{20}$; and $\frac{4}{5}$

changed to 20ths is $\frac{15}{20}$. Now $\frac{15}{20}$ and $\frac{18}{20}$ are $\frac{33}{20}$, or $1\frac{13}{20}$, which added to 5, the sum of the integers, gives $6\frac{13}{20}$. Hence, he sold $6\frac{13}{20}$ tons.

31. How do you find the sum of two or more mixed numbers?

32. A woodchopper put $3\frac{3}{4}$ cords of wood in one pile, and $6\frac{5}{8}$ cords in another. How much wood was in both?

33. A jeweler paid $\$24\frac{3}{4}$ for a silver watch, and sold it at a gain of $\$4\frac{7}{8}$. What did he get for it?

34. A boy sold $3\frac{2}{5}$ bushels of chestnuts one week, and $4\frac{3}{16}$ bushels the next. How many did he sell in both weeks?

35. If $4\frac{3}{4}$ yards of cloth are needed for a coat, $2\frac{3}{4}$ yards for a pair of pants, and $\frac{7}{8}$ of a yard for a vest, how many yards are needed in all?

36. A boy had \$4, earned $\$3\frac{5}{8}$ more, and his father gave him $\$1\frac{7}{10}$. How much had he then?

37. What is the sum of $\frac{1}{6}$ of 18, $\frac{1}{6}$ of 20, and $\frac{1}{6}$ of 27?

38. John has \$10, Charles has $\$5\frac{3}{10}$, and Henry has $\$3\frac{7}{8}$ more than both the others. How much have all?

39. If a mechanic's daily wages amounted to $\$2\frac{4}{5}$, and he made $\$\frac{7}{10}$ by overwork, what did he receive per day?

40. An agent sells 2 acres for one building lot, $\frac{3}{8}$ of an acre for a second, and $\frac{5}{16}$ for a third. How many acres does he sell?

174. Principles.

I. *Only fractional units of the same kind of things can be added.*

II. *Only similar fractions can be added.*

Exercises.

Find the sum of $\frac{7}{8}$ and $\frac{9}{10}$.

175. Model. — The least common multiple of the denominators of $\frac{7}{8}$ and $\frac{9}{10}$ is 40. $\frac{7}{8}$ changed to 40ths is $\frac{35}{40}$, and $\frac{9}{10}$ changed to 40ths *is* $\frac{36}{40}$. *Now* $\frac{35}{40}$ *and* $\frac{36}{40}$ are $\frac{71}{40}$, or $1\frac{31}{40}$, the sum required.

Find the sum of

1. $\frac{3}{9}$, $\frac{5}{6}$, and $\frac{7}{8}$.
2. $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{7}{12}$.
3. $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{9}{20}$.
4. $\frac{3}{9}$, $\frac{7}{12}$, and $\frac{11}{15}$.
5. $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{13}{16}$.
6. $3\frac{2}{3}$, $4\frac{1}{4}$, and 5.
7. $\frac{3}{5}$, $4\frac{2}{3}$, and $6\frac{2}{5}$.
8. 7, $2\frac{3}{8}$, and $5\frac{1}{6}$.
9. $5\frac{3}{4}$, $\frac{9}{7}$, and 3.
10. 10, $\frac{3}{22}$, and $4\frac{1}{33}$.
11. $4\frac{1}{10}$, 10, and $10\frac{1}{4}$.
12. $5\frac{3}{10}$, 9, and $3\frac{1}{15}$.

Section IV.

SUBTRACTION OF FRACTIONS.

1. What is the unit of 7 eighths? Of 5 eighths? What is the remainder of 7 eighths less 5 eighths?

2. How many ninths are 6 ninths less 3 ninths? $\frac{8}{9}$ less $\frac{5}{9}$?

3. What is the numerator of the remainder of $\frac{6}{7}$ less $\frac{3}{7}$? What is the denominator of the remainder?

4. How do you subtract one fraction from another fraction having the same denominator?

5. What is the fractional unit of $\frac{4}{5}$? Of $\frac{3}{6}$? Can $\frac{3}{4}$ be taken directly from $\frac{4}{5}$? Why?

6. How many 20ths in $\frac{4}{5}$? In $\frac{3}{4}$? Change $\frac{4}{5}$ and $\frac{3}{4}$ to 20ths. How many 20ths in their difference?

7. Change $\frac{5}{6}$ and $\frac{4}{9}$ to least similar fractions, and find their difference; $\frac{9}{10}$ and $\frac{7}{8}$; $\frac{8}{9}$ and $\frac{7}{12}$.

8. From a pile of wood containing $\frac{7}{8}$ of a cord were taken $\frac{3}{10}$ of a cord. How much was left?

176. Analysis.— Since $\frac{3}{10}$ of a cord were taken from $\frac{7}{8}$ of a cord, there was left the difference between $\frac{7}{8}$ of a cord and $\frac{3}{10}$ of a cord.

The least common multiple of the denominators is 40. $\frac{7}{8}$ changed to 40ths equals $\frac{35}{40}$, and $\frac{3}{10}$ changed to 40ths equals $\frac{12}{40}$. Now $\frac{35}{40}$ of a cord less $\frac{12}{40}$ of a cord equals $\frac{23}{40}$ of a cord. Hence, etc.

9. How do you subtract dissimilar fractions, or fractions having different denominators?

10. Find the difference between $\frac{3}{4}$ of a mile and $\frac{4}{5}$ of a mile.

11. What fraction increased by $\frac{4}{5}$ becomes $\frac{5}{6}$? Becomes $\frac{6}{7}$?

12. What fraction taken from $\frac{7}{8}$ leaves a remainder of $\frac{5}{6}$?

13. Find what fraction must be added to $\frac{5}{8}$ to make $\frac{7}{10}$.

14. If the sum of two fractions is $\frac{7}{12}$, and the less $\frac{1}{3}$, what is the greater? Take the less from the greater.

15. If the sum of two fractions is $\frac{11}{12}$, and the greater is $\frac{8}{9}$, what is the less? Take the less from the sum.

16. A grocer bought tea at $\$\frac{5}{8}$ a pound, and sold it at $\$\frac{7}{10}$. How much did he gain or lose on it?

17. A man worked $\frac{7}{12}$ of a day, and his son worked $\frac{3}{8}$. How much longer did the man work than his son?

18. A merchant owned $\frac{8}{9}$ of a vessel, and sold $\frac{5}{6}$ of it. What part of the vessel did he then own?

19. If James pays $\frac{7}{10}$ of the cost of a pair of skates, and Charles pays the balance, how much does Charles pay? How much less does Charles pay than James?

20. A boy having $3 spent $\$\frac{7}{10}$. How much remained?

177. Analysis. — $3 equal $2 plus $1, or $2 plus $\$\frac{10}{10}$, or $\$2\frac{10}{10}$. Since he spent $\$\frac{7}{10}$, he had remaining the difference between $\$2\frac{10}{10}$ and $\$\frac{7}{10}$, which is $\$2\frac{3}{10}$. Hence, etc.

21. If 4 miles be diminished by $\frac{7}{8}$ of a mile, what will remain?

22. Find the remainder of 5 tons diminished by $\frac{9}{20}$ of a ton.

23. What is the remainder of 6 yards less $\frac{7}{12}$ of a yard?

24. I owed my grocer $\$8\frac{9}{10}$, and handed him a $10-bill. How much change did I receive?

25. A tailor bought $6\frac{1}{8}$ yards of cloth, and used $3\frac{3}{4}$ yards to make a coat. How much was left?

178. Analysis. — Since a tailor bought $6\frac{1}{8}$ yards, and used $3\frac{3}{4}$ *yards, there remained* the difference between $6\frac{1}{8}$ yards and $3\frac{3}{4}$ yards.

The least common denominator of $\frac{1}{6}$ and $\frac{3}{4}$ is 12. $\frac{1}{6}$ equals $\frac{2}{12}$, and $\frac{3}{4}$ equals $\frac{9}{12}$. Since $\frac{9}{12}$ cannot be taken from $\frac{2}{12}$, take 1, or $\frac{12}{12}$, from 6 ones of the minuend, leaving 5 ones. $\frac{12}{12}$ and $\frac{2}{12}$ are $\frac{14}{12}$; and $\frac{14}{12}$ less $\frac{9}{12}$ are $\frac{5}{12}$, which added to 2, the difference of the integers, equals $2\frac{5}{12}$. Hence, etc.

26. If the minuend is $5\frac{1}{3}$ feet, and the subtrahend is $3\frac{2}{3}$ feet, what is the remainder?

27. If the difference is $2\frac{3}{4}$ miles, and the minuend is $6\frac{5}{8}$ miles, what is the subtrahend?

28. From a field containing $5\frac{3}{8}$ acres I sold $\frac{7}{10}$ of an acre. How much land was left in the field?

29. If the cost of making a pair of boots is $\$5\frac{2}{5}$, and the selling price is $\$7\frac{1}{4}$, what is the profit?

30. A laborer who earns $\$8\frac{3}{10}$ a week expends $\$6\frac{3}{8}$. How much a week does he save?

31. From a piece of beef weighing 10 pounds, a butcher sold $6\frac{3}{10}$ pounds. How many pounds remained?

Find the difference between

32. $\$\frac{7}{8}$ and $\$\frac{3}{4}$.	*37.* 5 sheets and $4\frac{2}{5}$ sheets.	*42.* $10\frac{1}{4}$ and $\frac{7}{8}$.
33. $\$\frac{5}{8}$ and $\$\frac{7}{12}$.	*38.* 6 quires and $3\frac{3}{8}$ quires.	*43.* $12\frac{2}{3}$ and $1\frac{1}{12}$.
34. $\$1\frac{1}{2}$ and $\$\frac{1}{20}$.	*39.* 7 reams and $4\frac{5}{16}$ reams.	*44.* $15\frac{3}{5}$ and $2\frac{9}{10}$.
35. $\$\frac{7}{20}$ and $\$\frac{7}{24}$.	*40.* 8 bales and $5\frac{7}{20}$ bales.	*45.* $18\frac{5}{6}$ and $5\frac{1}{12}$.
36. $\$3\frac{3}{4}$ and $\$1\frac{7}{8}$.	*41.* 9 bales and $6\frac{4}{25}$ bales.	*46.* $20\frac{7}{8}$ and $6\frac{3}{10}$.

47. If I pay $\$6\frac{3}{4}$ for a ton of coal, and $\$2\frac{4}{5}$ for wood, what change should I get from a ten-dollar bill?

179. Analysis. — Since I pay $\$6\frac{3}{4}$ for a ton of coal, and $\$2\frac{4}{5}$ for wood, I pay for both the sum of $\$6\frac{3}{4}$ and $\$3\frac{4}{5}$.

The least common denominator of $\frac{3}{4}$ and $\frac{4}{5}$ is 20. $\frac{3}{4}$ equals $\frac{15}{20}$, and $\frac{4}{5}$ equals $\frac{16}{20}$. Now $\$\frac{15}{20}$ and $\$\frac{16}{20}$ equal $\$\frac{31}{20}$, or $\$1\frac{11}{20}$, which, added to $8, the sum of the integers, is $\$9\frac{11}{20}$.

Since I had $10, and paid out $\$9\frac{11}{20}$, I had remaining the difference between $10 and $\$9\frac{11}{20}$. $10 equal $\$9\frac{20}{20}$; and $\$9\frac{20}{20}$ less $\$9\frac{11}{20}$ are $\$\frac{9}{20}$, the change required. Hence, etc.

48. A man owned $\frac{9}{10}$ of a flour-mill, and sold $\frac{1}{3}$ of the mill to one man, and $\frac{2}{5}$ of it to another. What part had he left?

49. The sum of three numbers is $8\frac{3}{5}$ tons. If two of them are $2\frac{2}{3}$ tons and $\frac{3}{4}$ of a ton, what is the third?

50. Henry had \$10, spent $\$2\frac{3}{10}$, and earned enough to make $\$11\frac{1}{2}$. What did he earn?

51. A boy had $\$9\frac{1}{2}$, then earned \$5, lacking $\$1\frac{5}{8}$. How much had he then? How much more than at first?

52. From two remnants of calico containing $3\frac{3}{8}$ yards and $4\frac{3}{4}$ yards, there were cut $8\frac{1}{12}$ yards. How much less than 1 yard remained? How much more than $\frac{2}{9}$ of a yard?

53. The sum of three numbers is $12\frac{3}{10}$. If one of the numbers is $5\frac{2}{3}$, and another is $4\frac{5}{9}$, what is the third?

54. John earned $\$5\frac{7}{8}$ one week and $\$6\frac{8}{10}$ the next. What did he then lack of having enough to pay \$15 for a coat?

55. Into three milk-cans were poured $15\frac{3}{4}$ quarts of milk. If one contained $5\frac{3}{4}$ quarts, and another $4\frac{3}{8}$ quarts, what did the other contain?

56. William is $10\frac{3}{4}$ years old, and James is $12\frac{5}{12}$ years old. How many years less is the sum of their ages than their father's age, which is $35\frac{1}{2}$ years?

180. Principles.

I. *Only fractional units of the same kind of things can be taken one from another.*

II. *Only similar fractions can be taken one from another.*

Exercises.

Find the remainder of $\frac{7}{10}$ less $\frac{7}{15}$.

181. Model. — The least common denominator of $\frac{7}{15}$ and $\frac{7}{10}$ is 30. $\frac{7}{10}$ equals $\frac{21}{30}$, and $\frac{7}{15}$ equals $\frac{14}{30}$. Now $\frac{21}{30}$ less $\frac{14}{30}$ equals $\frac{7}{30}$, the *remainder required.*

Find the result of

1. $\frac{5}{8}$ less $\frac{5}{12}$.	6. $10 - 8\frac{1}{4}$.	11. $7\frac{5}{6} + \frac{8}{9} - \frac{5}{12}$.
2. $\frac{7}{12}$ less $\frac{7}{15}$.	7. $9 - 7\frac{5}{6}$.	12. $3\frac{3}{10} - \frac{1}{2} + \frac{4}{5}$.
3. $\frac{8}{15}$ less $\frac{9}{20}$.	8. $8\frac{1}{3} - 2\frac{2}{7}$.	13. $5\frac{1}{8} + \frac{3}{6} - 1\frac{1}{12}$.
4. $\frac{9}{14}$ less $\frac{10}{21}$.	9. $7\frac{4}{5} - 3\frac{3}{4}$.	14. $7\frac{4}{5} - 2\frac{5}{6} + 3\frac{1}{10}$.
5. $\frac{11}{16}$ less $\frac{11}{24}$.	10. $6\frac{5}{6} - 4\frac{8}{9}$.	15. $9\frac{1}{6} - 4\frac{4}{7} - 3\frac{2}{3}$.

Section V.

MULTIPLICATION OF FRACTIONS.

1. *To Multiply a Fraction by an Integer.*

1. How many fourths are 3 times 1 fourth? 2 times $\frac{3}{4}$?

2. What part of a dollar are 2 times 3 fifths of a dollar?

3. How many sixths are 3 times $\frac{2}{6}$? 4 times $\frac{3}{6}$? 6 times $\frac{5}{6}$?

4. At $\$\frac{3}{5}$ a pound, what do 4 pounds of tea cost?

182. ***Analysis.*** — Since 1 pound of tea costs $\$\frac{3}{5}$, 4 pounds cost 4 times $\$\frac{3}{5}$, which are $\$\frac{12}{5}$, or $\$2\frac{2}{5}$. Hence, etc.

5. How many are 5 times $\frac{2}{7}$? 6 times $\frac{5}{8}$? 8 times $\frac{7}{9}$?

6. If a horse eats $\frac{3}{8}$ of a bushel of oats in a day, how much will 2 horses eat? 5 horses? 7 horses? 9 horses?

7. At $\$\frac{5}{8}$ a bushel, how much will 5 bushels of corn cost? 7 bushels? 10 bushels? 12 bushels?

8. What is the result, in its lowest terms, if the numerator of $\frac{4}{15}$ is multiplied by 3? If the denominator is divided by 3?

9. Show that multiplying the numerator of $\frac{3}{20}$ by 5 multiplies the fraction by 5. Give the reason.

10. Show that dividing the denominator of $\frac{3}{20}$ by 5 multiplies the fraction by 5. Give the reason.

11. Show in what two ways a fraction may be multiplied by an integer. Give an example of each.

12. At $\$6\frac{3}{4}$ a ton, what is the cost of 5 tons of coal?

183. Analysis. — Since 1 ton costs $\$6\frac{3}{4}$, 5 tons cost 5 times $\$6\frac{3}{4}$. 5 times $\$\frac{3}{4}$ are $\$\frac{15}{4}$, or $\$3\frac{3}{4}$; 5 times \$6 are \$30. Now \$30 and $\$3\frac{3}{4}$ are $\$33\frac{3}{4}$. Hence, etc.

13. If 1 rod is $5\frac{1}{2}$ yards, how many yards are 9 rods?

14. How much milk is needed to fill 5 cans, each of which holds $2\frac{1}{4}$ gallons? To fill 7 cans? To fill 9 cans?

15. At $\$7\frac{4}{5}$ a barrel, how much will 6 barrels of XX flour cost? 8 barrels? 9 barrels? 10 barrels?

Find the result of

16. 4 times $\frac{2}{5}$ of a ton.	*21.* $3\frac{2}{3}$ mills $\times$ 2.	*26.* 10 times $6\frac{3}{4}$.
17. 6 times $\frac{3}{4}$ of a foot.	*22.* $5\frac{3}{4}$ cents $\times$ 3.	*27.* 12 times $8\frac{5}{8}$.
18. 7 times $\frac{7}{8}$ of a year.	*23.* $7\frac{4}{5}$ dimes $\times$ 4.	*28.* 15 times $10\frac{7}{10}$.
19. 8 times $\frac{8}{9}$ of a yard.	*24.* $9\frac{7}{8}$ dollars $\times$ 5.	*29.* 20 times $12\frac{5}{12}$.
20. 9 times $\frac{9}{10}$ of a mile.	*25.* $11\frac{9}{10}$ eagles $\times$ 6.	*30.* 25 times $20\frac{9}{10}$.

184. Principle.

A fraction is multiplied by multiplying its numerator or dividing its denominator by any integer.

Exercises.

How much is 8 times $3\frac{3}{8}$?

185. Model. — 8 times $\frac{3}{8}$ are $\frac{24}{8}$, or $5\frac{1}{3}$; 8 times 3 are 24; and 24 plus $5\frac{1}{3}$ are $29\frac{1}{3}$.

Find the product of

1. 10 times $\frac{4}{5}$.	*6.* $10\frac{3}{4} \times 6$.	*11.* 7 times $11\frac{5}{7}$.	*16.* $20\frac{4}{5} \times 10$.
2. 12 times $\frac{7}{8}$.	*7.* $12\frac{5}{6} \times 8$.	*12.* 9 times $13\frac{5}{6}$.	*17.* $25\frac{9}{10} \times 12$.
3. 15 times $\frac{9}{10}$.	*8.* $18\frac{7}{8} \times 10$.	*13.* 15 times $20\frac{7}{10}$.	*18.* $30\frac{4}{15} \times 15$.
4. 18 times $\frac{7}{12}$.	*9.* $20\frac{9}{10} \times 12$.	*14.* 20 times $25\frac{7}{15}$.	*19.* $35\frac{7}{20} \times 20$.
5. 20 times $\frac{11}{15}$.	*10.* $25\frac{11}{12} \times 20$.	*15.* 25 times $30\frac{9}{20}$.	*20.* $40\frac{3}{25} \times 25$.

2. *To Multiply an Integer by a Fraction.*

1. How much is $\frac{1}{2}$ of 6 cents? $\frac{1}{3}$ of 8 feet? $\frac{1}{4}$ of 16 tons?

2. How much is 1 time 5 miles? $\frac{1}{2}$ of 1 time 5 miles? $\frac{1}{3}$ of 1 time 9 men? $\frac{1}{4}$ of 12 dollars? $\frac{1}{6}$ of 15 years?

3. Multiplying by $\frac{1}{3}$ is the same as dividing by what integer? Multiplying by $\frac{1}{5}$? By $\frac{1}{7}$? By $\frac{1}{9}$?

4. $\frac{3}{4}$ is how many times $\frac{1}{4}$? How much is $\frac{1}{4}$ of 8 dollars? $\frac{3}{4}$ of 8 dollars? $\frac{1}{5}$ of 10 days? $\frac{4}{5}$ of 12 hours?

5. Multiplying \$20 by $\frac{4}{5}$ is the same as dividing \$20 by what number, and then multiplying the quotient by what?

6. At \$20 a ton, what will $\frac{7}{8}$ of a ton of hay cost?

186. Analysis. — Since 1 ton costs \$20, $\frac{7}{8}$ of a ton will cost 7 times $\frac{1}{8}$ of \$20, or $\frac{7}{8}$ of \$20. $\frac{1}{8}$ of \$20 is $\$\frac{20}{8}$, and $\frac{7}{8}$ of \$20 are 7 times $\$\frac{20}{8}$, which are $\$\frac{140}{8}$, or \$17$\frac{4}{8}$, or \$17$\frac{1}{2}$. Hence, etc.

7. If a man walks 55 miles in 3 days, how far does he walk in 1 day? In 2 days?

8. A farmer raised 94 bushels of wheat on 4 acres. What is the average yield per acre? Of 5 acres?

9. If a man can do a piece of work in 40 days, in what time can he do $\frac{1}{4}$ of it? $\frac{3}{4}$ of it? $\frac{1}{8}$ of it? $\frac{5}{8}$? $\frac{1}{10}$ of it? $\frac{9}{10}$?

10. When wood is worth \$6 a cord, what must be paid for $\frac{5}{8}$ of a cord? For $\frac{7}{9}$ of a cord? $\frac{9}{10}$?

11. At the rate of \$7 for a ton of coal, what is the price of $\frac{3}{10}$ of a ton? Of $\frac{7}{10}$ of a ton? Of $\frac{9}{10}$ of a ton?

12. I bought 8 thousand feet of lumber, and used $\frac{4}{5}$ of it in building a barn. How many feet did I use?

13. The cleared land of a farm of 72 acres is $\frac{3}{9}$ of the whole farm. How many acres are woodland?

14. What is the result of $\frac{4}{5}$ of \$15? The product of $15 \times \frac{4}{5}$? $\frac{5}{8}$ of 20 feet? The product of 20 feet $\times \frac{5}{8}$?

15. Multiplying an integer by $\frac{3}{4}$ is the same as taking what part of the integer? Multiplying by $\frac{4}{5}$? By $\frac{5}{8}$? By $\frac{6}{7}$?

16. Show by an example that multiplying an integer by a fraction is the same as taking the fractional part of the integer.

17. Show in what two ways an integer may be multiplied by a fraction. Give an example of each way.

Find the product of

18. 10 miles multiplied by $\frac{3}{5}$.	*22.* $\frac{3}{7}$ of 21 years.	*26.* $15 \times \frac{5}{6}$.
19. 12 rods multiplied by $\frac{7}{8}$.	*23.* $\frac{5}{8}$ of 24 months.	*27.* $20 \times \frac{6}{7}$.
20. 15 yards multiplied by $\frac{8}{9}$.	*24.* $\frac{7}{12}$ of 48 weeks.	*28.* $25 \times \frac{7}{8}$.
21. 20 feet multiplied by $\frac{9}{10}$.	*25.* $\frac{9}{18}$ of 72 hours.	*29.* $30 \times \frac{8}{9}$.

30. At 15 cents a pound, what will 2 pounds of sugar cost? $\frac{1}{5}$ of a pound? $2\frac{1}{5}$ pounds? $\frac{3}{5}$ of a pound? $2\frac{3}{5}$ pounds?

31. What will $4\frac{3}{5}$ tons of coal cost at \$6 a ton?

187. Analysis. — Since 1 ton costs \$6, $4\frac{3}{5}$ tons cost $4\frac{3}{5}$ times \$6. 4 times \$6 are \$24; $\frac{3}{5}$ of \$6 are 3 times $\frac{1}{5}$ of \$6, or 3 times $\$\frac{6}{5}$, which are $\$1\frac{1}{5}$, or $\$3\frac{3}{5}$; and \$24 plus $\$3\frac{3}{5}$ are $\$27\frac{3}{5}$. Hence, etc.

32. Find the cost of $7\frac{3}{4}$ pounds of rice at 7 cents a pound.

33. How many miles will a yacht sail in $4\frac{7}{8}$ hours, at the rate of 10 miles an hour? 12 miles an hour?

34. What is the cost of $3\frac{5}{12}$ dozen lead-pencils at 20 cents a dozen? Of $5\frac{7}{12}$ dozen? $7\frac{11}{12}$ dozen?

35. How far will a railroad train run in $5\frac{3}{4}$ hours, at the rate of 24 miles an hour? In $6\frac{3}{8}$ hours? $7\frac{5}{12}$ hours?

36. A and B bought a reaper for \$75, A paying $\frac{3}{10}$ of the cost, and B $2\frac{1}{3}$ times as much. What did each pay?

188. Principle.

An integer is multiplied by a fraction by taking such part of the integer as the fraction indicates.

Exercises.

How much is $\frac{2}{3}$ of $\frac{5}{6}$ of 12 tons?

189. Model. — $\frac{5}{6}$ of 12 tons are 5 times $\frac{1}{6}$ of 12 tons, or 5 times $\frac{12}{6}$ of a ton, or $\frac{60}{6}$ of a ton, or 10 tons; and $\frac{2}{3}$ of 10 tons are 2 times $\frac{1}{3}$ *of 10 tons, or 2 times* $\frac{10}{3}$ of a ton, or $\frac{20}{3}$ of a ton, or $6\frac{2}{3}$ tons. Hence, etc.

Find the result of

1. $\frac{2}{3}$ of 10.	6. $\frac{1}{2}$ of $\frac{4}{5}$ of 40.	11. $8 \times 8\frac{3}{4}$.	16. $7\frac{2}{3}$ times 12.
2. $12 \times \frac{4}{5}$.	7. $\frac{1}{3}$ of $\frac{5}{6}$ of 54.	12. $9\frac{7}{8} \times 6$.	17. 8 times $10\frac{5}{6}$.
3. $\frac{4}{5}$ of 13.	8. $\frac{2}{5}$ of $\frac{6}{7}$ of 63.	13. $10 \times 7\frac{5}{6}$.	18. $9\frac{3}{4}$ times 14.
4. $14 \times \frac{5}{6}$.	9. $\frac{4}{7}$ of $\frac{7}{9}$ of 81.	14. $12\frac{7}{8} \times 12$.	19. 10 times $15\frac{7}{8}$.
5. $\frac{5}{6}$ of 15.	10. $\frac{5}{6}$ of $\frac{9}{10}$ of 90.	15. $18 \times 10\frac{8}{9}$.	20. $12\frac{3}{5}$ times 15.

3. *To Multiply a Fraction by a Fraction.*

1. If $\frac{1}{2}$ of an apple is cut into 2 equal pieces, what part of the whole apple is each piece?

2. What part of an apple is 1 half of $\frac{1}{4}$ of an apple?

3. What part of 1 is one third of $\frac{1}{4}$? One fourth of $\frac{1}{4}$?

4. How much is $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{5}$ of $\frac{1}{6}$?

5. If I cut one half of $\frac{2}{3}$ of a yard from a piece of ribbon, what fraction of a yard do I cut off?

6. How much is $\frac{1}{2}$ of 2 tons? $\frac{1}{2}$ of 4 feet? $\frac{1}{2}$ of 2 thirds? $\frac{1}{2}$ of 4 fifths? $\frac{1}{2}$ of $\frac{6}{7}$? $\frac{1}{2}$ of $\frac{8}{9}$? $\frac{1}{3}$ of $\frac{6}{7}$? $\frac{1}{3}$ of $\frac{9}{10}$?

7. How much is $\frac{1}{3}$ of 6 tons? $\frac{2}{3}$ of 6 tons? $\frac{1}{3}$ of 6 sevenths? $\frac{2}{3}$ of $\frac{6}{7}$? $\frac{1}{4}$ of 8 yards? $\frac{3}{4}$ of $\frac{8}{9}$? $\frac{3}{5}$ of $\frac{10}{11}$?

8. At $\$\frac{4}{5}$ a bushel, what is the cost of $\frac{7}{8}$ of a bushel of corn?

190. ***Analysis.*** — Since 1 bushel of corn costs $\$\frac{4}{5}$, the cost of $\frac{1}{8}$ of a bushel is $\frac{1}{8}$ of $\$\frac{4}{5}$.

$\frac{1}{8}$ of $\$\frac{4}{5}$ are 7 times $\frac{1}{8}$ of $\$\frac{4}{5}$. $\frac{1}{8}$ of $\$\frac{1}{5}$ is $\$\frac{1}{40}$; $\frac{1}{8}$ of $\$\frac{4}{5}$ is 4 times $\$\frac{1}{40}$, or $\$\frac{4}{40}$; and $\frac{7}{8}$ of $\$\frac{4}{5}$ are 7 times $\$\frac{4}{40}$, or $\$\frac{28}{40}$, or $\$\frac{7}{10}$. Hence, etc.

9. If a pound of tea costs $\$\frac{8}{10}$, what is the cost of $\frac{1}{4}$ of a pound? Of $\frac{3}{4}$? $\frac{1}{5}$ of a pound? $\frac{4}{5}$? $\frac{1}{8}$ of a pound? $\frac{7}{8}$?

10. If I have $\frac{6}{10}$ of an acre of land, and sell $\frac{1}{3}$ of it, what part of an acre do I sell? What part do I keep?

11. If a carter feeds his horse $\frac{2}{5}$ of $\frac{5}{8}$ of a bushel of oats in a day, what part of a bushel does he feed his horse?

12. If a family burns $\frac{3}{4}$ of a gallon of kerosene in a week, what part of a gallon will be burned in $\frac{5}{7}$ of a week?

13. A man owned $\frac{7}{8}$ of a factory, and sold $\frac{3}{4}$ of his share to his brother. What part did he then own?

14. If two men own a mill in equal shares, and one of them sells $\frac{2}{5}$ of his share, what part of the mill does he then own?

15. Since $\frac{3}{4}$ of $\frac{3}{5}$, or the product of $\frac{3}{4} \times \frac{3}{5}$ is $\frac{9}{20}$, how may the numerator of the product be found from the terms of the factors. How may the denominator of the product be found?

Find the product of

16. $\frac{4}{5}$ of a pound multiplied by $\frac{2}{3}$.

17. $\frac{5}{6}$ of an ounce multiplied by $\frac{3}{4}$.

18. $\frac{7}{8}$ of a dozen multiplied by $\frac{5}{6}$.

19. $\frac{8}{9}$ of a gallon multiplied by $\frac{7}{8}$.

20. $\frac{9}{10}$ of a ton by $\frac{5}{6}$.

21. $\frac{7}{12}$ of a gross by $\frac{8}{9}$.

22. $\frac{11}{21}$ of a barrel by $\frac{7}{10}$.

23. $\frac{13}{40}$ of an acre by $\frac{7}{20}$.

24. At $\$2\frac{1}{4}$ a yard, what is the cost of $\frac{2}{3}$ of a yard of cloth?

191. Analysis. — Since 1 yard of cloth costs $\$2\frac{1}{4}$, the cost of $\frac{2}{3}$ of a yard is $\frac{2}{3}$ of $\$2\frac{1}{4}$, or $\frac{2}{3}$ of $\$\frac{9}{4}$. $\frac{2}{3}$ of $\$\frac{9}{4}$ are 2 times $\frac{1}{3}$ of $\$\frac{9}{4}$, or 2 times $\$\frac{9}{12}$, or $\$1\frac{8}{12}$, or $\$1\frac{1}{2}$. Hence, etc.

25. At $\$6\frac{3}{4}$ a barrel, what will $\frac{1}{4}$ of a barrel of flour cost? $\frac{3}{4}$? $\frac{1}{8}$ of a barrel? $\frac{3}{8}$? $\frac{1}{6}$ of a barrel? $\frac{5}{6}$?

26. If a man can put up $3\frac{3}{5}$ rods of stone wall in a day, how much can he put up in $\frac{5}{6}$ of a day?

27. If a horse trots $8\frac{3}{4}$ miles an hour, how far does he trot in $\frac{4}{5}$ of an hour? In $\frac{6}{7}$ of an hour?

28. If a pound of sugar is worth $\frac{4}{5}$ of $17\frac{1}{2}$ cents, how much will $\frac{3}{4}$ of $1\frac{1}{7}$ pounds cost? $\frac{3}{5}$ of $2\frac{2}{9}$ pounds?

29. At $\$\frac{3}{4}$ a pound, what will $3\frac{2}{3}$ pounds of tea cost?

192. Analysis. — Since 1 pound of tea costs $\$\frac{3}{4}$, $3\frac{2}{3}$ pounds cost $3\frac{2}{3}$ times $\$\frac{3}{4}$, or $\frac{11}{3}$ of $\$\frac{3}{4}$. $\frac{1}{3}$ of $\$\frac{3}{4}$ is $\$\frac{1}{4}$, and $\frac{11}{3}$ of $\$\frac{3}{4}$ etc.

30. What are $3\frac{2}{3}$ times $\$\frac{4}{5}$? $4\frac{3}{4}$ times $\frac{3}{7}$ of a day? $5\frac{3}{5}$ times $\$\frac{3}{10}$? $6\frac{5}{8}$ times $\frac{5}{12}$ of a yard? $7\frac{6}{7}$ times $\frac{7}{18}$?

31. If a farmer can mow $\frac{7}{8}$ of an acre of grass in a day, how much can he mow in $2\frac{1}{4}$ days? In $3\frac{3}{7}$ days? In $2\frac{3}{10}$ days?

32. Emma is $15\frac{3}{4}$ years old, and her sister is $4\frac{2}{3}$ times as *old. What is the sister's* age? Find the sum of their ages.

33. What will $2\frac{1}{4}$ yards of silk cost at $\$2\frac{2}{3}$ a yard?

193. Analysis. — Since 1 yard of silk costs $\$2\frac{2}{3}$, $2\frac{1}{4}$ yards will cost $2\frac{1}{4}$ times $\$2\frac{2}{3}$, or $\frac{9}{4}$ of $\$\frac{8}{3}$. $\frac{1}{4}$ of $\$\frac{8}{3}$ is $\$\frac{8}{12}$, etc.

34. What are $4\frac{1}{2}$ times $2\frac{2}{5}$ years? $3\frac{2}{3}$ times $7\frac{1}{2}$ feet? $2\frac{1}{7}$ times $4\frac{1}{8}$ gallons? $4\frac{2}{3} \times 3\frac{3}{4}$?

35. If a teamster hauls $3\frac{3}{5}$ cords of wood in a day, how much will he haul in $2\frac{1}{2}$ days? In $6\frac{2}{3}$ days?

36. A merchant had $8\frac{3}{4}$ yards of broadcloth, and sold $\frac{5}{7}$ of it at $\$2\frac{2}{3}$ a yard. What did he receive for it?

37. How much will $\frac{7}{8}$ of $5\frac{1}{3}$ tons of coal cost at $\$6\frac{3}{4}$ a ton? What will $\frac{5}{9}$ of $6\frac{3}{4}$ tons cost?

Find the product of

38. $6\frac{2}{3}$ yards multiplied by $\frac{3}{4}$.

39. $6\frac{3}{4}$ tons multiplied by $\frac{5}{9}$.

40. $9\frac{3}{5}$ cords multiplied by $\frac{7}{8}$.

41. $\$\frac{9}{10}$ multiplied by $4\frac{2}{7}$.

42. $\frac{2}{3}$ of $\frac{6}{7}$ of $10\frac{1}{2}$ gallons.

43. $6\frac{1}{4}$ times $\frac{4}{5}$ of 15 bushels.

44. 12 bales $\times$ $\frac{2}{5}$ of $\frac{5}{6}$ of $4\frac{2}{7}$.

45. $\frac{3}{5}$ of 4 times $6\frac{1}{4} \times \frac{3}{4} \times 6$.

46. What number added to 4 times $\frac{2}{5}$ of 28 miles will make the number equal to $45\frac{1}{2}$ miles?

47. A laborer works 2 days for $\$1\frac{3}{4}$ and $\$1\frac{7}{8}$. At 2 dollars a day, how much more would he have received?

48. A painter used $\frac{2}{3}$ of $\frac{6}{7}$ of a keg of white-lead worth $\$2\frac{5}{8}$. What part of the keg did he use, and what was it worth?

49. I paid $\$6\frac{1}{4}$ for a ton of coal, and $1\frac{1}{3}$ times as much for a barrel of flour. How much did both cost?

50. George had $\$1\frac{3}{5}$, and paid $\frac{3}{4}$ of it for a knife. What did he gain or lose by selling it for $\$1\frac{3}{8}$?

51. If I buy a cow for $\$37\frac{1}{2}$, and sell her for $\frac{9}{10}$ of the cost, how much will I lose?

52. Suppose a coat costs $\frac{6}{7}$ of $\$9\frac{1}{5}$, and a vest costs $\frac{4}{7}$ as much. How much do both cost?

53. Mary is $3\frac{3}{4}$ years old, and Helen is 5 times as old as Mary. In how many years will Helen be 21 years old?

54. $\frac{3}{4}$ of $5\frac{1}{3}$ times $2\frac{2}{5}$ tens are how many more than $2\frac{3}{4}$ tens? How many less than $\frac{1}{2}$ of $20\frac{2}{3}$ tens?

55. If I buy $\frac{3}{8}$ of $10\frac{2}{3}$ yards of muslin at $\frac{3}{4}$ of 25 cents a yard, how much less than $\$\frac{9}{10}$ do I pay for it?

56. Harry's wages amount to $\$6\frac{3}{8}$ a week, and his expenses are $\$4\frac{7}{10}$. How much can he save in 4 weeks?

Definitions.

194. A ***Simple Fraction*** is a fraction whose numerator and denominator are both integers.

Thus, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{8}$, and $\frac{15}{7}$ are *simple fractions.*

195. A ***Compound Fraction*** is a fraction of a fraction; it consists of two or more fractions connected by *of.*

Thus, $\frac{2}{3}$ *of* $\frac{3}{4}$, and $\frac{5}{6}$ *of* $\frac{6}{8}$ *of* $2\frac{1}{7}$ are *compound fractions.*

196. Principle.

A fraction is multiplied by a fraction by taking such part of the fraction as the fractional multiplier indicates.

Exercises.

Find the product of $\frac{2}{3}$ of $2\frac{1}{2}$ times $3\frac{1}{5}$.

197. Model. — $2\frac{1}{2}$ equals $\frac{5}{2}$, and $3\frac{1}{5}$ equals $\frac{16}{5}$. $\frac{5}{2}$ times $\frac{16}{5}$, or $\frac{5}{2}$ of $\frac{16}{5}$, is 5 times $\frac{1}{2}$ of $\frac{16}{5}$, or 5 times $\frac{8}{5}$, which are $\frac{40}{5}$, or 8; and $\frac{2}{3}$ of 8 is 2 times $\frac{1}{3}$ of 8, or 2 times $\frac{8}{3}$, or $\frac{16}{3}$, or $5\frac{1}{3}$.

Find the value of

1. $\frac{1}{3}$ *of* $\frac{4}{5}$ *of* $2\frac{1}{7}$.
2. $\frac{1}{4}$ *of* $\frac{2}{5}$ *of* $6\frac{2}{3}$.
3. $\frac{1}{5}$ *of* $\frac{5}{6}$ *of* $8\frac{2}{5}$.
4. $\frac{2}{3}$ *of* $\frac{6}{7}$ *of* $9\frac{4}{5}$.
5. $\frac{3}{4}$ *of* $\frac{8}{9}$ *of* $10\frac{2}{7}$.
6. $7\frac{1}{2}$ *times* $\$2\frac{4}{5}$.
7. $3\frac{3}{4}$ *feet* $\times\ 8\frac{2}{3}$.
8. $6\frac{2}{3}$ *times* $\$5\frac{8}{10}$.
9. $8\frac{2}{5}$ *tons* $\times\ 6\frac{3}{7}$.
10. $9\frac{3}{5}$ *times* $\$9\frac{1}{6}$.
11. $\frac{3}{4}$ *of* $3\frac{1}{3} + 6\frac{1}{2}$.
12. $4\frac{1}{2} \times 2\frac{2}{3} - 11\frac{1}{8}$.
13. $2\frac{1}{5} + 3\frac{1}{4} \times 6\frac{2}{3}$.
14. $3\frac{3}{7} \times 3\frac{1}{4} + 4\frac{1}{3}$.
15. $4\frac{1}{6}$ *times* $(6\frac{1}{5} - 4\frac{2}{3})$.

Section VI.

DIVISION OF FRACTIONS.

1. *To Divide a Fraction by an Integer.*

1. If you divide 3 fourths of an apple equally among three boys, what part of $\frac{3}{4}$ does each boy get? What part of the apple?

2. Dividing 4 fifths by 4 is taking what part of $\frac{4}{5}$? Tell the quotient of $\frac{4}{5}$ divided by 4; $\frac{6}{7}$ divided by 3; $\frac{8}{9} \div 4$.

3. Dividing $\frac{5}{8}$ by 5 is taking what part of $\frac{5}{8}$? It is the same as multiplying $\frac{5}{8}$ by what fraction?

4. Dividing by 3 is the same as multiplying by what fraction? Dividing by 5? By 7? By 9? By 6? By 8?

5. How much is $\frac{8}{11}$ divided by 4, or $\frac{1}{4}$ of $\frac{8}{11}$? $\frac{10}{13}$ divided by 5, or $\frac{1}{5}$ of $\frac{10}{13}$? $\frac{14}{17} \div 7$, or $\frac{1}{7}$ of $\frac{14}{17}$, or $\frac{14}{17} \times \frac{1}{7}$?

6. Show that dividing the numerator of a fraction divides the value of the fraction. Give an example.

7. If $\frac{1}{4}$ of a cake is cut into 2 equal parts, what part of $\frac{1}{4}$ is each piece? What part of the cake?

8. How much is $\frac{1}{2}$ of $\frac{1}{5}$, or $\frac{1}{5}$ divided by 2? Of $\frac{2}{3}$ divided by 3, or $\frac{1}{3}$ of $\frac{2}{3}$? $\frac{3}{4} \div 5$, or $\frac{1}{5}$ of $\frac{3}{4}$, or $\frac{3}{4} \times \frac{1}{5}$? $\frac{4}{5} \div 6$, or $\frac{4}{5} \times \frac{1}{6}$?

9. Show that multiplying the denominator of a fraction divides the value of the fraction. Give an example.

10. At $\$\frac{3}{4}$ for 5 pounds of sugar, what is the price of 1 pound?

198. Analysis. — Since 5 pounds of sugar cost $\$\frac{3}{4}$, the price of 1 pound is $\frac{1}{5}$ of $\$\frac{3}{4}$, which is $\$\frac{3}{20}$. Hence, etc.

11. If $\frac{4}{5}$ of an acre of land is divided into 3 equal lots, what part of an acre is in each lot?

12. If $\frac{5}{6}$ of a vessel is owned in equal shares by 4 men, what part of the ship is each man's share?

13. If a man can do $\frac{7}{8}$ of a piece of work in 4 days, what part of *the work can* he do in 1 day?

14. At \$3 a yard, how many yards of cloth can be bought for \$13$\frac{4}{5}$?

199. Analysis. — At \$3 a yard, there can be bought for \$13$\frac{4}{5}$ as many yards as the number of times 3 is contained in 13$\frac{4}{5}$. Now $\frac{1}{3}$ of 13$\frac{4}{5}$ is $\frac{1}{3}$ of $\frac{69}{5}$, or $\frac{23}{5}$, which equals 4$\frac{3}{5}$. Hence, etc.

Or, 3 is contained in 13$\frac{4}{5}$ 4 times, with a remainder of 1$\frac{4}{5}$, or $\frac{9}{5}$; and $\frac{1}{3}$ of $\frac{9}{5}$ is $\frac{3}{5}$, which, added to 4, equals 4$\frac{3}{5}$. Hence, etc.

15. If a fruit dealer sold a dozen boxes of berries for \$2$\frac{1}{2}$, what was the price per box? If for \$3$\frac{1}{4}$?

16. How many times will 14$\frac{2}{5}$ gallons of vinegar fill a vessel that holds 3 gallons? A 4-gallon vessel?

17. If a man earns \$22$\frac{3}{4}$ in 10 days, how much does he earn in 1 day? In 4 days? In 6 days? 8 days?

What is the quotient of

18. $\frac{2}{3}$ of a bale divided by 6?

19. $\frac{4}{5}$ of a cord divided by 7?

20. $\frac{5}{8}$ of an acre divided by 8?

21. $\frac{9}{10}$ of a mile divided by 9?

22. 12$\frac{1}{2}$ tons ÷ 4 tons?

23. 13$\frac{1}{3}$ feet ÷ 8 feet?

24. 16$\frac{2}{3}$ days ÷ 10 days?

25. 18$\frac{3}{4}$ weeks ÷ 12 weeks?

200. Principle.

A fraction is divided by dividing its numerator or by multiplying its denominator by any integer.

Exercises.

Find the value of $\frac{8}{9}$ divided by 4.

201. Model. — The value of $\frac{8}{9}$ divided by 4 is $\frac{1}{4}$ of $\frac{8}{9}$, which is $\frac{2}{9}$.

Find the value of

1. $\frac{10}{12} \div 5$.
2. $\frac{12}{18} \div 6$.
3. $\frac{5}{10} \div 7$.
4. $\frac{6}{11} \div 8$.
5. $\frac{7}{12} \div 9$.
6. $10\frac{5}{6} \div 4$.
7. $12\frac{6}{6} \div 6$.
8. $15\frac{6}{7} \div 8$.
9. $18\frac{7}{8} \div 7$.
10. $20\frac{8}{9} \div 9$.
11. $22\frac{1}{2} \div \frac{1}{2}$ of 8.
12. $\frac{2}{3}$ of $18\frac{3}{4} \div 5$.
13. $30\frac{1}{4} \div \frac{3}{4}$ of 8.
14. $\frac{4}{5}$ of $26\frac{1}{4} \div 7$.
15. $45\frac{1}{3} \div \frac{5}{6}$ of 12.
16. $1\frac{7}{20} \div (8 \times \frac{3}{4})$.
17. $(10\frac{2}{3} \div 5) + 6\frac{1}{5}$.
18. $(\frac{16}{17} \div 8) + \frac{1}{5}$ of $1\frac{1}{4}$.
19. $6\frac{3}{4} \times 4 - 10\frac{1}{2} \div 7$.
20. $(\frac{3}{10} + \frac{4}{5}) \div \frac{3}{4}$ of 8.

2. To Divide an Integer by a Fraction.

1. How many fourths of a cake in 1 cake? In 2 cakes?

2. How many times $\$\frac{1}{4}$ in \$1? In \$2? In \$3? In \$4?

3. 1 is how many times $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$?

4. Dividing by $\frac{1}{2}$ is the same as multiplying the dividend by what integer? Dividing by $\frac{1}{4}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$?

5. What is the quotient of $1 \div \frac{1}{4}$? Of $1 \div \frac{3}{4}$? The quotient of $1 \div \frac{3}{4}$ is what part of the quotient of $1 \div \frac{1}{4}$?

6. 1 is how many times $\frac{2}{3}$? How many times $\frac{3}{4}$? $\frac{4}{5}$? $\frac{6}{7}$? $\frac{8}{9}$?

7. 3 is how many times $\frac{1}{5}$? How many times $\frac{2}{5}$? 4 is how many times $\frac{1}{6}$? $\frac{5}{6}$? 5 is how many times $\frac{2}{3}$? $\frac{4}{5}$? $\frac{6}{7}$?

8. What is the quotient of $6 \div \frac{7}{8}$? It is the same as 6 multiplied by what fraction? How, then, may an integer be divided by a fraction?

9. At $\$\frac{4}{5}$ a yard, how many yards of linen will \$6 buy?

202. *Analysis.* — Since 1 yard of linen costs $\$\frac{4}{5}$, there can be bought for \$6 as many yards as the number of times $\$\frac{4}{5}$ are contained in \$6. 6 equals $\frac{30}{5}$; and 4 fifths are contained in 30 fifths $7\frac{1}{2}$ times.

Or, $\$\frac{4}{5}$ are contained in \$1 $\frac{5}{4}$ times; and $\$\frac{4}{5}$ are contained in \$6 6 times $\frac{5}{4}$ times, or $\frac{30}{4}$ times, or $7\frac{1}{2}$ times. Hence, etc.

10. If a fruit-jar holds $\frac{3}{4}$ of a gallon, how many jars will hold 3 gallons? 6 gallons? 9 gallons? 12 gallons?

11. If $\frac{5}{8}$ of a yard of cloth will make a vest, how many vests will 5 yards make? 10 yards? 15 yards?

12. A merchant paid \$30 for a piece of linen at $\$\frac{8}{10}$ a yard. How many yards were in the piece?

13. At $\$\frac{7}{8}$ a bushel for oats, how many bushels can be bought for \$6? For \$8? For \$10? For \$20?

14. At $\$1\frac{4}{5}$ a bushel, how many bushels of wheat can be bought for \$18?

203. *Analysis.* — Since one bushel of wheat costs $\$1\frac{4}{5}$, for \$18 there can be bought as many bushels as the number of times $\$1\frac{4}{5}$, or $\$\frac{9}{5}$, are contained in \$18, or $\frac{90}{9}$ of a bushel, or 10 bushels. Hence, etc.

15. How many coats can be made from 15 yards of cloth, if each contains $2\frac{1}{2}$ yards? From 20 yards?

16. How many yards of cloth can be made from 18 pounds of wool, allowing $1\frac{2}{3}$ pounds to the yard?

17. How many rolls of wall-paper, each containing $4\frac{1}{2}$ square yards, are required to cover 27 square yards of wall?

18. If a boy who earns $\$6\frac{1}{4}$ a week receives \$75 for his work, how many weeks has he worked?

19. Into how many fields, each containing $6\frac{4}{5}$ acres, can a farm of 68 acres be divided? A farm of 102 acres?

20. Show that the quotient of an integer by a fraction equals the product of the integer by the fraction inverted.

What is the quotient of

21. 12 gross $\div \frac{3}{4}$ of a gross?
22. 15 days $\div \frac{7}{8}$ of a day?
23. 18 quarts $\div \frac{9}{10}$ of a quart?
24. 22 chains $\div 1\frac{1}{12}$ of a chain?
25. 12 feet $\div \frac{3}{4}$ of 5 feet?
26. 15 yards $\div \frac{4}{5}$ of 6 yards?
27. 20 bales $\div \frac{5}{8}$ of 10 bales?
28. 25 quires $\div \frac{7}{8}$ of 25 quires?

204. Principle.

Multiplying the dividend multiplies the quotient, and dividing the dividend divides the quotient.

Exercises.

Find the quotient of 8 divided by $\frac{2}{3}$.

205. Model. — Since 8 divided by 1 equals 8, 8 divided by $\frac{1}{3}$ equals 9 times 8, or 72; and 8 divided by $\frac{2}{3}$ equals $\frac{1}{3}$ of 72, or 9.

Find the value of

1. $10 \div \frac{2}{5}$.
2. $12 \div \frac{3}{4}$.
3. $18 \div \frac{6}{7}$.
4. $21 \div \frac{7}{8}$.
5. $27 \div \frac{7}{10}$.
6. $18 \div 2\frac{2}{3}$.
7. $24 \div 3\frac{3}{4}$.
8. $36 \div 4\frac{1}{5}$.
9. $48 \div 6\frac{6}{7}$.
10. $75 \div 8\frac{4}{7}$.
11. $\frac{2}{3}$ *of* $36 \div \frac{5}{6}$.
12. $45 \div \frac{4}{5}$ *of* 9.
13. $\frac{3}{4}$ *of* $52 \div 1\frac{6}{7}$.
14. $50 \div \frac{5}{7}$ *of* $10\frac{1}{2}$.
15. $\frac{2}{3}$ of $10\frac{2}{7} \div 2\frac{4}{7}$.
16. 8 *times* $\frac{3}{4} \div 1\frac{7}{20}$.
17. $4\frac{4}{5} + (10 \div 2\frac{2}{3})$.
18. $(21 \div 2\frac{1}{3}) - 7\frac{4}{5}$.
19. $8\frac{3}{4} + (20 \div 3\frac{2}{6})$.
20. $(7\frac{1}{4} + 3\frac{1}{5}) \div 2\frac{1}{2}$.

3. *To Divide a Fraction by a Fraction.*

1. How many times are 2 fifths of a ton contained in 4 fifths of a ton? 2 sixths in 4 sixths? $\$\frac{3}{8}$ in $\$\frac{6}{8}$?

2. What is the quotient of 8 tenths ÷ 4 tenths? Of $\frac{9}{10} \div \frac{3}{10}$?

3. At $\$\frac{3}{20}$ each, how many melons can be bought for $\$\frac{6}{20}$? For $\$\frac{9}{20}$? For $\$\frac{12}{20}$? For $\$\frac{15}{20}$? For $\$\frac{18}{20}$?

4. A dealer received $\frac{3}{4}$ of a dollar for some ribbon at $\$\frac{1}{4}$ a yard. How many yards did he sell?

5. If $\frac{5}{8}$ of a yard of cloth will make a vest, how many vests can be made from $6\frac{2}{8}$ yards? From $9\frac{3}{8}$ yards?

6. How many packages of tea, each containing $\frac{4}{5}$ of a pound, can be made from $9\frac{3}{5}$ pounds? From $12\frac{4}{5}$ pounds?

7. When the divisor and the dividend are similar fractions, how is the quotient found?

Find the quotient of

8. $\frac{8}{9}$ of a yard divided by $\frac{4}{9}$.

9. $\frac{10}{12}$ of a gross divided by $\frac{5}{12}$.

10. $\frac{15}{20}$ of a ton divided by $\frac{3}{20}$.

11. $\frac{36}{40}$ of an acre divided by $\frac{9}{40}$.

12. $7\frac{1}{5}$ cords ÷ $\frac{4}{5}$ of a cord.

13. $8\frac{3}{4}$ miles ÷ $1\frac{1}{4}$ miles.

14. $9\frac{3}{7}$ years ÷ $\frac{6}{7}$ of a year.

15. $10\frac{7}{8}$ pecks ÷ $3\frac{5}{8}$ pecks.

16. How many pounds of coffee at $\frac{3}{8}$ of a dollar a pound can be bought for $\frac{3}{4}$ of a dollar?

206. Analysis. — Since 1 pound of coffee costs $\frac{3}{8}$ of a dollar, for $\frac{3}{4}$ of a dollar there can be bought as many pounds as the number of times $\frac{3}{8}$ is contained in $\frac{3}{4}$. Now $\frac{3}{4}$ equals $\frac{6}{8}$; and $\frac{3}{8}$ is contained in $\frac{6}{8}$ 2 times. Hence, etc.

17. At $\$\frac{3}{16}$ a yard, how many yards of muslin can be bought for $\$\frac{3}{8}$? For $\$\frac{5}{8}$? $\$\frac{3}{4}$? $\$1\frac{1}{2}$? $\$2\frac{1}{8}$?

18. If a farmer received $\$4\frac{1}{2}$ for some corn at $\$\frac{3}{4}$ a bushel, how many bushels did he sell? If he received $\$3\frac{1}{8}$? $\$4\frac{1}{12}$?

19. If $1\frac{1}{4}$ gallons of oil can be put in one jug, how many such jugs are needed to hold $7\frac{1}{2}$ gallons? How many are needed to hold $10\frac{1}{8}$ gallons? 15 gallons?

20. When the dividend and the divisor are not similar fractions, how may the quotient be found?

21. How often is $\frac{1}{2}$ contained in 1? How often in $\frac{1}{2}$? $\frac{1}{4}$ in 1? $\frac{1}{4}$ in $\frac{1}{2}$? $\frac{1}{5}$ in $\frac{1}{2}$? $\frac{1}{5}$ in $\frac{1}{3}$? $\frac{1}{4}$ in $\frac{1}{5}$?

22. How often is $\frac{2}{3}$ contained in $\frac{3}{4}$?

207. Analysis. — Since 1 is contained in $\frac{3}{4}$ $\frac{3}{4}$ times, $\frac{1}{3}$ is contained in $\frac{3}{4}$ 3 times $\frac{3}{4}$ times, or $\frac{9}{4}$ times; and $\frac{2}{3}$ is contained in $\frac{4}{3}$ $\frac{1}{2}$ of $\frac{9}{4}$ times, or $\frac{3}{8}$ times. Hence, etc.

23. Find the quotient of $\frac{3}{4} \div \frac{3}{8}$; $\frac{2}{5} \div \frac{5}{6}$; $\frac{3}{7} \div \frac{3}{8}$; $\frac{4}{9} \div \frac{3}{10}$.

24. If $\frac{3}{8}$ of a yard of cloth cost $\$\frac{4}{5}$, what will 1 yard cost?

25. Find the cost of 1 quart of wine at $\$1\frac{1}{2}$ for $\frac{7}{8}$ of a quart.

26. If a mason can do $\frac{7}{8}$ of a piece of work in one day, in what time can he do $\frac{3}{4}$ of it? $\frac{3}{5}$ of it? $\frac{5}{6}$ of it?

27. If the current of a river moves $\frac{9}{10}$ of a mile in $\frac{2}{3}$ of an hour, how far does it move in 1 hour? If it moves $1\frac{1}{2}$ of a mile?

28. If the quotient of $\frac{3}{4} \div \frac{4}{5}$ is $1\frac{5}{16}$, how may the numerator of the quotient be found from the terms of the fraction? How may the denominator of the quotient be found?

29. Show in what two ways the quotient of a fraction divided by a dissimilar fraction may be found.

30. At $\$\frac{3}{4}$ a pound, how many pounds of tea can be bought for $\$2\frac{3}{8}$?

208. Analysis. — Since 1 pound of tea costs $\$\frac{3}{4}$, for $\$2\frac{3}{8}$ there can be bought as many pounds as the number of times $\$\frac{3}{4}$ are contained in $\$2\frac{3}{8}$.

$\$2\frac{3}{8}$ equal $\$\frac{19}{8}$, and $\$\frac{3}{4}$ equal $\$\frac{6}{8}$; $\$\frac{6}{8}$ are contained in $\$\frac{19}{8}$ as often as 6 is contained in 19, which is $3\frac{1}{6}$ times. Hence, etc. Or, (207).

31. What is the quotient of $3\frac{2}{3}$ days divided by $\frac{3}{4}$ of a day?

32. How long will it take to earn $\$4\frac{3}{4}$ at $\$\frac{7}{8}$ a day?

33. At $\$\frac{9}{10}$ a yard, how many yards of cassimere must a merchant sell to receive $\$5\frac{4}{5}$? To receive $\$6\frac{7}{8}$?

34. If $7\frac{1}{2}$ years is $\frac{5}{12}$ of Harry's age, how old is he?

35. A teamster hauled $8\frac{2}{5}$ cords of wood in $\frac{7}{8}$ of a day. How many cords a day could he haul at that rate?

36. If a canal-boat averages $2\frac{1}{2}$ miles an hour, how long will it be in running $8\frac{1}{3}$ miles?

209. Analysis. — Since a canal-boat runs $2\frac{1}{2}$ miles an hour, to run $8\frac{1}{3}$ miles will require as many hours as the number of times $2\frac{1}{2}$ miles are contained in $8\frac{1}{3}$ miles.

$2\frac{1}{2}$ miles equal $\frac{5}{2}$ of a mile, and $8\frac{1}{3}$ miles equal $\frac{25}{3}$ of a mile. Also, $\frac{1}{2}$ of a mile equal $\frac{1}{6}$ of a mile, and $\frac{25}{3}$ of a mile equal $\frac{50}{6}$ of a mile; and $\frac{1}{6}$ of a mile are contained in $\frac{50}{6}$ of a mile, etc. Hence, etc. Or, (207).

37. If a man can cut $1\frac{1}{8}$ cords of wood in a day, in how many days can he cut $4\frac{1}{4}$ cords? $6\frac{1}{2}$ cords? 10 cords?

38. How many weeks will $8\frac{1}{4}$ bushels of oats last a horse that eats $2\frac{1}{2}$ bushels a week? How long will $12\frac{1}{16}$ bushels last?

39. At $\$3\frac{1}{4}$ a pair, how many pair of shoes will $\$19\frac{1}{2}$ buy? How many pair can be bought for $\$22\frac{3}{4}$? For $\$32\frac{1}{2}$?

40. A farmer cut $12\frac{1}{2}$ tons of hay from a field that yielded $1\frac{3}{4}$ tons to the acre. How many acres were in the field?

41. If $6\frac{3}{4}$ bushels of wheat cost $\$10\frac{4}{5}$, what is the price of 1 bushel? If $7\frac{1}{2}$ bushels cost $\$11\frac{1}{4}$?

42. Explain how a fraction is divided by a mixed number. A mixed number by a mixed number.

Find the quotient of	How often are
43. $\frac{4}{5}$ divided by $2\frac{1}{3}$.	*47.* $3\frac{1}{3}$ feet contained in $6\frac{2}{3}$ feet?
44. $\frac{5}{8}$ divided by $3\frac{1}{4}$.	*48.* $4\frac{1}{5}$ days contained in $2\frac{1}{3}$ days?
45. $1\frac{7}{8}$ divided by $4\frac{4}{5}$.	*49.* $6\frac{3}{4}$ hours contained in $10\frac{1}{8}$ hours?
46. $2\frac{4}{5}$ divided by $5\frac{5}{6}$.	*50.* $8\frac{3}{4}$ dollars contained in $4\frac{1}{6}$ dollars?

51. What number divided by $\frac{4}{5}$ gives a quotient $2\frac{1}{2}$ less than $4\frac{1}{6}$? A quotient $3\frac{1}{3}$ more than $5\frac{1}{4}$?

52. If the quotient is 7, the remainder is $1\frac{4}{5}$, and the divisor is $6\frac{1}{2}$, what is the dividend? If the divisor is $7\frac{2}{3}$?

53. If the dividend is $40\frac{3}{5}$, the quotient 7, and the remainder $5\frac{1}{2}$, what is the divisor? If the quotient is 8?

54. If the dividend is $52\frac{2}{3}$, the divisor $6\frac{3}{4}$, and the quotient 7, what is the remainder? If the divisor is $7\frac{1}{4}$?

55. The product of two factors is $33\frac{1}{3}$, and one of the factors is $6\frac{1}{4}$. Find the other factor. $\frac{2}{3}$ of the other factor.

56. If the product of two factors is $26\frac{1}{4}$, and one of the factors is $\frac{2}{3}$ of $3\frac{3}{8}$, what is $\frac{3}{5}$ of the other factor?

57. The product of three factors is $50\frac{2}{5}$, and two of the factors are $4\frac{2}{3}$ and $2\frac{1}{4}$. What is the third factor?

58. A newsboy bought 10 papers at $3\frac{1}{2}$ cents each, and sold them for 48 cents. What did he gain on each?

59. If a man earns $\$2\frac{1}{4}$ a day, and his expenses average $\$1\frac{4}{5}$, how many days must he work to save $\$20\frac{5}{8}$?

60. A farmer having \$25, gave $\frac{3}{4}$ of his money for clover-seed at $\$3\frac{3}{4}$ a bushel. How many bushels did he buy?

210. Principle.

Multiplying the dividend multiplies the quotient, and dividing the dividend divides the quotient.

Exercises.

What is the quotient of $\frac{7}{8}$ divided by $\frac{5}{8}$?

211. Model. — Since 1 is contained in $\frac{7}{8}$ $\frac{7}{8}$ times, $\frac{1}{8}$ is contained in $\frac{7}{8}$ 6 times $\frac{7}{8}$, or $\frac{42}{8}$; and $\frac{5}{8}$ is contained in $\frac{7}{8}$ $\frac{1}{5}$ of $\frac{42}{8}$ times, or $\frac{42}{40}$ times, or $\frac{21}{20}$ times, or $1\frac{1}{20}$ times (207).

Or, by changing the fractions to similar fractions (206).

Find the value of

1. $\frac{3}{4}$ of $18 \div 2\frac{1}{2}$.
2. $16\frac{2}{3} \div \frac{3}{5}$ of 15.
3. $\frac{4}{5}$ of $15 \div 3\frac{1}{5}$.
4. $18\frac{3}{4} \div \frac{5}{6}$ of $3\frac{3}{5}$.
5. $\frac{5}{6}$ of $17 \div 3\frac{2}{5}$.
6. $\frac{2}{3}$ of $10 \div \frac{3}{4}$ of 8.
7. $\frac{3}{4}$ of $11 \div \frac{3}{8}$ of 22.
8. $\frac{2}{5}$ of $3\frac{1}{3} \div \frac{3}{4}$ of $2\frac{1}{5}$.
9. $\frac{3}{5}$ of $6\frac{1}{4} \div \frac{5}{6}$ of $2\frac{4}{7}$.
10. $\frac{2}{7}$ of $4\frac{2}{3} \div \frac{3}{8}$ of $6\frac{2}{3}$.
11. $(7 \times \frac{3}{5}) \div 8\frac{2}{5}$.
12. $8\frac{1}{3} - (10 \div 1\frac{2}{3})$.
13. $(15 \div 1\frac{3}{7}) - 7\frac{5}{6}$.
14. $7\frac{5}{7} + (\frac{4}{7} \div \frac{2}{21})$.
15. $(11\frac{1}{4} \div \frac{5}{9}) - 12\frac{3}{5}$.

Section VII.

RELATIONS OF NUMBERS.

1. *To Find what Part One Number is of Another.*

1. 2 is how many times 1? 3 is how many times 1?

2. How much is 1 divided by 2? 1 divided by 3? $1 \div 4$?

3. 1 is what part of 2? What part of 3? Of 4? Of 5? Of 6?

4. 2 is what part of 3?

212. Analysis. — Since 1 is $\frac{1}{3}$ of 3, 2 is 2 times $\frac{1}{3}$, or $\frac{2}{3}$, of 3.

5. 3 is what part of 5? 4, of 7? 5, of 8? 6, of 9?

6. What part of 7 is 5? Of 9 is 7? Of 10 is 6? Of 12 is 8?

7. 6 tons are what part of 8 tons? 8 pounds of 14 pounds? 10 cents of 25 cents? 9 eggs of a dozen? \$20 of \$50?

8. What part of 12 feet are 6 feet? Of 18 yards are 9 yards? Of 25 miles are 18 miles? Of 40 bushels are 25 bushels?

9. If an acre of land can be bought for \$50, what part of an acre can be bought for \$5? For \$10? For \$25?

10. 12 inches are a foot, and 3 feet are a yard. What part, then, of a yard are 9 inches? 18 inches? 24 inches?

11. What part of 5 is $\frac{3}{4}$?

213. Analysis. — Since 1 is $\frac{1}{5}$ of 5, $\frac{1}{4}$ is $\frac{1}{5}$ of $\frac{1}{5}$ of 5, or $\frac{1}{20}$ of 5; and $\frac{3}{4}$ is 3 times $\frac{1}{20}$ of 5, or $\frac{3}{20}$ of 5. Hence, etc.

12. $\frac{3}{5}$ is what part of 6? $\frac{5}{6}$, of 7? $\frac{6}{7}$, of 8? $\frac{7}{8}$, of 10?

13. What part of 12 is $\frac{5}{7}$? Of 13 is $\frac{7}{8}$? Of 15 is $\frac{9}{10}$?

14. $2\frac{1}{2}$ is what part of 6? $3\frac{1}{3}$, of 10? $11\frac{1}{3}$, of 15? $6\frac{1}{4}$, of 25?

15. \$$2\frac{1}{2}$ are what part of \$3? $2\frac{1}{2}$ feet, of 12 feet? $4\frac{1}{4}$ miles, of 15 miles? $6\frac{2}{3}$ miles, of 30 miles? $7\frac{1}{2}$ miles, of 45 miles?

16. What part of 12 dozen are $2\frac{1}{2}$ dozen? Of 20 yards are $3\frac{1}{2}$ yards? Of 25 gallons are $8\frac{1}{3}$ gallons? Of \$30 are \$$9\frac{1}{2}$?

17. Frank has 12 dollars, and Henry has $9\frac{3}{5}$. Henry's money is what part of Frank's?

18. What part of $\frac{3}{4}$ is $\frac{2}{3}$?

214. Analysis. — Since $\frac{2}{3}$ is $\frac{2}{3}$ of 1, it is 4 times $\frac{2}{3}$, or $\frac{8}{3}$ of $\frac{1}{4}$; and $\frac{2}{3}$ is $\frac{1}{3}$ of $\frac{8}{3}$, or $\frac{8}{9}$, of $\frac{3}{4}$. Hence, etc.

Or, $\frac{2}{3}$ equals $\frac{8}{12}$, and $\frac{3}{4}$ equals $\frac{9}{12}$; and the relation of $\frac{2}{3}$ to $\frac{3}{4}$ is the same as 8 to 9, or $\frac{8}{9}$. Hence, etc.

19. $\frac{3}{4}$ is what part of $\frac{3}{5}$? $\frac{2}{5}$, of $\frac{5}{6}$? $\frac{4}{5}$, of $\frac{6}{7}$?

20. What part of $\frac{5}{6}$ is $\frac{3}{4}$? Of $\frac{7}{8}$ is $\frac{5}{6}$? Of $\frac{8}{9}$ is $\frac{6}{7}$?

21. $\frac{3}{5}$ of an ounce are what part of $2\frac{1}{2}$ ounces? $\frac{5}{8}$ of a mile of $3\frac{1}{4}$ miles? $\frac{7}{10}$ of a ton, of $5\frac{1}{2}$ tons?

22. What part of $6\frac{1}{4}$ cents are $3\frac{1}{8}$ cents? Of $\$7\frac{1}{2}$ are $\$2\frac{1}{2}$? Of $16\frac{2}{3}$ cents are $6\frac{1}{4}$ cents? Of $\$18\frac{3}{4}$ are $\$12\frac{1}{2}$?

23. What part of 15 days are $\frac{4}{5}$ of 10 days? $\frac{3}{5}$ of 12 days? $\frac{4}{7}$ of 15 days? $\frac{2}{3}$ of $5\frac{1}{4}$ days? $\frac{5}{8}$ of $9\frac{3}{5}$ days?

24. If $5\frac{1}{2}$ yards are one rod, what part of a rod are $2\frac{3}{4}$ yards? What part are $3\frac{2}{3}$ yards?

215. The ***relation*** of one number to another is the value of one number compared with the value of another; or,

It is the number of times one of two numbers contains the other, or the part one number is of the other.

216. General Principle.

Only similar numbers, or numbers of the same kind, can be compared with each other.

Exercises.

1. What is the relation of 12 to $\frac{3}{4}$?

217. Model. — The relation of 12 to $\frac{3}{4}$ is the quotient of 12 divided by $\frac{3}{4}$; 12 divided by $\frac{1}{4}$ is 48, and 12 divided by $\frac{3}{4}$ is $\frac{1}{3}$ of 48, or 16.

2. $\frac{3}{4}$ of $\frac{8}{9}$ is what part of $1\frac{1}{4}$?

218. Model. — $\frac{3}{4}$ of $\frac{8}{9}$ is such part of $1\frac{1}{4}$ as the quotient of $\frac{3}{4}$ of $\frac{8}{9}$ *divided by* $1\frac{1}{4}$ is of 1. $\frac{3}{4}$ of $\frac{8}{9}$ equals $\frac{2}{3}$, and $1\frac{1}{4}$ equals $\frac{7}{4}$; $\frac{2}{3}$ divided *by 1, etc.*

What part of

1. 18 is 9?	6. 20 is $\frac{4}{5}$? $1\frac{4}{5}$? $2\frac{1}{2}$?	11. $\frac{2}{4}$ of $\frac{5}{9}$ is $3\frac{2}{3}$?
2. 32 is 12?	7. $18\frac{3}{4}$ is $\frac{3}{5}$? $3\frac{3}{4}$? $6\frac{1}{4}$?	12. $12\frac{1}{2}$ is $\frac{6}{7}$ of $10\frac{1}{2}$?
3. 40 is 32?	8. 25 is 7? $1\frac{7}{8}$? $12\frac{1}{2}$?	13. $\frac{2}{3}$ of $2\frac{2}{5}$ is $\frac{1}{2}$ of $2\frac{2}{3}$?
4. 12 is $\frac{7}{8}$?	9. $31\frac{1}{4}$ is $\frac{5}{4}$? $3\frac{1}{8}$? 25?	14. 25 $\times$ $\frac{4}{5}$ is $\frac{3}{4}$ of $5\frac{3}{5}$?
5. 15 is $1\frac{4}{5}$?	10. $30\frac{1}{4}$ is $\frac{3}{2}$? 25? $17\frac{2}{7}$?	15. 4 times $3\frac{3}{4}$ is $\frac{2}{5}\times 7\frac{1}{2}$?

2. *To Find a Number when a Fractional Part of it is given.*

1. 3 is $\frac{1}{2}$ of what number?

219. ***Analysis.*** — Since 3 is $\frac{1}{2}$ of some number, $\frac{2}{2}$ of the number, or the number, is 2 times 3, or 6. Hence, etc.

2. $8\frac{1}{2}$ is $\frac{1}{4}$ of what number? $\frac{1}{5}$ of what? $\frac{1}{8}$? $\frac{1}{7}$? $\frac{1}{6}$?

3. $\frac{2}{3}$ of 11 is $\frac{1}{5}$ of what number? $\frac{1}{6}$ of what? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$?

4. If $\frac{1}{5}$ of a ream of paper costs 75 cents, what is the cost of a ream? If $\frac{1}{8}$ of a ream costs $37\frac{1}{2}$ cents?

5. If 9 is $\frac{3}{4}$ of some number, what is $\frac{1}{4}$ of the number? What is $\frac{4}{4}$ of the number, or the number?

6. 8 is $\frac{4}{5}$ of what number?

220. ***Analysis.*** — Since 8 is $\frac{4}{5}$ of some number, $\frac{1}{5}$ of the number is $\frac{1}{4}$ of 8, or 2; and $\frac{5}{5}$ of the number, or the number, is 5 times 2, or 10. Hence, etc.

7. 12 is $\frac{6}{7}$ of what number? $\frac{8}{9}$ of what? $1\frac{2}{13}$ of what?

8. 36 is $\frac{6}{7}$ of what number? $\frac{9}{10}$ of what? $1\frac{2}{13}$ of what?

9. $\frac{8}{9}$ is $\frac{2}{3}$ of what number? $\frac{4}{5}$ of what? $\frac{6}{7}$ of what? $\frac{8}{9}$?

10. $2\frac{2}{3}$ is $\frac{3}{4}$ of what number? $\frac{4}{5}$ of what? $\frac{8}{9}$ of what? $\frac{9}{10}$?

11. $\frac{3}{4}$ of $3\frac{3}{7}$ is $\frac{6}{11}$ of what number? $\frac{9}{13}$ of what? $1\frac{1}{5}$ of what?

12. If $\frac{5}{8}$ of the cost of a barrel of flour is $\$7\frac{1}{2}$, what is the cost of a barrel? What is the cost of $2\frac{1}{2}$ barrels?

13. $\frac{3}{4}$ of 12 is $\frac{6}{7}$ of what number?

221. Analysis. — $\frac{1}{4}$ of 12 is 3, and $\frac{3}{4}$ of 12 is 3 times 3, or 9.

Since 9 is $\frac{6}{7}$ of some number, $\frac{1}{7}$ of the number is $\frac{1}{6}$ of 9, or $\frac{3}{2}$, or $1\frac{1}{2}$; and $\frac{7}{7}$ of the number, or the number, is 7 times $1\frac{1}{2}$, or $\frac{21}{2}$, or $10\frac{1}{2}$. Hence, etc.

14. $\frac{3}{4}$ of 16 is $\frac{4}{5}$ of what number? $\frac{6}{7}$ of what? $\frac{8}{9}$ of what?

15. $\frac{4}{5}$ of 30 is $\frac{6}{7}$ of what number? $\frac{8}{11}$ of what? $1\frac{2}{3}$ of what?

16. $\frac{5}{8}$ of $3\frac{3}{7}$ is $\frac{5}{9}$ of what number? $\frac{4}{11}$ of what? 2 times what?

17. Harry is 15 years old, and $\frac{2}{3}$ of his age is $\frac{5}{6}$ of his sister's age. How old is his sister? What is $\frac{6}{7}$ of her age?

18. $\frac{3}{4}$ of 20 acres are $\frac{5}{11}$ of the number of acres in my farm. How many acres in my farm?

19. A boy earned $\$8\frac{2}{5}$, and $\frac{6}{7}$ of his wages amounted to $\frac{9}{15}$ of his father's wages. What did his father earn?

20. 28 is $\frac{7}{9}$ of how many times 12?

222. Analysis. — Since 28 is $\frac{7}{9}$ of some number, $\frac{1}{9}$ of the number is $\frac{1}{7}$ of 28, or 4; and $\frac{9}{9}$ of the number, or the number, is 9 times 4, or 36. Now 36 is as many times 12 as the number of times 12 is contained in 36, which is 3 times. Hence, etc.

21. 45 is $\frac{9}{10}$ of how many times 5? Of how many times 10?

22. $\frac{3}{4}$ of 24 is $\frac{6}{7}$ of how many times 7? Times 14?

23. $\frac{5}{9}$ of 54 is $\frac{6}{8}$ of how many times 10? Times 20?

24. $\frac{7}{9}$ of 63 is $\frac{7}{8}$ of how many times 6? 9? 12? 18?

25. I paid \$60 for a cow, and this sum was $\frac{4}{7}$ of the cost of my horse. What did I pay for my horse?

26. If \$25 are $\frac{5}{8}$ of 9 times the cost of a yard of silk, what is the price of the silk per yard?

27. After John had spent $\frac{5}{8}$ of his money for a suit of clothes, he had \$24. How much had he at first?

28. A real-estate dealer, after selling $\frac{4}{7}$ of his farm, had 30 acres left. How many acres were in the farm at first?

29. If $\frac{2}{3}$ of a yard of cloth cost $\$\frac{3}{4}$, what will $\frac{2}{5}$ of a yard *cost? What will* $\frac{5}{8}$ of a yard cost?

30. If $\frac{3}{5}$ of 6 yards of silk cost \$7$\frac{1}{5}$, what will $\frac{3}{4}$ of 9 yards cost? $\frac{4}{5}$ of 12 yards?

Exercises.

$\frac{1}{7}$ of 42 is $\frac{1}{3}$ of 3 times what number?

223. Model. — $\frac{1}{7}$ of 42 is 4 times $\frac{1}{7}$ of 42, or 4 times 6, which are 24; 24 is $\frac{1}{3}$ of 9 times $\frac{1}{4}$ of 24, or 9 times 6, which are 54; and 54 is 3 times $\frac{1}{3}$ of 54, which is 18.

1. $\frac{4}{9}$ *of* 27 *is* $\frac{6}{7}$ *of what?*
2. $\frac{5}{8}$ *of* 64 *is* $\frac{5}{9}$ *of what?*
3. $\frac{7}{9}$ *of* 72 *is* $\frac{8}{11}$ *of what?*
4. $\frac{9}{10}$ *of* 80 *is* $\frac{9}{13}$ *of what?*
5. $\frac{7}{8}$ *of* 64 *is* $\frac{8}{9}$ *of* 4 *times what?*
6. $\frac{8}{5}$ *of* 40 *is* $\frac{16}{5}$ *of* 5 *times what?*
7. $\frac{9}{8}$ *of* 48 *is* $\frac{9}{10}$ *of* 7 *times what?*
8. $\frac{11}{10}$ *of* 60 *is* $\frac{11}{12}$ *of* 8 *times what?*

General Review of Fractions.

1. The parts are $4\frac{1}{2}$ tons and $5\frac{2}{3}$ tons. Find the sum.

2. Find $\frac{2}{3}$ of the sum of $2\frac{1}{4}$ feet, 3 feet, and $4\frac{1}{3}$ feet.

3. Find 2 times the difference between 5 miles and $4\frac{7}{8}$ miles. Find $\frac{1}{2}$ of $2\frac{2}{5}$ times the difference.

4. The greater of two numbers is $6\frac{1}{3}$, and the less is $4\frac{3}{5}$. Find the difference. Find $2\frac{1}{2}$ times the difference.

5. The less of two numbers is \$$4\frac{2}{5}$, and their difference is \$$3\frac{1}{4}$. Find the greater number, and multiply it by 4.

6. The minuend is $7\frac{2}{3}$ yards, and the subtrahend is $6\frac{3}{4}$ yards. Find the remainder, and divide it by $\frac{2}{3}$ of 6.

7. If the minuend is $6\frac{3}{4}$ gallons, and the remainder is $3\frac{5}{6}$ gallons, find $\frac{2}{5}$ of the subtrahend.

8. The subtrahend is $7\frac{3}{5}$ months, and the remainder is $3\frac{3}{4}$ months. How much is the minuend increased by $\frac{3}{4}$ of 8 months? The minuend diminished by $\frac{3}{5}$ of $2\frac{1}{2}$ months?

9. The sum of three numbers is 12 feet, and two of the numbers are $4\frac{1}{3}$ *feet and* $5\frac{3}{4}$ feet. What is the other number?

10. If the whole is $15\frac{2}{5}$ miles, and two of three parts are 7 miles and $6\frac{7}{10}$ miles, what is the third part?

11. What is the product of the two factors $4\frac{1}{2}$ and $\frac{3}{4}$ of 5?

12. Find the continued product of \$$2\frac{1}{2}$, $\frac{3}{4}$ of $1\frac{3}{5}$, and 3 times $1\frac{1}{3}$. Of \$$4\frac{2}{5}$, $\frac{2}{3}$ of $2\frac{1}{7}$, and 5 times $2\frac{4}{5}$.

13. If the multiplicand is $4\frac{1}{2}$ cords, and the multiplier is $1\frac{1}{3}$, what is the product? $\frac{1}{4}$ of $2\frac{1}{2}$ times the product?

14. The product is \$$8\frac{3}{4}$, and the multiplicand \$$\frac{5}{7}$. Find the multiplier. Divide the multiplier by $5\frac{4}{5}$.

15. If the product is \$$7\frac{1}{5}$, and the multiplier is $1\frac{5}{7}$, what is the multiplicand? The multiplicand increased by \$$6\frac{3}{8}$?

16. The product of two factors is $\frac{2}{3}$ of 20 tons, and one of the factors is $1\frac{3}{5}$. What is $\frac{3}{8}$ of the other factor?

17. If the product of three factors is $8\frac{1}{2}$ bales, and two of the factors are $1\frac{1}{7}$ and $3\frac{1}{2}$, what is the other factor?

18. The product of three factors is $\frac{2}{3}$ of \$$7\frac{7}{8}$, and two of the factors are \$$1\frac{4}{5}$ and \$$1\frac{7}{18}$. Find two times the third factor.

19. Find the quotient of $\frac{2}{3}$ of $7\frac{1}{2}$ cords divided by 3 times $\frac{3}{4}$.

20. If the dividend is $6\frac{2}{3}$, and the divisor is $1\frac{7}{8}$, what is the quotient? From the quotient take $\frac{1}{2}$ of $5\frac{1}{3}$.

21. The divisor is \$$8\frac{2}{3}$, and the quotient is $2\frac{1}{2}$. Find the dividend. Find 6 times $\frac{2}{5}$ of the dividend.

22. If the quotient is $9\frac{1}{3}$ feet, and the dividend is $5\frac{3}{5}$ feet, what is the divisor? Find $\frac{2}{3}$ of 6 times the divisor.

23. The divisor is $2\frac{2}{3}$ quarts, the quotient $1\frac{3}{4}$, and the remainder is $1\frac{3}{4}$ quarts. Find the dividend, and multiply it by 3.

24. The dividend is $11\frac{4}{15}$ acres, the remainder is $1\frac{2}{3}$ acres, and the quotient is $2\frac{2}{5}$ acres. Find the divisor.

25. The sum of two numbers is $12\frac{1}{2}$ years, and their difference is $1\frac{1}{6}$ years. Find the greater number. Find the less.

26. John and James together have \$$17\frac{5}{12}$, and John has \$$2\frac{1}{12}$ *more than* James. How much money has each of them?

27. If 2 be added to both terms of $\frac{5}{7}$, will the value of the fraction be increased or diminished, and how much?

28. If $4\frac{1}{3}$ dozen eggs cost $\$1\frac{5}{8}$, what is the cost of 1 dozen? Of $2\frac{1}{2}$ dozen? Of $3\frac{1}{3}$ dozen?

29. If 3 tons of coal cost $\$16\frac{1}{2}$, what will 5 tons cost? If $\frac{7}{8}$ of a ton cost \$6, what will 6 tons cost?

30. How many pounds of tea worth $\$\frac{9}{10}$ a pound will cost as much as $4\frac{1}{2}$ pounds of coffee at $\$\frac{3}{8}$ a pound?

31. How much less than \$12 will 9 pounds of tea cost at $\$\frac{7}{8}$ a pound? How much more than $\frac{2}{3}$ of 4 times $\$2\frac{1}{8}$?

32. After I had spent $\frac{2}{3}$ of my money, I found that \$20 equalled $\frac{4}{5}$ of what I had remaining. How much had I at first?

33. A, B, and C own a yacht worth \$3750. A owns $\frac{2}{7}$ of it, and B $\frac{2}{5}$ of it, and C the remainder. How much is each man's share worth?

34. A man deposited his money in 4 banks. In the first he deposited $\frac{1}{3}$ of it, in the second $\frac{2}{5}$, in the third $\frac{1}{8}$, and in the fourth \$1200. How much did he deposit in all?

35. If 5 men can do a piece of work in $7\frac{2}{3}$ days, in what time can 1 man do the same amount? In what time can 3 men do it? 4 men? 6 men? 8 men? 10 men?

36. If a man engages to do a piece of work in 100 days, what part ought he to finish in 10 days? In $12\frac{1}{2}$ days? In $16\frac{2}{3}$ days? In $37\frac{1}{2}$ days? In $66\frac{2}{3}$ days?

37. If A owns $\frac{3}{5}$ of a tract of 80 acres of land, and sells B $\frac{3}{4}$ of his tract, how many acres has he left?

38. I paid $\$3\frac{1}{3}$ for a yard of cloth. What would $\frac{3}{5}$ of a yard of such cloth cost? What would $3\frac{1}{2}$ yards cost?

39. What must a flour-dealer pay for $3\frac{1}{4}$ barrels of flour at the rate of $\$2\frac{1}{2}$ for $\frac{3}{8}$ of a barrel? For $4\frac{4}{5}$ barrels?

40. If $\frac{3}{5}$ of $7\frac{1}{2}$ times the distance between two places is $31\frac{1}{2}$ miles, what is the distance between them?

41. If $2\frac{3}{4}$ yards of cloth cost $\$3\frac{1}{2}$, what will $5\frac{1}{2}$ yards cost?

42. A bought $\frac{3}{4}$ of $\frac{6}{7}$ of a piece of land, and B bought $\frac{8}{10}$ of the remainder. Which bought the more, and how much?

43. If $\frac{3}{5}$ of 8 yards of cloth cost $\$14\frac{2}{5}$, what will $\frac{3}{4}$ of 9 yards cost? How much will 2 times $3\frac{1}{3}$ yards cost?

44. An apple-woman bought apples at the rate of 3 for 2 cents, and sold them at the rate of 5 for 4 cents. Find the gain on 15 apples. The gain on $3\frac{1}{2}$ dozen apples.

45. How many yards of carpeting $\frac{2}{3}$ of a yard wide are equal to 8 yards $\frac{3}{4}$ of a yard wide?

46. A man gave \$45 to his two children, so that $\frac{3}{4}$ of Harry's share equalled $\frac{6}{7}$ of Mary's share. What did each receive? Which had the greater share, and how much?

47. If 3 men can do a piece of work in $9\frac{3}{4}$ days, in what time can 4 men do it? In what time can 8 men do it?

48. If $\frac{2}{5}$ of $3\frac{1}{3}$ times my age is 44 years, how old am I?

49. $\frac{3}{8}$ of A's age is $\frac{4}{5}$ of B's age, and the sum of their ages is 47 years. Find the difference of their ages.

50. If Frank had 4 times as much money as he has, $\frac{2}{3}$ of his money would be \$36. How much has he?

51. If to $\frac{5}{8}$ of John's age 10 years be added, the sum will be $\frac{5}{4}$ of his age. How old was John $10\frac{3}{4}$ years ago?

52. If to A's money there be added $\frac{3}{4}$ and $\frac{3}{5}$ of his money, the sum will be \$47. How much less than $\$27\frac{3}{8}$ has he?

53. A man is 45 years old, and $\frac{4}{9}$ of his age is $\frac{4}{7}$ of his wife's age. What is his wife's age? The sum of their ages?

54. The difference between $\frac{2}{3}$ of A's age and $\frac{2}{5}$ of his age is 4 years. How old is B, whose age is $\frac{2}{3}$ of $\frac{7}{10}$ of A's?

55. If I sell a cow for \$45, and I receive for her $\frac{9}{8}$ of the price which I paid for her, what price did I pay?

56. By selling a horse for \$88, I cleared $\frac{2}{9}$ of the price *which I paid* for him. What did he cost me?

57. A man sold a wagon for \$75, and by so doing received only $\frac{5}{8}$ of its cost. What did he pay for it?

58. I paid \$30 for a cow, and $\frac{9}{10}$ of the cost of the cow was $\frac{3}{8}$ of the cost of my horse. How much did I pay for both?

59. If 28 dollars are $\frac{4}{5}$ of your money, how many books at \$1$\frac{3}{4}$ each can you buy with it?

60. I sold a watch and chain for \$135, receiving for the chain $\frac{2}{7}$ as much as for the watch, and clearing on each $\frac{1}{5}$ of the cost. What did I pay for each?

Section VIII.

DECIMALS.

1. Notation and Numeration.

1. If a cake is divided into 10 equal parts, what is each part called? What are 3 parts? 5 parts?

2. How much is $\frac{1}{10}$ of 1? $\frac{3}{10}$ of 1? $\frac{5}{10}$? What is the fractional unit of 2 tenths? 4 tenths? Of any number of tenths?

3. If a cake or any unit is divided into 10 equal parts, and each tenth into 10 equal parts, or the unit into 100 equal parts, what is each part called? What are 5 parts? 25? 50?

4. How much is $\frac{1}{10}$ of $\frac{1}{10}$ of 1? $\frac{1}{100}$ of 1? $\frac{5}{10}$ of $\frac{1}{10}$ of 1?

5. What is the fractional unit of 5 hundredths? Of 25 hundredths? Of $\frac{7}{100}$? Of $\frac{75}{100}$? Of any number of hundredths?

6. If a unit is divided into 100 equal parts, and each hundredth into 10 equal parts, or the unit into 1000 equal parts, what is each part called? What are 5 parts called? 25? 125?

7. How much is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of 1? $\frac{1}{10}$ of $\frac{1}{100}$ of 1? $\frac{1}{1000}$ of 1? $\frac{5}{10}$ of $\frac{1}{100}$ of 1? $\frac{25}{100}$ of $\frac{1}{10}$ of 1? $\frac{125}{1000}$ of 1?

8. What is the fractional unit of 5 thousandths? Of 25 thousandths? Of $\frac{125}{1000}$? Of $\frac{750}{1000}$? Of any number of thousandths?

9. What part of 1 is 1 tenth? What part of 1 tenth is 1 hundredth? What part of 1 hundredth is 1 thousandth?

10. How many thousandths in 1 hundredth? In 1? How many hundredths in 1 tenth? In 1? How many tenths in 1?

224. A ***Decimal Fraction*** is one or more of the equal decimal divisions of a unit.

Thus, *1 tenth*, *5 hundredths*, $\frac{2}{10}$, $\frac{25}{100}$, $\frac{175}{1000}$ are *decimal fractions.*

Decimal fractions are commonly called *Decimals.*

Note. — The decimal division of a unit is the division of a unit into 10 equal parts (*tenths*); and these again into 10 equal parts (*hundredths*); and these again into 10 equal parts (*thousandths*); etc.

225. Since in decimals the values of the fractional units increase and decrease by the scale of 10, they are governed by the same principles as are integers; and they may be expressed by the same system of notation, by extending the orders of decimal units below ones.

226. Decimals are distinguished from integers by the *Decimal Point.*

The ***Decimal Point*** is a period ***(.)*** placed *after ones* or *before tenths.*

227. A ***Decimal Unit*** is *one* of the equal decimal divisions of the unit 1.

Thus, *1 tenth* is a *decimal unit;* also, 1 hundredth, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc.

228. Orders of Decimal Units are classes of units formed from the decimal division of the unit 1.

Thus, *tenths* are decimal units of the *first order;* hundredths of the second; etc.

229. A decimal unit of the first order, or ***one***

tenth, is expressed by writing the figure 1 with the decimal point before it; thus, *.1.*

In the same manner are expressed

2 tenths, $\frac{2}{10}$, *.2*	*4 tenths,* $\frac{4}{10}$, *.4*
3 tenths, $\frac{3}{10}$, *.3*	*5 tenths,* $\frac{5}{10}$, *.5*
6 tenths, $\frac{6}{10}$, *.6*	*8 tenths,* $\frac{8}{10}$, *.8*
7 tenths, $\frac{7}{10}$, *.7*	*9 tenths,* $\frac{9}{10}$, *.9*

230. When a decimal is expressed by a single figure, it denotes a number of tenths.

231. A ***decimal unit of the second order,*** or ***one hundredth,*** is expressed by writing the figure 1 with a cipher and the decimal point before it; thus, *.01.*

In the same manner are expressed

2 hundredths, or $\frac{2}{100}$, *.02*	*6 hundredths, or* $\frac{6}{100}$, *.06*
3 hundredths, or $\frac{3}{100}$, *.03*	*7 hundredths, or* $\frac{7}{100}$, *.07*
4 hundredths, or $\frac{4}{100}$, *.04*	*8 hundredths, or* $\frac{8}{100}$, *.08*
5 hundredths, or $\frac{5}{100}$, *.05*	*9 hundredths, or* $\frac{9}{100}$, *.09*

232. When a decimal is expressed by two figures, the first figure to the right of the decimal point denotes a number of tenths, and the second figure a number of hundredths, and the decimal is read as so many hundredths.

Thus, *.23* is 2 tenths 3 hundredths; but 2 tenths equal 20 hundredths, and 20 hundredths and 3 hundredths are *23 hundredths.*

233. A ***decimal unit of the third order,*** or ***one thousandth,*** is expressed by writing the figure 1 with two ciphers and the decimal point before it; thus, *.001.*

In the same manner are expressed

2 thousandths, or $\frac{2}{1000}$, *.002*	*6 thousandths, or* $\frac{6}{1000}$, *.006*
3 thousandths, or $\frac{3}{1000}$, *.003*	*7 thousandths, or* $\frac{7}{1000}$, *.007*
4 thousandths, or $\frac{4}{1000}$, *.004*	*8 thousandths, or* $\frac{8}{1000}$, *.008*
5 thousandths, or $\frac{5}{1000}$, *.005*	*9 thousandths, or* $\frac{9}{1000}$, *.009*

234. When a decimal is expressed by three figures, the third figure to the right of the decimal point denotes a number of thousandths, and the decimal is read as so many thousandths.

235. A decimal unit of the fourth order, or *one ten-thousandth*, is expressed, *.0001; one hundred-thousandth, .00001; one millionth, .000001;* etc.

236. The orders of decimal units are denoted by the ***places*** in which the figures used to express a decimal stand.

237. A figure in the *first* decimal place denotes *tenths*, or units of the first decimal order; in the *second* decimal place, *hundredths*, or units of the second decimal order; etc.

Table of Orders.

Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	ONES.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
7	6	5	4	3	2	1.	2	3	4	5	6	7
INTEGERS.							DECIMALS.					

238. A ***Decimal*** is a decimal fraction expressed without the denominator.

Thus, *.5* is a *decimal;* so, also, is .007, .095, .1003, etc.

239. A ***Mixed Decimal*** is a number expressed by an integer and a decimal.

Thus, *4.05*, read *4 and 5 hundredths*, is a *mixed decimal.*

240. A ***Complex Decimal*** is a decimal with a common fraction annexed.

Thus, .45$\frac{2}{4}$, read 45$\frac{2}{4}$ *hundredths*, is a *complex decimal.*

241. ***United States Money*** is the currency of the United States.

It is based upon the decimal scale, and is, therefore, a *Decimal Currency.*

242. The orders or denominations of United States money are *mill, cent, dime, dollar,* and *eagle.*

Table of United States Money.

10 mills (*m.*)	are	*1 cent,*	*ct.* or *c.*
10 cents	"	*1 dime,*	*d.*
10 dimes	"	*1 dollar,*	$.
10 dollars	"	*1 eagle,*	*E.*

Note. — The denominations *eagles* and *dimes* are not used in business: *eagles* being regarded as *tens of dollars,* and *dimes* as *tens of cents.*

Thus, *12 eagles* is expressed as *120 dollars;* and *5 dimes* as *50 cents.*

243. The dollar is the principal unit of United States money.

Dollars are written as integers, with the dollar sign prefixed.

Thus, *5 dollars* is written $*5;* 275 dollars is written $275.

244. Since 100 cents are 1 dollar, 1 cent is 1 hundredth of a dollar; and *cents* are expressed as *hundredths* of a dollar.

Thus, *1 cent* is expressed $*.01;* 7 cents, $.07; 75 cents, $.75.

245. Since 10 mills are 1 cent, 1 mill is 1 tenth of a cent, or 1 thousandth of a dollar; and *mills* are expressed as *thousandths* of a dollar.

Thus, *1 mill* is expressed $*.001;* 5 mills, $.005; 12 cents 5 mills, $.125.

246. Principles.

I. *Ten units of any decimal order are one unit of the next higher decimal order.*

II. *The successive orders of decimal units decrease in value tenfold from left to right.*

III. *The value of a decimal is not changed by annexing ciphers to, or omitting ciphers from, the right of the decimal.*

Exercises.

1. One is how many tenths? One tenth is how many hundredths? One hundredth is how many thousandths? How many thousandths equal one hundredth? How many hundredths equal one tenth? How many tenths equal one?

2. Name the units of the first decimal order; the third; the second; the fourth. What decimal order are tenths? Thousandths? Hundredths? Ten-thousandths?

3. How is one tenth expressed? One thousandth? One hundredth? How many decimal figures are needed to express tenths? Thousandths? Hundredths? What does a decimal expressed by one figure denote? By two figures? Three?

4. In what place in a decimal expressed by figures are tenths written? Thousandths? Hundredths? What order of decimal units does a figure written in the first decimal place denote? In the third? The second? The fourth?

5. What effect in the value of a decimal is produced by placing a cipher between the numerator of a decimal and the decimal point? Why? Two ciphers? Three ciphers? Why? By omitting a cipher from between the numerator and the decimal point? Why? Two ciphers? Three? Why?

6. Express and read as decimals, or as mixed decimals,

1. $\frac{1}{10}$; $\frac{5}{10}$.	3. $\frac{25}{100}$; $\frac{80}{100}$.	5. $\frac{25}{1000}$; $\frac{612}{1000}$.	7. $50\frac{5}{100}$; $100\frac{1}{100}$.
2. $\frac{4}{100}$; $\frac{8}{100}$.	4. $\frac{5}{100}$; $\frac{9}{100}$.	6. $9\frac{5}{10}$; $10\frac{8}{100}$.	8. $\frac{5}{1000}$; $6000\frac{6}{1000}$.

7. Tell what order of decimal units each figure in the following decimals denotes; tell how many of each order are expressed; read each decimal, or express it in words: —

.1	.01	\$.20½	.001	.012	\$.409⅜	.0009	.0805½.
.3	.23	\$.03¼	.123	.004	\$.090¾	.0087	.6003⅜.
.5	.07	\$.45⅛	.075	.206	\$.600⅝	.0905	.0092⅜.

.7; .49; \$.06¼; .307; .430; \$.007⅝; .8763; .8601⅝.

1.5; 2.07; 10.2; 30.04; 300.2; 500.04; \$400.05; \$500.006.

300; 300.03; 40.04; .005; 500.005; .0505; \$6000.006; \$8.008.

8. Tell how to express by figures the following decimals: —

Five tenths; eight and six tenths; fifty and five tenths; twenty-five hundredths; three hundredths; nine and five hundredths; seventy and seven hundredths; two hundred and two hundredths; five hundred five and five hundredths; six hundred sixteen and sixteen hundredths; $2\frac{1}{2}$ tenths; 6 and $6\frac{1}{2}$ tenths; $12\frac{1}{2}$ hundredths; $31\frac{1}{4}$ hundredths; 50 and $6\frac{1}{2}$ tenths; 100 and $8\frac{1}{8}$ hundredths; $12\frac{1}{2}$ cents; $6\frac{1}{4}$ cents; 90 dollars $66\frac{2}{3}$ cents; one hundred dollars $7\frac{1}{8}$ cents.

Six thousandths; sixty-five thousandths; seven hundred fourteen thousandths; eight hundred thirty-six thousandths; fifty and fifteen thousandths; one hundred five and fifty thousandths; five thousand and five thousandths; five thousand five and five hundred five thousandths; $7\frac{1}{2}$ thousandths; $18\frac{3}{4}$ thousandths; 6 and $137\frac{1}{2}$ thousandths; 4000 and 404 thousandths; 8000 and $8\frac{2}{3}$ thousandths; $6\frac{1}{4}$ mills; 18 cents $7\frac{1}{2}$ mills; 75 dollars $8\frac{1}{3}$ cents; 200 dollars $2\frac{1}{2}$ mills.

9. In the decimal .5, place a cipher between the decimal point and the numerator, and tell how the value of the decimal is affected.

247. Model. — If a cipher is placed between the decimal point and the numerator in .5, the decimal becomes .05, and the value is divided by 10; since the *number* of decimal units remains the *same*, while the *value* of each is made but $\frac{1}{10}$ *as great* by being removed one decimal place to the right.

Tell how the value of the following decimals is affected

By placing a cipher before the numerator of

1. .2; .3; .5; .7; .9; .12.
2. .25; .46; .37; .68; .125.

By rejecting a cipher from before the numerator of

3. .2; .03; .04; .06; .025; .075.
4. .013; .024; .035; .0046; .0057.

By annexing a cipher to the numerator of

1. .2; .4; .6; .8; .12; .23.
2. .35; .40; .57; .120; .306.

By omitting a cipher from the numerator of

3. .30; .50; .70; .130; .240; .370.
4. .360; .470; .580; .690; .7080.

In each of the preceding instances annex or reject two ciphers instead of one; three ciphers; etc.

2. *Reduction of Decimals.*

1. How many tenths in 1? In 2? In 3? In 4?

2. How many hundredths in 1? In 2? In 3? In 4? In 6?

3. In 1 dollar are how many cents? In $3? In $5? In $7?

4. In 2 are how many tenths? Hundredths? Thousandths?

5. In 2 dollars are how many dimes? How many cents? How many mills?

6. How many hundredths in 3 tenths? In 5 tenths? .9?

7. How many cents in 5 dimes? In 7 dimes? In 9 dimes?

8. In 40 hundredths how many tenths? In .50? In $.90?

9. How many thousandths in .25? Hundredths in .750?

10. In .5 how many hundredths? How many thousandths?

11. How many hundredths in .800? How many tenths?

12. How many cents in 4 dollars? How many mills?

13. In 6000 mills how many cents? How many dollars?

14. How many cents are $3.75?

248. Analysis. — Since 1 dollar is 100 cents, 3 dollars are 3 times 100 cents, or 300 cents; 300 cents and 75 cents are 375 cents. Hence, $3.75 are 375 cents.

15. Express 5 dollars as cents; 7 dollars; $8.50; $12.75.

16. Express 25 cents as mills; 12 cents 5 mills; $8.50; $9.875.

17. How are 275 cents expressed as dollars?

249. Analysis. — Since 100 cents are 1 dollar, 275 cents are as many dollars as the number of times 100 cents are contained in 275 cents, which is 2 times, with 75 cents remaining. Hence, 275 cents are 2 dollars 75 cents, or, decimally, $2.75.

18. Express 445 cents as dollars; 7000 mills; 8125 mills.

19. Express as higher orders 40 mills; 475 mills; 5625 mills.

20. Explain how dollars are changed to cents; cents to mills; dollars to mills.

21. Explain how mills are changed to cents; cents to *dollars; mills to* dollars.

22. How are thousands changed to hundreds? Hundreds to tens? Tens to ones? Ones to tenths? Tenths to hundredths? Hundredths to thousandths?

23. How are dollars changed to dimes? Dimes to cents? Cents to mills? Mills to cents? Cents to dimes? Dimes to dollars?

24. How may an integer or a decimal be changed to lower orders of units? To higher orders? State the principle.

25. Write 2 tenths as a decimal fraction; 4 tenths; .6; $.8.

26. Express .5 as a decimal fraction, and change it to halves.

27. How many halves in $\frac{50}{100}$? In .50? In .500?

28. In $\frac{25}{100}$ how many fourths? In $\frac{75}{100}$? In .50? In $.75?

29. Express .8 as a common fraction in its lowest terms.

30. Express $.25 as a fraction of a dollar; $.50; $.75.

31. Change .16$\frac{2}{3}$ to a fraction in its lowest terms.

250. Analysis. — Since .16$\frac{2}{3}$ is 16$\frac{2}{3}$ hundredths, it may be expressed $\frac{16\frac{2}{3}}{100}$, which equals $\frac{50}{300}$, or $\frac{1}{6}$. Hence, etc.

32. Express $.12$\frac{1}{2}$ as a fractional part of a dollar. Express $.62$\frac{1}{2}$; $.75; $.06$\frac{1}{4}$; $.08$\frac{1}{3}$.

33. When a decimal is expressed as a common fraction, what is made the numerator of the fraction? What is made the denominator?

34. Show how a decimal may be changed to a common fraction. Show that the value is not changed.

35. In 1 are how many tenths? In $\frac{1}{2}$ of 1, or $\frac{1}{2}$, how many?

36. How many hundredths in 1? In $\frac{1}{4}$ of 1, or $\frac{1}{4}$, how many?

37. How many cents in $\frac{1}{2}$ of a dollar? In $\frac{1}{4}$ of a dollar?

38. How many tenths in 2 divided by 5, or $\frac{2}{5}$?

251. Analysis. — Since 1 is 10 tenths, 2 is 2 times 10 tenths, or 20 tenths; and $\frac{1}{5}$ of 20 tenths is 4 tenths, or, decimally, .4. Hence, etc.

39. Express $\frac{4}{5}$ as tenths; $\frac{7}{10}$; $\frac{8}{4}$ as hundredths; $\frac{9}{20}$.

40. What decimal of a yard is $\frac{7}{8}$ of a yard? $\frac{3}{16}$ of a yard?

41. How may $\$\frac{3}{4}$ be expressed as the decimal of a dollar?

252. Analysis. — $\frac{3}{4}$ of a dollar equal 3 times $\frac{1}{4}$ of a dollar. Since 1 dollar is 100 cents, $\frac{1}{4}$ of a dollar is $\frac{1}{4}$ of 100 cents, or 25 cents; and $\frac{3}{4}$ of a dollar are 3 times 25 cents, or 75 cents, or \$.75. Hence, etc.

42. In $\$\frac{3}{4}$ how many cents? How many mills in $\frac{4}{5}$ of a cent? Express decimally the value of $\$\frac{5}{8}$; $\$\frac{3}{16}$; $\$\frac{2}{3}$.

43. Which term of a common fraction is the dividend? Which is the divisor? How is a common fraction changed to a decimal?

44. Since 10 mills are 1 cent, what part of a cent is 1 mill? 3 mills are what part of a cent? 5 mills? 7 mills?

45. Since 100 cents are 1 dollar, what part of a dollar is 1 cent? 5 cents are what part? 25 cents? 50 cents?

46. Since 1000 mills are 1 dollar, what part of a dollar is 1 mill? 7 mills are what part? 75 mills? 625 mills?

47. How many sheets of paper at 1 cent a sheet can be bought for \$1? For \$1.25? For \$2.50?

48. How many bushels of apples at \$1 a bushel can be bought for 500 cents? For 700 cents? For 800 cents?

49. By selling apples at 1 cent each, a boy took in 1 dollar 10 cents a day. How many apples did he sell?

50. At \$1 each, how many books can I buy with 625 cents, and how many cents will I have left?

51. At $\$.12\frac{1}{2}$ a pound, what will 12 pounds of sugar cost?

253. Analysis. —Since 1 pound costs $\$.12\frac{1}{2}$, or $\frac{12\frac{1}{2}}{100}$, or $\frac{12.5}{100}$, or $\frac{1}{8}$ of a dollar, 12 pounds cost 12 times $\$\frac{1}{8}$, or $\$\frac{12}{8}$, or $\$1\frac{4}{8}$, or $\$1\frac{1}{2}$, or \$1.50. Hence, etc.

52. At $\$.16\frac{2}{3}$ a yard, what will 8 yards of muslin cost? At \$.25 a yard? At $33\frac{1}{3}$ cents a yard?

53. What is the cost of 10 pounds of flour at $\$.06\frac{1}{4}$ a pound? *The cost of* 12 pounds? Of 20 pounds? Of 32 pounds?

54. If 12 dozen eggs cost \$2, what is the price per dozen?

254. Analysis. — Since 12 dozen eggs cost \$2, the price per dozen is $\frac{1}{12}$ of \$2, or $\frac{1}{6}$ of \$1, or \$$\frac{1}{6}$, or \$.16$\frac{2}{3}$. Hence, etc.

55. If 16 pounds of mutton cost \$2, what is the price per pound? If 16 pounds cost \$3? If 20 pounds of beef cost \$5?

56. If 24 quarts of berries cost \$1.50, what is the price per quart? If they cost \$2? If they cost \$3? If \$4?

57. If a piece of linen containing 12 yards cost \$7.50, what was the price per yard? If 16 yards cost \$9?

58. At \$.16$\frac{2}{3}$ a yard, how many yards can be bought for \$2?

255. Analysis. — Since 1 yard costs \$.16$\frac{2}{3}$, or $\frac{16\frac{2}{3}}{100}$, or $\frac{50}{300}$, or $\frac{1}{6}$ of a dollar, for \$2 there can be bought as many yards as the number of times \$$\frac{1}{6}$ is contained in \$2, or 12 yards. Hence, etc.

59. For \$3 how many pounds of raisins can be bought at \$.12$\frac{1}{2}$ a pound? At \$.20? At \$.25? At \$.33$\frac{1}{3}$? At \$.37$\frac{1}{2}$?

60. At \$1.25 each, how many silk handkerchiefs can be bought for \$5? For \$7.50? For \$10? For \$12.50?

256. Principles.

I. *The value of a decimal is not changed by annexing ciphers to, or omitting ciphers from, the right of the decimal.*

II. *A decimal fraction is a decimal with the denominator expressed.*

III. *The value of a fraction is the quotient of the numerator divided by the denominator.*

Exercises.

Tell, with your reason, how to express

1. .2 as hundredths; .4; .6.	*4. \$5 as cents.*	*7. 75 cents as mills.*
2. .25 as thousandths; .75.	*5. \$6.25 as mills.*	*8. 250 cents as dollars.*
3. .700 as hundredths; as tenths.	*6. \$7.50 as cents.*	*9. 2006$\frac{1}{4}$ mills as dol.*

Express 3.06$\frac{1}{4}$ as a mixed fraction.

257. Model. — Since .06¼ is 6¼ hundredths, it may be expressed $\frac{6\frac{1}{4}}{100}$, which equals $\frac{25}{400}$, or $\frac{1}{16}$; and $\frac{1}{16}$, united with the integer 3, gives $3\frac{1}{16}$, the mixed number required.

Express as fractions in their lowest terms, or as mixed numbers

1. .5; .8; .25.	4. .12½; .37½.	7. .037½; .001⅞.
2. .6; .75; .625.	5. .16⅔; .18¾.	8. $.006¼; $.012½
3. $.50; $.008.	6. $.06¼; $.62½.	9. $7.18¾; $10.07½.

Express 4¾ as a mixed decimal.

258. Model. — ¾ equals 3 divided by 4; and since 1 is 10 tenths, 3 is 3 times 10 tenths, or 30 tenths. ¼ of 30 tenths is 7 tenths, with 2 tenths remaining.

2 tenths are 20 hundredths; ¼ of 20 hundredths is 5 hundredths.

The result is 7 tenths 5 hundredths, or .75, which, united with the integer 4, gives *4.75*, the mixed decimal required.

Express as decimals or as mixed decimals

1. ¼; ⅖; ⅜; $\frac{5}{10}$.	4. $⅞; $⅘; $$\frac{9}{20}$.	7. 2⅞; 3⅘; 5$\frac{3}{10}$.
2. ¾; ⅝; $\frac{7}{10}$; $\frac{3}{20}$.	5. $$\frac{11}{25}$; ⅗ of $10⅗.	8. $6¼; $8⅞; 9\frac{7}{10}$.
3. ¾ of ⅚; ⅗ of $\frac{7}{12}$.	6. $$\frac{21}{50}$; 2½ times 8\frac{7}{10}$.	9 $.7½; $.06¼; $.00⅛

3. *Applications of Decimals.*

1. How many tenths are 3 tenths and 4 tenths? $\frac{4}{10}$ and $\frac{5}{10}$?
2. How many tenths are 8 tenths less 3 tenths? $\frac{9}{10}$ less $\frac{5}{10}$?
3. What is the sum of $\frac{2}{10}$ and $\frac{7}{10}$? Of .4 and .6?
4. Find the difference between $\frac{8}{10}$ and $\frac{3}{10}$. .9 and .4.
5. What is the sum of $\frac{4}{100}$ and $\frac{25}{100}$? The sum of .05 and .75?
6. Find the difference between $\frac{75}{100}$ and $\frac{9}{100}$. 1 and .75.
7. Find the sum of $\frac{8}{10}$ and $\frac{5}{100}$. The sum of .7 and .25.
8. What is the difference between $\frac{5}{10}$ and $\frac{6}{100}$? .75 and .5?
9. *How many* hundredths and tenths in .07 and .18?

10. Find how many hundredths and tenths in .5 less .25.

11. What is the sum of $\frac{3}{10}$, $\frac{5}{100}$, $\frac{7}{1000}$? Of .4, .6, and .008?

12. How much is $\frac{5}{10}$ less $\frac{5}{100}$? .06 less .006? .8 — .008?

13. What is the sum of 8 and $\frac{1}{2}$ expressed decimally? Of 12 and $\frac{3}{4}$? Of 15 and .75? Of \$8 and \$$\frac{1}{4}$? Of \$10 and \$.75?

14. Find the difference between 7 and $\frac{1}{4}$ expressed decimally. Between 10 and .75. Between \$8 and \$$\frac{3}{4}$. Between \$12 and \$.25. Between \$12.50 and \$$8\frac{9}{10}$.

15. When only can integral units be added or subtracted? When only can fractions be added or subtracted? Decimal units?

16. How many decimal places in the sum of tenths and tenths? Of tenths and hundredths? Of hundredths and thousandths? Tenths and thousandths?

17. In the difference between tenths and hundredths, how many decimal places? Between hundredths and thousandths? Tenths and thousandths?

18. If a farmer sells .10 of a lot of wheat to one miller, and .25 to another, what part does he sell to both?

19. If I offer a \$1 bill in payment for a pair of gloves that cost but \$.75, how much change should I receive?

20. A gardener paid \$1.50 for a spade, and \$1.25 for a rake. How much did he pay for both?

21. If a lady pays \$.25 for sugar, \$.30 for coffee, and \$.75 for tea, what does she pay for them all?

22. What is the remainder of \$1 less $\frac{7}{8}$? \$2 less \$.06$\frac{1}{4}$?

23. A carter bought a bushel of corn for \$.87$\frac{1}{2}$, and a bushel of oats for \$.62$\frac{1}{2}$. What did both cost him?

24. From a barrel of oil, containing 31.5 gallons, $4\frac{1}{4}$ gallons were drawn off. How many gallons remained?

25. What part of a dollar must be added to \$9.99 to make \$10? How much is 1 tenth less 1 thousandth?

26. What is the weight of three bars of silver weighing respectively .75 of a pound, 2.5 pounds, and .37½ of a pound?

27. James had a five-dollar bill, and paid $2.25 for some books, and $2⅛ for some paints. What had he remaining?

28. If I pay $6.50 for a barrel of flour, and $2¾ for groceries, what change from a $10 bill should I receive?

29. A silversmith had 12 ounces of silver, from which he made a set of teaspoons weighing 3.75 ounces, and a set of tablespoons weighing 7.5 ounces. How much had he left?

30. From a piece of carpeting containing 30½ yards were sold one piece measuring 18.5 yards, and another measuring 8.25 yards. How many yards remained?

Find the sum and the difference of

31. 8 yards and .25 of a yard.
32. 36 gallons and 1.5 gallons.
33. 7.5 rods and 18⅛ rods.
34. .37½ of a dollar and $6¼.
35. 5 bushels and 2.25 bushels.
36. 10¼ feet and .62½ of a foot.
37. $10 and .87½ of a dollar.
38. 14.5 days and 12¾ days.

39. How many tenths are 2 times 4 tenths? 3 times $\frac{3}{10}$?

40. How many tenths in ½ of 4 tenths? ⅓ of $\frac{6}{10}$? ¼ of .8?

41. How much is 3 times $\frac{2}{10}$? 2 times .4? 4 times .5?

42. How much is ¼ of $\frac{16}{100}$? ⅐ of $\frac{35}{100}$? ⅛ of .64?

43. Find the product of 3 times $\frac{5}{100}$; 4 times .06; 5 times .09.

44. What is the quotient of .25 divided by 5? .72 divided by 8? 8 divided by 4? .024 divided by 6?

45. What is the product of $\frac{5}{10}$ multiplied by $\frac{7}{10}$? .4 × .4?

46. Divide $\frac{9}{10}$ by $\frac{3}{10}$; $\frac{72}{100}$ by $\frac{9}{100}$; .9 by .3; .72 by .08.

47. Multiply $\frac{3}{100}$ by 2; $\frac{3}{100}$ by $\frac{2}{10}$; .03 by .2; .03 by .02.

48. Divide $\frac{24}{100}$ by 8; $\frac{24}{100}$ by $\frac{8}{10}$; .24 by .8; .024 by .08.

49. Find the product of 5 times $.08; 9 times $.005.

50. *How much* is ⅛ of $$\frac{24}{100}$$? ⅐ of $.63? $.075 divided by 5?

51. Find the product of the two factors .04 and .7.

52. The product of two factors is .028, and one of the factors is .7. What is the other factor?

53. How many ciphers in the product of the denominators of any two decimal fractions? Decimal places in the product of any two decimals?

54. The dividend is the product of what two factors? The dividend contains as many decimal places as what two factors?

55. How many decimal places in the product of tenths multiplied by ones? Tenths by tenths? Hundredths by tenths? How many decimal places are there always in the product of any two decimals?

56. How many decimal places in the quotient of tenths divided by ones? Tenths by tenths? Hundredths by tenths? Thousandths by tenths? How many decimal places are there always in the quotient?

57. If a grocer sells A sugar at $.10 a pound, how much will 3 pounds cost? 5 pounds? 7 pounds? 10 pounds?

58. If a boy sells 4 papers of gum-drops for $.80, how much does he receive for one paper?

59. At $.06 a pound, what will 10 pounds of nails cost? If $.35 are paid for 5 pounds, what is the price per pound?

60. At $.24 a dozen for eggs, what must be paid for .5 of a dozen? For 1.5 dozen? .25 of a dozen? 1.25 dozen?

61. In 1 rod are 5.5 yards. How many yards long is a chain that measures 5 rods? 8 rods?

62. If 2.5 yards of linen cost $1.25, what is the price per yard? If .25 of a yard of cloth cost $1.25?

63. What is the product of 5 multiplied by 5 thousandths? Of 25 hundredths by 4 thousandths? Of $.005 by .10?

64. If a pound of tea costs $.75, what is the cost of .5 of a pound? Of .25 of a pound? Of 2 pounds? 2.5 pounds?

65. If 4 quarts of cranberries are sold for 60 cents, what part of a dollar does one quart cost? 5 quarts cost what decimal part of a dollar? 8 quarts?

66. I paid $.75 fare on a railroad, at an average of 3 cents a mile. How far did I travel?

67. A housekeeper paid $5.25 for 100 pounds of flour. What price per pound did she pay? Find the cost of 6 pounds.

68. If 2 pounds of sugar cost $.33⅓, what will 9 pounds cost?

69. Frank bought 6 quires of paper at $.25 a quire, and had $.50 left. How much money had he at first?

70. Harry bought 7 melons at $.18 each, and had but a dollar and nine cents in his pocket. How much did he need to pay for them?

71. A lady paid $.75 for oranges at $.05 each. After giving away 10 of them, how many had she left?

72. I had $10 in my pocket, and paid all but $2.25 for 5 yards of cloth. What was the price of the cloth per yard?

Find the product of	Find the quotient of
73. .5 times .25 of a ton.	*77.* $7.25 divided by 100.
74. $.005 multiplied by 100.	*78.* 1.44 dozen divided by 12.
75. .25 times .05 of a gallon.	*79.* $1.50 divided by $.03.
76. $.125 multiplied by 80.	*80.* .875 of a ton divided by .25.

259. Principles.

I. *Only similar orders of decimal units can be added or subtracted.*

II. *The number of decimal places in the product equals the number of decimal places in both factors.*

III. *The number of decimal places in the dividend equals the number of decimal places in the divisor and the quotient together,*

Exercises.

1. From the product of .5 times .25 take .05.

260. Model. — 5 times 25 are 125; and since hundredths multiplied by tenths produce thousandths, the product must contain three decimal places, or as many decimal places as are in both factors, giving *.125*.

Now 125 thousandths less 5 hundredths, or 50 thousandths, are 75 thousandths, or *.075*, the result required.

2. To the quotient of .025 divided by .5 add .5.

261. Model. — 25 divided by 5 equals 5; and since thousandths divided by tenths produce hundredths, the quotient must contain two decimal places, or as many decimal places as are in the dividend less the number in the divisor, giving *.05*.

Now 5 tenths, or 50 hundredths, added to 5 hundredths are 55 hundredths, or *.55*, the result required.

Find the value of

1. $(.05 + .75) \times 50$.	*5.* $(.125 \div .5) \times 40$	*9.* $100 \times .05 + 50$.
2. $(.005 + .07) \times 12$.	*6.* $500 \times (.05 \div .25)$.	*10.* $(50 + .05) \times 10$.
3. $(.125 \times 80) + .25$.	*7.* $(.075 \times 1.1) \times 80$.	*11.* $1.25 \times (.7\frac{1}{2} + 1.25)$.
4. $(75 \times .001) + .5$.	*8.* $.008 \div (4 \times .25)$.	*12.* $(60\frac{1}{2} \times .5) - 25$.

3. Give your reasoning for fixing or placing the decimal point in the result of each of the preceding examples.

CHAPTER IV.

DENOMINATE NUMBERS.

1. Notation and Numeration.

262. A ***Denominate Unit*** is *one* of any name or denomination.

Thus, *1 yard* is a denominate unit; so, also, is 1 ton, 1 mile, etc.

263. A ***Denominate Number*** is one or more units of any denomination.

Thus, *1 foot* is a denominate number; so, also, is 3 quarts, etc.

264. A ***Simple Denominate Number*** is a number expressed in units of only one denomination.

Thus, *5 yards* is a simple denominate number; so, also, is 7 feet.

265. A ***Compound Denominate Number*** is a number expressed in units of two or more denominations of the same nature.

Thus, *5 yards 2 feet* is a compound number; also, 8 gallons 3 quarts.

266. Compound Denominate Numbers are generally called *Compound Numbers.*

MEASURES.

267. A ***Measure*** is a fixed unit used in comparing or determining quantity.

Thus, *1 foot* is a *measure of length;* 1 pound is a measure of weight; 1 dollar is a measure of value.

Measures may be divided into *six classes:*—

1. *Measures of Extension.*
2. *Measures of Capacity.*
3. *Measures of Weight.*
4. *Measures of Money.*
5. *Measures of Circles.*
6. *Measures of Time.*

MEASURES OF EXTENSION.

268. ***Extension*** may be a line, a surface, or a solid.

A ***Line*** is that which has length only.

A ***Surface*** is that which has length and breadth.

*A **Solid,** or **Body,** is that which has length, breadth, and thickness.*

269. Measures of extension are of three classes: *measures of lines*, *measures of surfaces*, and *measures of solids*.

I. Linear Measures.

270. *Linear Measures* are used in measuring lines and distances.

The units or denominations of linear measures are *inch*, *foot*, *yard*, *rod*, and *mile*.

Table.

12 inches (in.)	are	*1 foot, ft.*
3 feet	"	*1 yard, yd.*
5½ yards, or 16½ feet	"	*1 rod, rd.*
320 rods	"	*1 mile, mi.*

In measuring *cloth*, or goods sold by the yard, the linear yard is divided into *halves*, *quarters*, *eighths*, and *sixteenths*.

II. Surface Measures.

271. A ***Square*** is a surface having four equal straight sides, and four equal corners or angles.

A ***Square Inch*** is a surface 1 inch long and 1 inch wide.

A ***Square Foot*** is a surface 1 foot long and 1 foot wide.

272. *Surface Measures,* or ***Square Measures,*** are used in ascertaining the extent of surfaces; as of *boards*, *plastering*, *land*, etc.

The units or denominations of surface or square measures are *square inch*, *square foot*, *square yard*, *square rod*, *acre*, and *square mile*.

Table.

144 square inches (sq. in.)	are	*1 square foot, sq. ft.*
9 square feet	"	*1 square yard, sq. yd.*
30¼ square yards	"	*1 square rod, sq. rd.*
160 square rods	"	*1 acre, A.*
640 acres	"	*1 square mile, sq. mi.*

In measuring land, 16 square rods are 1 square chain (*sq. ch.*), and 10 square chains are 1 acre.

III. SOLID MEASURES.

273. A ***Cube*** is a solid having six equal square sides, called faces.

A ***Cubic Inch*** is a solid 1 inch long, 1 inch wide, and 1 inch thick.

A ***Cubic Foot*** is a solid 1 foot long, 1 foot wide, and 1 foot thick.

274. ***Solid*** or ***Cubic Measures*** are used in ascertaining the contents or volumes of solids; such as *boxes, timber, rooms,* etc.

The units or denominations of solid or cubic measures are *cubic inch, cubic foot,* and *cubic yard.*

Table.

1728 cubic inches (*cu. in.*)	are	*1 cubic foot, cu. ft.*
27 cubic feet	"	*1 cubic yard, cu. yd.*

In measuring wood, 128 cubic feet are 1 cord (*cd.*).

MEASURES OF CAPACITY.

275. ***Capacity*** means extent of room or space.

Thus, the *capacity* of a box is the *extent of space* within the six inner surfaces of the box; the capacity of a barrel is the extent of space within the bounding inner surfaces of the barrel.

The measures of capacity are of two classes: *measures of liquids* and *measures of dry substances.*

Note. — The measures of capacity are all cubic measures; each unit of capacity representing a certain number of cubic inches.

I. LIQUID MEASURES.

276. ***Liquid Measures*** are used in measuring liquids; such as *water, milk, oil, liquors,* etc.

The units or denominations of liquid measure are *gill, pint, quart,* and *gallon.*

Table.

4 gills (gi.)	are	*1 pint, pt.*
2 pints	"	*1 quart, qt.*
4 quarts	"	*1 gallon, gal.*

In measuring the capacity of cisterns, etc., 31½ gallons are 1 barrel (*bbl.*); 63 gallons are 1 hogshead (*hhd.*).

The gallon, liquid measure, contains *231 cu. in.*

II. DRY MEASURES.

277. Dry Measures are used in measuring dry substances; such as *grain, fruit, vegetables, lime, coal,* etc.

The units or denominations of dry measure are *pint, quart, peck,* and *bushel.*

Table.

2 pints (pt.)	are	*1 quart, qt.*
8 quarts	"	*1 peck, pk.*
4 pecks	"	*1 bushel, bu.*

The gallon, or half-peck, or 4 quarts, dry measure, contain *268 4/5 cubic inches.* 1 bushel contains *2150.42* cubic inches.

MEASURES OF WEIGHT.

278. Weight is the quantity of heaviness in a body.

The measures of weight are of two classes: *Troy weights* and *avoirdupois weights.*

I. TROY WEIGHTS.

279. Troy Weights are used in weighing gold, silver, etc.

The units or denominations of Troy weights are *grain, pennyweight, ounce,* and *pound.*

Table.

24 grains (gr.)	are	*1 pennyweight, pwt.*
20 pennyweights	"	*1 ounce, oz.*
12 ounces	"	*1 pound, lb.*

The Troy pound weighs *5760* Troy grains.

Note.* — *Apothecaries' Weights are used by physicians in prescribing, and by apothecaries in mixing dry medicines.

Table.

20 grains (gr.)	are	*1 scruple, sc. or* ℈.
3 scruples	"	*1 dram, dr. or* ʒ.
8 drams	"	*1 ounce, oz. or* ℥.
12 ounces	"	*1 pound, lb. or* ℔.

The pound, the ounce, and the grain are the same in Troy and Apothecaries' Weights. The *only* difference between the two is in the *division of the ounce.*

II. AVOIRDUPOIS WEIGHTS.

280. Avoirdupois Weights are used in weighing all coarse and heavy articles; such as *groceries, iron, coal,* etc.

The units or denominations of avoirdupois weights are *ounce, pound, hundred-weight,* and *ton.*

Table.

16 ounces (oz.)	are	*1 pound,*	*lb.*
100 pounds	"	*1 hundred-weight,*	*cwt.*
20 hundred-weight	"	*1 ton,*	*T.*

In weighing coal and iron, and in a few other cases, the gross hundred-weight of 112 pounds, and the gross ton of 2240 pounds are used.

The pound avoirdupois weighs *7000* grains Troy.

MEASURES OF MONEY.

281. Money is the standard of price or value used in trade.

Money is of two kinds: *coin* and *paper money.*

282. Currency is whatever circulates in trade and commerce as money.

Currency is either coin or paper money.

I. UNITED STATES MONEY.

283. United States Money is the legal currency *of the United States.*

The units or denominations of United States money are *mill, cent, dime, dollar,* and *eagle.*

Table.

10 mills (m.)	are	*1 cent, ct. or c.*
10 cents	"	*1 dime, d.*
10 dimes	"	*1 dollar, $.*
10 dollars	"	*1 eagle, E.*

284. The ***Coins*** of the United States by the New Coinage Act of 1873 are of gold, silver, nickel, and bronze.

The ***Gold Coins*** are the double-eagle, eagle, half-eagle, quarter-eagle, three-dollar piece, and dollar.

The ***Silver Coins*** are the trade-dollar, half-dollar, quarter-dollar, dime, and twenty-cent piece (1875).

The ***Nickel Coins*** are the five-cent piece and the three-cent piece.

The ***Bronze Coin*** is the cent.

II. ENGLISH MONEY.

285. ***English Money*** is the legal currency of Great Britain.

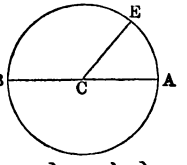
The units or denominations of English money are *farthing, penny, shilling,* and *pound.*

Table.

4 farthings (far. or qr.)	are	*1 penny, d.*
12 pence	"	*1 shilling, s.*
20 shillings	"	*1 pound, £.*

The pound, or gold sovereign, is worth in U. S. money $\$4.866\frac{1}{2}$.

MEASURES OF THE CIRCLE.



286. A ***Circle*** is a plane or flat surface bounded by a curved line, which is everywhere at an equal distance from a point within called the centre.

A ***Circumference*** is the line that bounds a circle.

A ***Degree*** is one of the 360 equal parts into which the circumference of every circle is divided.

An ***Angle*** is the opening between two straight lines that meet at the same point.

Thus, the lines E C and A C, which meet at C, form the angle A C E.

287. Circular or ***Angular Measures*** are used to measure angles, to reckon latitude, longitude, time, etc.

The units or denominations of circular or angular measures are *second, minute, degree,* and *circumference.*

Table.

60 seconds ($''$)	are	*1 minute,*	$'$.
60 minutes	"	*1 degree,*	°.
360 degrees	"	*1 circumference,*	*C.*

MEASURES OF TIME.

288. Time is a limited or definite portion of duration.

The units or denominations of time are *second, minute, hour, day,* and *year.*

Table.

60 seconds (sec.)	are	*1 minute,*	*min.*
60 minutes	"	*1 hour,*	*hr.*
24 hours	"	*1 day,*	*da.*
365 days	"	*1 common year,*	*yr.*
366 days	"	*1 leap year,*	*yr.*
100 years	"	*1 century,*	*C.*

7 days are 1 week (*wk.*). For many purposes 4 weeks are called a month (*mo.*); and 52 weeks a year. In most business transactions, 30 days are considered as a month, and 12 months as a year.

289. The ***Solar Year*** is the time required for the earth to revolve around the sun, and is 365 da. 5 hr. 48 min. 49.7 sec. in length.

Hence, by counting only 365 days as a year, almost $\frac{1}{4}$ of a day is *lost every year,* or 1 day every four years.

This 1 day is, therefore, added every fourth year to February, giving it 29 days, and making what is called *leap year*, which contains 366 days.

290. In ***counting*** certain articles, the units or denominations *dozen, gross,* and *great gross* are used.

Table.

12 things	are	*1 dozen,*	*doz.*
12 dozen	"	*1 gross,*	*gro.*
12 gross	"	*1 great gross,*	*grt. gro.*

291. In ***buying and selling paper,*** the units or denominations *sheet, quire, ream, bundle,* and *bale* are used.

Table.

24 sheets	are	*1 quire.*	*2 reams*	are	*1 bundle.*
20 quires	"	*1 ream.*	*5 bundles*	"	*1 bale.*

292. Principles.

I. *The units of denominate numbers increase and decrease by a varying scale.*

II. *The successive orders of units increase in value from the right to the left.*

Exercises.

1. How many ones make one ten? One is how many tenths? How many tens make one hundred? One tenth is how many hundredths? How many inches are one foot? Square inches one square foot? Cubic feet one cubic yard? Gills one pint? Quarts one peck? Grains one pennyweight? Ounces one pound? Hours one day? Single things one dozen?

2. Name the units of the first order in integers; in decimals. The units of the second order in integers; in decimals. The units of the first order in linear measures; of the second order. Name the orders of units in each measure of extension; in each measure of capacity; each measure of weight; each measure of money, or value; measures of the circle; measures of time.

3. What is the scale of increase and decrease in integers? In decimals? In linear measures? In square measures? In cubic

measures? Dry and liquid measures? Troy and avoirdupois weights? United States and English money? Measures of the circle? Measures of time?

4. Tell what orders of units are denoted in the following denominate numbers; tell how many of each order; read each number, or express it in words: —

6 in.	*9 gr.*	*4 $\frac{3}{4}$ ft.*	*3 cd. 5 cu. ft.*	*5 mi. 7 rd. 10 ft.*
8 sq. ft.	*10 oz.*	*9.5 A.*	*4 bu. 7 $\frac{1}{2}$ qt.*	*10 hhd. 4 gal. 2 qt.*
9 cu. yd.	*$15.*	*7 $\frac{3}{8}$ T.*	*£5 15.5 s.*	*5 T. 8 cwt. 5 lb.*
2 gi.	*11 d.*	*40.5°.*	*75° 45 $\frac{1}{2}'$.*	*£10 5 s. 10 d.*
3 pk.	*21 hr.*	*6 $\frac{5}{12}$ doz.*	*5 yr. 20 da.*	*365° 50′ 37.5″.*

5. Express as denominate or compound numbers, with proper abbreviations,

Seven yards; ten square yards; six cubic feet; eight pints; ten bushels; three pennyweights; twelve grains; twenty tons; seventy-five cents; two shillings; fifteen degrees; five hours; $7\frac{1}{2}$ minutes; 10.5 degrees; $6\frac{3}{4}$ pence; $25\frac{3}{4}$ cents; $75\frac{3}{8}$ hundred-weight; $10\frac{1}{8}$ grains; $7\frac{1}{16}$ quarts; 12.5 hogsheads; $7\frac{3}{10}$ acres; 11.25 cubic yards.

Twenty-five dollars ten cents five mills; ten pounds six pence three farthings; six degrees two minutes one second; one hundred tons ninety pounds four ounces; six ounces four pennyweights ten grains; thirty barrels one pint two gills; eighty bushels two pecks one pint; twelve cords four cubic feet one cubic inch; five miles ten rods one foot; seventy acres fourteen square yards one hundred square inches; 10 miles 7 yards $7\frac{1}{3}$ inches; 10 square yards 7.25 square feet; 50 cords $9\frac{3}{10}$ cubic feet; 60 hogsheads 10 gallons 2.5 gills; 100 tons 5 hundred-weight $7\frac{3}{4}$ pounds.

2. *Reduction of Denominate Numbers.*

1. How many ones in 1 ten? In 2 tens? In 3 tens?

2. Change 1 hundred to tens; to ones. 1 to tenths; to hundredths.

3. How many tens in 10 ones? In 30 ones? In 50 ones?

4. *Change 100* ones to tens; to hundreds. 200 ones; 300 ones.

5. How many inches in 1 foot? In 2 feet? 3 feet?

6. Change 1 yard to feet; to inches. 36 in. to feet; to yards.

7. By what process is any integral unit changed to lower orders? Any decimal unit? What is the uniform multiplier?

8. By what process are integral units changed to higher orders? Decimal units? What is the uniform divisor?

9. By what process is any denominate unit changed to lower orders, or denominations? Is the multiplier uniform, or does it vary according to the measures?

10. By what process are denominate units changed to higher orders, or denominations? Is the divisor uniform, or is it as varying as the measures?

11. At 4 cents a pint, how much will a milkman receive for 6 gallons of milk?

293. Analysis. — Since 1 gallon is 4 quarts, 6 gallons are 6 times 4 quarts, or 24 qt.; and since 1 quart is 2 pints, 24 quarts are 24 times 2 pints, or 48 pt. And,

At 4 cents a pint, he will receive for 48 pt. 48 times 4 cents, or 192 cents, or 1 dollar 92 cents, or $1.92. Hence, etc.

12. How many pint-bottles are needed to hold 3 gal. of wine?

13. If a carter feeds his horse 1 peck of corn a day, how many days will 4 bu. of corn last him? 8 bu.? 12 bu.?

14. A boy sold 2 bushels of peanuts a day for 3 days, at 10 cents a quart. How much did he receive for them?

15. At 5 cents a square foot, how much will it cost to paint a wall containing 12 square yards? 20 sq. yd.?

16. If a housekeeper uses 1 pint of cream a day, how many gallons will she use in 60 days?

294. Analysis. — Since a housekeeper uses 1 pint a day, in 60 days she will use 60 times 1 pint, or 60 pt. And,

Since 2 pints are 1 quart, 60 pt. are as many quarts as the number of times 2 pt. are contained in 60 pt., or 30 qt.; and,

Since 4 quarts are 1 gallon, 30 qt. are as many gallons as the number of times 4 qt. are contained in 30 qt., or 7½ gal. Hence, etc.

17. A grocer sold 96 quarts of beans one week. How many bushels did he sell? If he had sold 100 qt.? 150 qt.?

18. If a boy sells 10 pints of chestnuts for 8 days, how many bushels does he sell? What did they cost at $2 a bushel?

19. At 5 cents a pint for milk, how many gallons of milk can be bought for 2 dollars? For 3 dollars?

20. What is the weight in pounds of 12 silver teaspoons, each weighing 30 pwt.? At $2 an ounce, what are they worth?

21. A druggist has on hand 2 gal. 3 qt. 1 pt. of laudanum. How long will it last him if he sells on an average 1 pt. a day?

295. Analysis. — Since 1 gallon is 4 quarts, 2 gallons are 2 times 4 quarts, or 8 quarts; 8 qt. and 3 qt. are 11 qt.

Since 1 quart is 2 pints, 11 quarts are 11 times 2 pints, or 22 pints; 22 pt. and 1 pt. are 23 pt. And,

Since he sold 1 pint a day, 23 pints will last him as many days, etc. Hence, etc.

22. If a farmer sells 1 bu. 3 pk. 5 qt. of clover-seed at 8 cents a quart, how much does he receive for it?

23. How many pound-papers can be filled from 4 cwt. 25 lb. of corn-starch? Find the value at $.20 per paper.

24. How much freight must be paid on 2 T. 4 cwt. of goods at 1 cent a pound? At 2 cents? At $.02½?

25. If the weight of 1 pint of water is a pound, what is the weight of a gallon of water? The weight of a barrel?

26. A lady made 2 gal. 3 qt. of marmalade, and put it into half-pint bottles. How many bottles were needed to contain it? What was it worth at $.12½ a bottle?

27. I ordered a gold watch-case to weigh 3 oz.; but the case delivered weighed only 56 pwt. What did it fall short of the required weight?

28. I fed my hens 1 quart of corn each day for 100 days. How much corn did I feed them?

296. Analysis. — Since I fed my hens 1 quart each day, in 100 *days I fed them 100* times 1 qt., or 100 qt.

Since 8 quarts are 1 peck, 100 qt. are as many pecks as the number of times 8 qt. are contained in 100 qt., or 12 pecks, with 4 qt. remaining. And,

Since 4 pecks are 1 bushel, 12 pk. are as many bushels as the number of times 4 pk. are contained in 12 pk., or 3 bu.

Hence, I fed them 3 bu. 4 qt.

29. A merchant bought 300 yd. of English goods at 1 shilling per yard. How much did he pay for the goods?

30. An apothecary sold 50 doses of medicine, each 5 grains. Express the weight in higher denominations.

31. At 8 cents a quart, what quantity of chestnuts can I buy for 2 dollars? For 4 dollars? For 5 dollars?

32. How much milk does a milkman serve in filling a half-pint measure 50 times? 75 times?

33. Find the length of a roll of gold wire weighing 3 lb. 8 oz., at the average weight of 1 grain to the foot.

34. If a vest requires 6 buttons, how many dozen buttons are needed to trim 50 vests? How many gross to trim 6 doz. vests? What are they worth at $\$.16\frac{2}{3}$ a dozen?

35. How long is a string which measures $\frac{8}{9}$ of a yard?

297. Analysis. — Since 1 yard is 3 feet, $\frac{8}{9}$ of a yard must be $\frac{8}{9}$ of 3 feet, or $\frac{24}{9}$, or $\frac{8}{3}$, of a foot; and since 1 foot is 12 inches, $\frac{8}{3}$ of a foot must be $\frac{8}{3}$ of 12 in., or $\frac{96}{3}$, or 32 in. Hence, etc.

36. At $37\frac{1}{2}$ cents a quart for molasses, what will $3\frac{3}{4}$ gal. cost?

37. How many ounces in $3\frac{3}{4}$ pounds of sugar? In $3\frac{3}{4}$ pounds of silver? In $3\frac{3}{4}$ pounds of opium put up in doses?

38. How much more or less will $\frac{3}{4}$ of a ream of paper cost at $\$.20$ a quire, than $\frac{3}{4}$ of a gross of lead-pencils at $\$.30$ a doz.?

39. A grocer bought $\frac{3}{5}$ of a cwt. of sugar for $\$6\frac{3}{4}$, and sold it at $12\frac{1}{2}$ cents a pound. How much did he gain?

40. How much is gained by buying $\frac{7}{12}$ of a gross of buttons at $\$.75$ a gross, and selling them at $7\frac{1}{2}$ cents a dozen?

41. At 25 cents a peck, what will .75 bu. of corn-meal cost?

298. Analysis. — Since 1 bushel is 4 pecks, .75 of a bushel is .75 of 4 pecks, or 3 pk.; and since 1 peck costs 25 cents, etc.

42. At $6\frac{1}{4}$ cents an ounce, what is the cost of .5 lb. of tea?

43. How many bottles, each holding 1.5 pt., are needed to contain 1.5 gal. of wine? To contain 2.25 gal.?

44. At $.25 per pound for coffee, how much will a bag of coffee containing .5 of a cwt. cost? At $\$\frac{3}{8}$ per pound?

45. Express $\frac{3}{4}$ pk. of oats as a part of a bushel.

299. Analysis. — Since 4 pecks are 1 bushel, there is $\frac{1}{4}$ as many bushels as pecks, or $\frac{1}{4}$ of $\frac{3}{4}$, or $\frac{3}{16}$ bu. Hence, etc.

46. What part of a quart of oil can be put in a can that holds $\frac{2}{5}$ of a pint? What part of a gallon?

47. If a boy wastes $\frac{1}{2}$ hr. in idleness in the morning, and $\frac{1}{4}$ hr. in the afternoon, what part of a school-day of 5 hr. is lost?

48. If a wine-bottle holds $\frac{3}{4}$ of a pint of wine, what part of a gallon does it contain? Find its value at $\$4\frac{1}{2}$ a gallon?

49. What decimal part of a pound is paid for an English book that cost .5 of a shilling?

50. If a rule measures .75 of a foot, what is its length, expressed as a decimal part of a yard?

51. Into one bottle a druggist put $\frac{3}{8}$ pt. of hair-oil, and .2 qt. into another. Which contained the greater part of a gal.?

52. What is the value of $\frac{3}{7}$ gal. in lower denominations?

300. Analysis. — Since 1 gallon is 4 quarts, $\frac{3}{7}$ gal. is $\frac{3}{7}$ of 4 quarts, or $\frac{12}{7}$ qt., or $1\frac{5}{7}$ qt.; since 1 quart is 2 pints, $\frac{5}{7}$ qt. is $\frac{5}{7}$ of 2 pints, or $\frac{10}{7}$ pt., or $1\frac{3}{7}$ pt.; and since 1 pt. is 4 gills, $\frac{3}{7}$ pt. is $\frac{3}{7}$ of 4 gills, or $\frac{12}{7}$ gi., or $1\frac{5}{7}$ gi. Hence, $\frac{3}{7}$ gal. equals 1 qt. 1 pt. $1\frac{5}{7}$ gi.

53. If the hills in a row of corn are $\frac{1}{3}$ of a rod apart, what is the distance between them in feet and inches?

54. How many square feet and square inches of paper are needed to cover the top of a box measuring $2\frac{5}{7}$ sq. yd.?

55. How many rods and feet of fencing are needed to enclose my lot, which measures $\frac{1}{12}$ of a mile around?

56. Express in lower denominations the value of .75 of a yd.

57. If a set of silver spoons weighs .125 of a pound, what is the weight in ounces and pennyweights?

58. Change 1 pk. 3 qt. to the fraction of a bushel.

301. Analysis. — Since 8 quarts are 1 peck, there must be $\frac{1}{8}$ as many pecks as there are quarts, or $\frac{1}{8}$ of 3, or $\frac{3}{8}$ pk., which, added to 1 peck, equals $1\frac{3}{8}$ pk., or $\frac{11}{8}$ pk.; and since 4 pk. are 1 bushel, there must be $\frac{1}{4}$ as many bushels as pecks, or $\frac{1}{4}$ of $\frac{11}{8}$, or $\frac{11}{32}$ bu. Hence, etc.

59. Express 1 pt. 3 gi. of vinegar as a fraction of a quart; as a decimal of a gallon. Find the value at \$.30 a gal.

60. At \$1 a bushel, what is the value of 2 bu. 2 pk. 3 qt. of wheat? Of 4 bu. 3 pk. 7 qt.? 5 bu. 2 pk. 6 qt.?

61. How much must I pay for 3 lb. 10 oz. of coffee, the cost being 32 cents a pound? For 4 lb. 5 oz.?

62. At \$.32 a gallon, what is the cost of 2 gal. 1 qt. $2\frac{1}{2}$ pt. of milk? Of 3 gal. 2 qt. $1\frac{1}{4}$ pt.?

63. $3\frac{2}{3}$ ounces of gold are worth \$55. What is the value of $3\frac{2}{3}$ pounds at the same rate? $\frac{3}{4}$ of .5 of a pound?

64. Which cost the greater sum, 2 bu. 4 qt. of chestnuts at 10 cents a quart, or 2 bu. 12 qt. at \$3 a bushel?

302. Principle.

Denominate numbers are changed to units of lower orders by multiplication, and to units of higher orders by division.

Exercises.

1. Change

1. *2 rd. 2 yd. 3 in. to in.*	6. *3 cwt. 5 lb. to oz.*	11. $\frac{4}{5}$ *gal. to gi.*
2. *4 sq. yd. 6 sq. ft. to sq. in.*	7. *4° 50′ 50″ to ″.*	12. $1\frac{1}{2}$ *bu. to pt.*
3. *10 gal. 2 qt. 1 $\frac{1}{4}$ pt. to gi.*	8. *1 gr. 10 doz. to ones.*	13. *.005 lb. to gr.*
4. *5 bu. 6 qt. $\frac{2}{5}$ pt. to pints.*	9. $\frac{1}{5}$ *yd. to ft.; to in.*	14. *.025 T. to lb.*
5. *2 lb. 2 oz. 2 pwt. to gr.*	10. $\frac{1}{4}$ *sq. yd. to sq. in.*	15. *1.05 gal. to pt.*

2. Change to integers of higher denominations

1. *63 in.; 87 in.; 66 ft.*
2. *360 sq. in.; 1089 sq. ft.*
3. *23 gi.; 127 pt.; 135 gi.*
4. *970 gr. T.; 428 oz. avr.*

Change

5. *18 in. to yd.; to rd.*
6. *72 sq. in. to sq. yd.*
7. *22 gi. to qt.; to gal.*
8. *48 pt. to pk.; to bu.*
9. *144 gr. to oz.; to lb.*

10. $\frac{1}{2}$ *pt. to gal.*
11. *.5 in. to yd.*
12. *.06 qt. to bu.*
13. $\frac{5}{8}$ *dr. to lb.*
14. *25 min. to hr.*

3. Change to integers of lower denominations

1. $\frac{3}{8}$ *yd.;* $\frac{5}{9}$ *yd.;* $\frac{3}{16}$ *ft.*
2. $\frac{1}{2}$ *sq. yd.;* $\frac{5}{8}$ *sq. yd.*
3. $\frac{4}{5}$ *gal.;* $\frac{1}{2}$ *bbl.;* $\frac{5}{7}$ *qt.*
4. $\frac{5}{32}$ *lb. av.;* $\frac{3}{32}$ *lb. Troy.*
5. *.25 T.;* $12\frac{1}{2}$ *hr.; 05 bu.*

Express in the form of a fraction or decimal, or mixed number

6. *2 ft. 6 in. as ft.; as yards.*
7. *2 qt. 1 pt. as qt.; as gallons.*
8. *2 bu. 3 pk. 2 qt. 1 pt. as bushels.*
9. *5 cwt. 25 lb. as cwt.; as tons.*
10. *2 yd. 1 ft. 6 in. as yd.; as rods.*

3. Applications of Denominate Numbers.

1. How many ones are 7 ones and 5 ones? How many tens and ones? 9 ones and 6 ones are how many?

2. How many ones are 9 ones less 4 ones? 1 ten 5 ones, or 15 ones, less 7 ones are how many?

3. How many inches are 8 inches and 6 inches? How many feet and inches? 10 inches and 8 inches?

4. How many inches are 11 inches less 6 inches? 1 foot 5 inches, or 17 inches, less 9 inches are how many?

5. What is the sum of 2 tens 6 ones and 3 tens 8 ones? Of 2 tenths 6 hundredths and 3 tenths 8 hundredths? Of 2 feet 6 inches and 3 feet 8 inches?

6. What is the remainder of 6 tens 4 ones less 3 and 8 ones? Of 6 tenths 4 hundredths less 3 tenths 8 hundredths? Of 6 feet 4 inches less 3 feet 8 inches?

7. In integral numbers, how many units of any order are

one unit of the next higher order? In decimal numbers? In denominate numbers?

8. What is the general principle of adding and of subtracting integral, decimal, and denominate numbers?

9. If a grocer puts 25 lb. 8 oz. of butter in a crock weighing 3 lb. 10 oz., what is the weight of both?

303. Analysis. — Since the butter weighs 25 lb. 8 oz., and the crock 3 lb. 10 oz., the weight of both is the sum of 25 lb. 8 oz. and 3 lb. 10 oz.

25 lb. 8 oz. and 3 lb. are 28 lb. 8 oz.; 28 lb. 8 oz. and 10 oz. are 28 lb. 18 oz., or 29 lb. 2 oz. Hence, etc.

10. From a can containing 10 gal. 2 qt. of oil there were drawn off 5 gal. 3 qt. How much remained in the can?

304. Analysis. — Since the can contained 10 gal. 2 qt. of oil, and there were drawn off 5 gal. 3 qt., there remained the difference between 10 gal. 2 qt. and 5 gal. 3 qt.

10 gal. 2 qt. less 5 gal. are 5 gal. 2 qt.; 5 gal. 2 qt., or 4 gal. 6 qt., less 3 qt. are 4 gal. 3 qt. Hence, etc.

11. If a grain-dealer sells 4 bu. 3 pecks to one man, and 3 bu. 2 pk. to another, how much does he sell to both?

12. From a piece of beef weighing 16 lb. 4 oz., a butcher sold 9 lb. 8 oz. How much remained in the piece?

13. A stationer has 4 gro. 10 doz. lead-pencils in one package, and 6 gro. 4 doz. in another. How many in both?

14. How much longer is a piece of muslin measuring 33 yd. 1 ft. than a piece measuring 28 yd. 2 ft.?

15. How much wood is in two piles, one containing 6 cd. 100 cu. ft., and the other containing 8 cd. 40 cu. ft.?

16. If the side-wall of a room contains 22 sq. yd. 3 sq. ft., and the windows measure 9 sq. yd. 8 sq. ft., what surface is to be covered by paper?

17. Into a tub containing $\frac{1}{2}$ bbl. of water, I poured $\frac{3}{4}$ of a gallon. How much water was there in it then?

18. From a field of 2 A. 4 sq. ch. was sold a lot measuring $\frac{4}{5}$ of an acre. How much then remained?

19. What is the weight of three bars of silver weighing respectively $2\frac{3}{4}$ lb., $1\frac{7}{8}$ lb., and $3\frac{1}{2}$ lb. av.? Troy?

20. From a piece of ribbon measuring $3\frac{7}{8}$ yd., a lady cut off $6\frac{3}{4}$ ft. How long was the remnant?

Find the sum of	Find the difference between
21. 8 yd. 2 ft. and 7 yd. 2 ft.	*25.* 8 A. and 6 A. 100 sq. ch.
22. 10 gal. 3 qt. and 9 gal. 1 pt.	*26.* 10 da. 12 hr. and 7 da. 18 hr.
23. 17 bu. 2 pk. and 11 bu. 3 pk.	*27.* 60° 10′ 20″ and 45° 20′ 30″.
24. $\frac{3}{4}$ lb. + $\frac{7}{8}$ oz.; £.5 + .25 s.	*28.* $\frac{7}{8}$ yd. — $\frac{7}{8}$ ft.; .25 bu. — .5 pk.

29. How many ones are 3 times 6 ones? Tens and ones?

30. One half of 1 ten 2 ones, or 12 ones, is how many ones?

31. How many quarts are 5 times 3 quarts? How many gallons and quarts?

32. One half of 1 gal. 2 qt., or 6 qt., is how many quarts?

33. What is the product of 3 times 2 tens 6 ones? 3 times 2 tenths 6 hundredths? 3 times 2 feet 6 inches?

34. How much is one third of 7 tens 8 ones? One third of 7 tenths 8 hundredths? One third of 7 feet 6 inches?

35. If a pair of young chickens weighs 3 lb. 9 oz., how much do 4 such pair weigh?

305. Analysis. — If one pair of chickens weighs 3 lb. 9 oz., 4 pair weigh 4 times 3 lb. 9 oz.

4 times 3 lb. are 12 lb.; 4 times 9 oz. are 36 oz., or 2 lb. 4 oz.; 12 lb. and 2 lb. 4 oz. are 14 lb. 4 oz. Hence, etc.

36. If a horse eats 13 bu. 3 pk. of oats in 5 weeks, how much does he eat in 1 week?

306. Analysis. — If a horse eats 13 bu. 3 pk. of oats in 5 weeks, in 1 week he will eat $\frac{1}{5}$ of 13 bu. 3 pk.

$\frac{1}{5}$ of 13 bu. is 2 bu., with 3 bu. remaining; 3 bu. are 12 pk., and 12 *pk. plus* 3 pk. are 15 pk.; $\frac{1}{5}$ of 15 pk. is 3 pk., which, united with 2 *bu., gives 2 bu. 3* pk. Hence, etc.

37. Frank is 6 years 8 months old, and his mother is 4 times as old. How old is his mother?

38. If a livery-stable keeper feeds 10 bu. 3 pk. of corn in 6 days, how much does he feed per day?

39. If a can of fruit weighs 2 lb. 8 oz., what is the weight of 6 such cans? 7 cans? 8 cans? 10 cans?

40. If 16 bu. 3 pk. of apples are needed to fill 6 barrels, how many are needed to fill each barrel?

41. How much linseed-oil will a painter use in 8 days, if he uses 2 gal. 2 qt. 1 pt. each day? In 10 days?

42. If a steamboat runs 57 mi. 30 rd. in 5 hours, how far does it run in 1 hr.? If 73 mi. 70 rd. in 6 hr.?

43. At $.50 a pound, what is the cost of 3 packages of tea, each 2 lb. 8 oz.? 5 packages, each 3 lb. 12 oz.?

44. If 4 bu. 1 pk. 4 qt. of corn cost $3.50, what is the price per bushel? If it cost $3.85? If $4.20?

45. If one barrel holds 2 bu. 3 pk. of apples, how many barrels are needed to hold 22 bu.? To hold $24\frac{3}{4}$ bu.?

46. If a field of grass will average 1 T. 15 cwt. to the acre, how much can be cut from 14 acres?

47. If from a barrel of vinegar 4 gal. 3 qt. be drawn, and the remainder put in 8 cans, what will each contain?

48. If a man can walk 7 miles in 2 hours, how far can he walk in 5 days, walking 10 hours a day?

What is the result of

49. 6 times £5 10 s. 6 d.?

50. 7 times 4 qt. 1 pt. 3 gi.?

51. 8 times 8 bu. 3 pk. 2 qt.?

52. 9 times 10 T. 10 cwt. 20 lb.?

53. $\frac{1}{5}$ of 10 yd. 2 ft. 6 in.?

54. $\frac{1}{8}$ of 12 lb. 2 oz. 8 pwt.?

55. $\frac{2}{7}$ of 20 gal. 0 qt. 1 pint?

56. $\frac{4}{9}$ of 25 bu. 2 pk. 3 qt.?

57. If a row contains 6 squares, each 1 in. long and 1 in. wide, how many square inches are in 5 such rows?

58. How many sq. in. in a pane of glass 8 in. long, 6 in. wide?

59. How many square feet of carpet are needed to cover a hall 10 ft. 6 in. long, 4 ft. wide? How many sq. yd.?

60. At 3 cents a square foot, or foot board measure, what is the cost of a board 15 ft. long, 6 in. wide?

61. The surface of a table 6 feet long contains 30 square feet. What is the width of the table?

62. What is the length of a field 20 rd. wide, and containing 600 sq. rd.? Area is 480 sq. rd., width 15 rd.?

63. How long is a floor, covered by 24 yards of yard-wide carpet, the width being 12 feet?

Find the area of	Find the required dimension,
64. Length 5 yd. by 4 yd. 6 in.	*67.* Area 42 sq. ft., width 5 ft. 3 in.
65. Length $7\frac{1}{2}$ ft. by 3 ft. 6 in.	*68.* Area 63 sq. yd., width $6\frac{3}{4}$ ft.
66. Length 40 rd. by 5 rd. 3 yd.	*69.* Area 24 A., breadth 10 ch.

70. What is the solidity of a block of marble 8 feet long, 3 feet wide, and 2 feet thick?

71. What is the solidity of a piece of marble 10 ft. long, 4 ft. wide, 3 ft. thick? 11 ft. 3 in. by 6 ft. by 4 ft.?

72. Find the cubical contents of a cistern 12 ft. 3 in. long, 6 ft. deep, 4 ft. wide. 15 ft. 6 in. long, 8 ft. wide, 6 ft. deep.

73. The solidity of a pile of wood is 128 cu. ft., the length is 8 ft., and the width is 4 ft. How high is it?

74. A pile of stones contains 64 cu. ft., the length being 8 ft., and the height 2 ft. How wide is it?

75. How long is a bin whose capacity is 135 cu. ft., the depth being 4 ft. 6 in., and width 3 ft.?

Find the solidity of a solid whose dimensions are	Find the required dimension of a solid whose content is
76. 8 ft. 6 in. by 6 ft. by 4 ft.	*79.* 216 cu. ft., 8 ft. long, 6 ft. wide.
77. 10 yd. 2 ft. by 8 ft. by 3 ft.	*80.* 339 cu. ft., 6 ft. wide, 6 ft. deep
78. 12 ft. 9 in. by 9 ft. by 2 ft.	*81.* 540 cu. ft., 9 ft. long, 8 ft. deep

CHAPTER V.

PERCENTAGE.

Notation and Numeration.

1. What is $\frac{1}{100}$ of \$100? $\frac{2}{100}$? $\frac{15}{100}$? .25? .50? .75?

2. How much is $\frac{3}{100}$ of \$100? $\frac{4}{100}$ of \$50? .08 of 25?

3. What part of 100 is 1? Of \$100 is \$1? Of \$100 are \$7?

4. 5 ft. are what part of 50 ft.? How many hundredths?

5. How many hundredths of \$100 are \$6? \$7? \$8½?

6. 5 ft. are how many hundredths of 100 ft.? Of 200 ft.?

7. How many hundredths of any number equal the number?

8. How many hundredths of any number equal ½ of it?

9. What part of 100 hundredths are 50 hundredths?

10. What part of a number are 50 hundredths of it? .07 of it?

Definitions.

307. Per Cent. is an abbreviation of the Latin *per centum*, and means *by the hundred.*

Thus, *6 per cent.* means *6 of every hundred;* 12½ per cent. is 12½ hundredths, or 12½ of every 100; 25 per cent. is .25, or 25 of 100; etc.

308. Percentage is the process of computing by the hundred.

309. The ***Sign of per cent.*** is %. It is read *per cent.*

310. Since per cent. is a number of hundredths, it is usually expressed in the form of a decimal; but it may be expressed either as a decimal or a common fraction. Thus,

7% is .07, or $\frac{7}{100}$.	$7\frac{1}{2}$% is .075, $\frac{7\frac{1}{2}}{100}$, or $\frac{75}{1000}$.
100% is 1.00, $\frac{100}{100}$, or 1.	$12\frac{1}{2}$% is .125, $\frac{12\frac{1}{2}}{100}$, or $\frac{1}{8}$.
125% is 1.25, $\frac{125}{100}$, or $1\frac{1}{4}$.	$\frac{3}{4}$% is .0075, $\frac{\frac{3}{4}}{100}$, or $\frac{3}{10000}$.

Exercises.

1. Express or write in the form of decimals: —

3%; 10%; 25%; 75%; 100%; 125%; $6\frac{1}{4}\%$; $8\frac{1}{3}\%$; $12\frac{1}{2}\%$; $66\frac{2}{3}\%$; $100\frac{1}{2}\%$; $118\frac{3}{4}\%$; $\frac{1}{2}\%$; $\frac{3}{4}\%$; $\frac{5}{8}\%$; .5%; .25%; $.7\frac{1}{2}\%$; $.06\frac{1}{4}\%$.

2. Express or write in the form of decimals and of fractions: —

5%; 15%; 50%; 100%; 25%; $2\frac{1}{2}\%$; $56\frac{1}{4}\%$; $87\frac{1}{2}\%$; $100\frac{7}{8}\%$; $112\frac{1}{2}\%$; $133\frac{1}{3}\%$; $\frac{1}{4}\%$; $\frac{3}{8}\%$; $\frac{1}{4}\%$; $\frac{3}{4}\%$; .5%; .75%; $.07\frac{1}{2}\%$; $.16\frac{2}{3}\%$.

3. Read and express as rate with the sign %: —

.05; .15; .25; 1; $.12\frac{1}{2}$; $.16\frac{2}{3}$; $.56\frac{1}{4}$; 1.25; $1.12\frac{1}{2}$; $.01\frac{1}{2}$; $.06\frac{1}{4}$; $1.04\frac{1}{8}$; $1.37\frac{1}{2}$; $.00\frac{1}{2}$; $.00\frac{3}{8}$; $1.00\frac{3}{4}$; $1.00\frac{7}{8}$; 2.50; .005; $.002\frac{1}{2}$; 2.

4. What per cent. of a number is $\frac{1}{4}$ of it?

311. Model. — Since the whole of any number is 100%, or $\frac{100}{100}$ of itself, $\frac{1}{4}$ of the number is $\frac{1}{4}$ of 100%, or 25%.

What % of a number is $\frac{1}{4}$ of it? $\frac{1}{5}$? $\frac{1}{8}$? $\frac{7}{8}$? $\frac{1}{10}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{3}{8}$? $\frac{7}{12}$? $\frac{3}{10}$? $\frac{4}{15}$? $\frac{7}{25}$? .05? $.06\frac{1}{4}$? $.12\frac{1}{2}$? .25? 2 times the number?

5. What part of a number is 5% of it?

312. Model.—Since 5% is $\frac{5}{100}$, 5% of a number is $\frac{5}{100}$, or $\frac{1}{20}$, of it.

What part of a number is 6% of it? $12\frac{1}{2}\%$? 25%? $37\frac{1}{2}\%$? 50%? $60\frac{2}{3}\%$? 100%? 250%? $\frac{1}{2}\%$? $\frac{7}{8}\%$? .5%? .25%? $.16\frac{2}{3}\%$? 200%?

GENERAL CASES OF PERCENTAGE.

313. In the operations and applications of percentage, at least three terms or elements are considered: —

The *Base*, the *Rate*, and the *Percentage*.

314. The ***Base*** is the number of which the per cent., or number of hundredths, is taken.

Thus, in the expression 5% of $125, the *base* is *$125*.

315. The ***Rate*** is the number of hundredths taken.

Thus, in the expression 5% of 75 yards, the *rate* is *5 hundredths*.

316. The ***Percentage*** is the result obtained by taking *any per cent. of* the base.

Thus, in the expression 5% of 40 yards is 2 yards, the *base* is *40 yards*, the *rate %* is *.05*, and the *percentage* is *2 yards*.

317. The ***Amount*** is the sum of the base plus the percentage.

318. The ***Difference*** is the remainder of the base less the percentage.

319. Principles.

I. *The percentage of any number is such part of that number as the given rate is of 100%.*

II. *The rate is such part of 100% as the percentage is of the base.*

III. *The base is as many times the percentage as 100% is times the given rate.*

Exercises.

1. Express in decimal form

The amount of 1 at 3%; 12%; 25%; 85%; 100%; 150%; $6\frac{1}{4}$%; $12\frac{1}{2}$%; $33\frac{1}{3}$%; $137\frac{1}{2}$%; $1\frac{1}{2}$%; $\frac{1}{4}$%; $\frac{3}{4}$%; $\frac{7}{8}$%; $100\frac{3}{8}$%; 1.5%; 100.5%.

2. Express in both fractional and decimal form

The difference of 1 at 5%; 15%; 52%; 90%; $8\frac{1}{3}$%; $37\frac{1}{2}$%; $30\frac{3}{8}$%; $66\frac{2}{3}$%; $87\frac{1}{2}$%; $\frac{1}{8}$%; $\frac{1}{4}$%; $\frac{2}{5}$%; $\frac{3}{8}$%; $\frac{7}{10}$%; .5%; .25%; .75%.

3. Express in both forms

The amount and the difference of 1 at 6%; 13%; 36%; 80%; 100%; $1\frac{1}{2}$%; $3\frac{1}{4}$%; $5\frac{2}{5}$%; $7\frac{1}{8}$%; $18\frac{3}{4}$%; $30\frac{3}{10}$%; $87\frac{1}{2}$%; $\frac{1}{8}$%; $\frac{1}{4}$%; $\frac{1}{5}$%; .5% .25%.

1. To Find the Percentage, and the Amount or the Difference.

1. What is 5% of 80 feet?

320. ***Analysis.*** — Since 5% is $\frac{5}{100}$, 5% of 80 feet is $\frac{5}{100}$, or $\frac{1}{20}$ of 80 feet, which is 4 feet. Hence, etc.

2. What is 6% of 50? Of 200? Of $250? Of 300 A.?

3. Find 8% of 60; $12\frac{1}{2}$% of 80 acres; $16\frac{2}{3}$% of 120 men.

How much is

4. 5% of \$40? 6%?
5. 7% of 50 bu.? 9%?
6. 9% of 30 yd.? 11%?
7. 12% of 75 mi.? 20%?
8. $6\frac{1}{4}$% of \$$\frac{3}{4}$? $12\frac{1}{2}$%? $\frac{1}{2}$%?
9. $8\frac{1}{3}$% of $\frac{2}{3}$ lb.? $33\frac{1}{3}$%? $\frac{1}{4}$%?
10. $16\frac{2}{3}$% of $\frac{3}{8}$ bu.? 50%? $\frac{1}{8}$%?
11. $66\frac{2}{3}$% of .75 yr.? $37\frac{1}{2}$%? $\frac{2}{3}$%?

12. A farmer raised 150 bushels of corn, and kept 20% of it to feed his horses. How many bushels did he keep?

13. A merchant sold 50% of a piece of muslin containing $33\frac{1}{3}$ yards. How many yards did he sell?

14. Henry had \$75, and expended $66\frac{2}{3}$% of it for a watch. How much did he pay for the watch?

15. From a hogshead of molasses $16\frac{2}{3}$% was drawn off. How many gallons were drawn off?

16. What is the amount of 50 ft. increased by 6% of itself?

321. Analysis. — Since 6% is $\frac{6}{100}$, 6% of 50 ft. is $\frac{6}{100}$, or $\frac{3}{50}$ of 50 ft., or 3 ft.; and 50 ft. plus 3 ft. are 53 ft. Hence, etc.

17. How many men are 200 men increased by 5% of that number? By 6%? By 10%? 50%? 100%?

18. How many feet are 400 ft. diminished by 20% of itself? By 25%? By 50%? 75%? 100%?

Find the amount of

19. 15 A. at 10%; 25%.
20. 25 bu. at 8%; $12\frac{1}{2}$%.
21. 50 mi. at 7%; $18\frac{3}{4}$%.
22. 48 doz. at 5%; $37\frac{1}{2}$%.

Find the difference of

23. $\frac{3}{8}$ hhd. at 5%; $8\frac{1}{3}$%.
24. $\frac{2}{3}$ bu. at 25%; $87\frac{1}{2}$%; $\frac{1}{2}$%.
25. .5 cwt. at 10%; $1\frac{1}{9}$%; $\frac{1}{4}$%.
26. $\frac{5}{12}$ doz. at $\frac{1}{5}$%; $6\frac{1}{4}$%; $\frac{4}{5}$%.

27. A drover had 48 head of cattle, and sold 25% of them. How many cattle had he left?

28. If the rent of a house, for which a mechanic pays \$12 per month, be raised 20%, what must he then pay?

29. If I borrow \$150, and pay 6% for the use of it a year, *how much* do I owe at the end of the year?

30. A horse dealer bought a horse for $225, and sold him at a loss of 20%. Find the selling price.

31. If calico is bought at 8 cents a yard, and is sold at a profit of $12\frac{1}{2}$%, for how much is it sold?

32. From a farm of 80 acres were sold $12\frac{1}{2}$% to one man, and 20% of the remainder to another. How many acres were then in the farm?

2. *To Find the Rate when the Base and the Percentage are given.*

1. What per cent. of $75 are $25?

322. ***Analysis.***—Since $75 are 100%, $25, which are $\frac{1}{3}$ of $75, are $\frac{1}{3}$ of 100%, or $33\frac{1}{3}$% of $75. Hence, etc.

2. What part of 48 is 8? What per cent. of 48 is 8?

3. What part of $50 are $12\frac{1}{2}$? What % of $50 are $12\frac{1}{2}$?

What per cent. of

4. 30 lb. are 15 lb.? Are 20 lb.?

5. 40 oz. are $13\frac{1}{3}$ oz.? Are 25 oz.?

6. 50 A. are $6\frac{1}{4}$ A.? Are $37\frac{1}{2}$ A.?

7. $.75 are $.18\frac{3}{4}$? Are $.62\frac{1}{2}$?

8. $1\frac{7}{8}$ are $\frac{3}{4}$? Are $\frac{3}{16}$?

9. $\frac{3}{4}$ bu. is $\frac{1}{2}$ bu.? Are $\frac{3}{8}$ bu.?

10. .5 lb. is .1 lb.? Is $\frac{1}{5}$ lb.?

11. $16\frac{2}{3}$ yr. are $4\frac{1}{6}$ yr.? $\frac{5}{6}$ yr.?

12. What per cent. of a number is $\frac{1}{4}$ of it? $\frac{1}{8}$ of it? $\frac{3}{8}$ of it? $\frac{7}{10}$ of it? .05 of it? $.12\frac{1}{2}$ of it? $.66\frac{2}{3}$ of it?

13. If I pay $6 for the use of $100 for a year, what per cent. do I pay? If I pay $7? If $8? $7\frac{1}{2}$?

14. If a man pays $5 an acre for the use of land worth $75 an acre, what per cent. of the value does he pay?

15. A farmer had 120 sheep, and sold all but 25% of them. How many did he sell, and what % did he keep?

16. A flour dealer sold some damaged wheat so as to lose $\frac{1}{8}$ of what it cost him. What % of the cost did he receive?

17. If I buy land at $50 per acre, and sell it at $62\frac{1}{2}$, what % above cost do I receive for it?

18. A boy having \$.75, spent \$.12½ for a ball. What % of his money had he remaining?

19. If a milkman adds a gill of water to every quart of milk that he serves, what % does he add?

20. An agent asked \$5 for a book, but sold it for \$4.59. What % of his asking price did he receive?

3. To Find the Base when the Rate and the Percentage are given.

1. 8 feet are 25% of how many feet?

323. Analysis. — Since 25%, or $\frac{25}{100}$, of some number is 8 ft., 1%, or $\frac{1}{100}$, of the number is $\frac{1}{25}$ of 8 ft., or $\frac{8}{25}$ ft.; and 100%, or $\frac{100}{100}$, of the number is 100 times $\frac{8}{25}$, which are $\frac{800}{25}$ ft., or 32 ft. Hence, etc.

2. 50 is $\frac{25}{100}$ of what number? 50 is 25% of what?

3. \$4½ are $\frac{6}{100}$ of what number? \$4½ are 6% of what?

Of what are

4. 12 mi. 6%? 12½%? 25%?

5. 25 yd. 8%? 6¼%? 16⅔%?

6. 37½ ft. 10%? 8⅓%? 12.5%?

7. 6¼ in. 2%? ½%? ⅛%? ⅝%?

8. 2¼ bu. 12½%? 1%? ½%?

9. 4½ pk. 100%? 1%? ¾%?

10. 5 qt. 125%? ⅜%? 1.25%?

11. 1.5 pt. 200%? 1½%? ⅝%?

12. Find the base, if the rate is 6¼% and the percentage \$½.

13. If \$10 are ½% of my bank deposit, what is the amount of my deposit? If \$20 are ⅓% of it?

14. If A pays \$12 a year for the use of a sum of money, at the rate of 6%, how much money does he use?

15. A man loaned another a sum of money. What was the sum loaned, if \$75 were 16⅔% of the amount?

16. If a man owns 25% of a mill, and sells 33⅓% of his share for \$1000, what is the value of the mill?

17. A drover bought 25 cattle, and this number was 62½% *of what he had* at first? How many had he after buying?

18. From $\frac{4}{5}$ of my money I spent $12\frac{1}{2}\%$ of my money, and then had \$27 left. How much had I at first?

19. Harry is 16 years old, and $37\frac{1}{2}\%$ of his age is $66\frac{2}{3}\%$ of Frank's age. How old is Frank?

20. A farmer sold 20% of his wheat to one miller, and $37\frac{1}{2}\%$ of the remainder to another, and had 70 bu. left. How many bushels had he raised?

4. To Find the Base when the Amount or the Difference and the Rate are given.

1. What number increased by 25% of itself amounts to 75?

324. Analysis. — Since the number is 100% of itself, 75 is 100% plus 25%, or 125%, of the number. If 75 is 125%, or $\frac{125}{100}$, of some number, 1%, or $\frac{1}{100}$ of the number is, etc. (323).

2. 150 is $\frac{120}{100}$ of what number? 150 is 120% of what?

3. \$75 are 125% of what number? 20% more than what?

4. What number increased by $6\frac{1}{4}\%$ is 85 ft.? Is \$34?

5. What number diminished by 20% of itself equals 60?

325. Analysis. — Since the number is 100% of itself, 60 is 100% less 20%, or 80%, of the number. If 60 is 80%, or $\frac{80}{100}$, of some number, 1%, or $\frac{1}{100}$ of the number is, etc.

6. 150 is $\frac{75}{100}$ of what number? 150 is 75% of what?

7. \$75 is 60% of what number? 20% less than what?

What number increased by	What number diminished by
8. 10% of itself is 55?	*12.* 20% of itself is 64?
9. $12\frac{1}{2}\%$ is \$72? Is $\$\frac{9}{16}$?	*13.* $37\frac{1}{2}\%$ is 5 bu.? $\frac{3}{4}$ bu.? 1.5?
10. $8\frac{1}{3}\%$ is 39 ft.? Is $2\frac{1}{2}$ yd.?	*14.* $6\frac{1}{4}\%$ is 45 qt.? $\frac{3}{4}$ pt.? 4.5 pt.?
11. $\frac{1}{2}\%$ is $\$50\frac{1}{4}$? Is $\frac{201}{240}$ ft.?	*15.* $\frac{1}{8}\%$ is $\$79\frac{9}{10}$? \$99$\frac{7}{8}$? $1.59\frac{4}{5}$?

16. 42 miles are $12\frac{1}{2}\%$ less than how many miles? $16\frac{2}{3}\%$?

17. If the amount is \$200, and the rate is 25%, what is the base? If the difference is \$200, and the rate 25%?

18. If a horse cost \$120, and this sum is 20% less than the cost of a carriage, what was the cost of the carriage?

19. A grocer sells tea at \$.98 a pound, and this sum is $16\frac{2}{3}\%$ more than cost. Find the cost.

20. After selling 75% of a piece of cloth, a tailor had remaining $7\frac{1}{2}$ yd. How much was in the piece at first?

21. A clerk expended 75% of his salary for his support, and saved \$20 a month. What was his salary?

22. If a grocer sells coffee at $31\frac{1}{2}$ cents, at an advance of 25% over the cost, what was the cost?

23. A man on a journey traveled $37\frac{1}{4}\%$ of the whole distance by stage, and the remaining 120 miles by railroad. How far did he travel?

Section II.

APPLICATIONS OF PERCENTAGE.

1. Profit and Loss.

326. ***Profit*** and ***Loss*** are terms used to denote the gain and the loss in business transactions.

327. The corresponding terms of Profit and Loss and Percentage are —

1. The ***Cost*** or ***Sum Invested*** is the *Base.*

2. The ***Rate*** of ***Profit*** or ***Loss*** is the *Rate.*

3. The ***Profit*** or ***Loss*** is the *Percentage.*

4. The ***Selling Price,*** or cost plus the profit, is the *Amount.*

The ***Selling Price,*** or cost minus the loss, is the *Difference.*

1. At a gain of $12\frac{1}{2}\%$ on the cost of an article, what part of the cost equals the gain?

328. ***Analysis.*** — Since the gain is $12\frac{1}{2}\%$, it is $\frac{12\frac{1}{2}}{100}$, or $\frac{1}{8}$ of the cost. Hence, etc.

2. At a gain or loss of 5%, what part of the gain or loss equals the cost? At 10%? 25%? $6\frac{1}{4}\%$? $8\frac{1}{3}\%$? $37\frac{1}{2}\%$? $66\frac{2}{3}\%$?

3. At a gain or loss of $12\frac{1}{2}\%$ on an investment, what part of the money invested equals the gain or loss? $16\frac{2}{3}\%$?

4. A dealer bought a barrel of flour for $\$6\frac{1}{4}$, and sold it at a gain of 20%. Find the gain. The selling price.

329. ***Analysis.*** — Since the gain was 20%, it was $\frac{20}{100}$, or $\frac{1}{5}$ of the cost; and $\frac{1}{5}$ of $\$6\frac{1}{4}$ is $\$1\frac{1}{4}$. The selling price was the sum of $\$6\frac{1}{4}$ and $\$1\frac{1}{4}$, or $\$7\frac{1}{2}$. Hence, etc.

Find the profit or loss, and selling price of

5. Coffee bought at \$.25, and sold at a gain of 25%.

6. Coffee bought at $\$.31\frac{1}{4}$, and sold at a loss of 20%.

7. Drugs that cost \$50, and were sold at a gain of 125%.

8. An investment of \$2000 on which there was a loss of $16\frac{2}{3}\%$; on which there was a gain of $6\frac{1}{4}\%$.

9. A ton of hay that cost $\$1\frac{1}{5}$ per cwt., at a gain of 10%.

10. How must calico that costs 10 cents a yard be sold to give a profit of 8%? 1%? $12\frac{1}{2}\%$? $6\frac{1}{4}\%$? $8\frac{1}{3}\%$? 25%?

11. A real-estate agent bought a city lot for \$200, and sold it at an advance of 250%. What was the selling price?

330. If a grocer buys sugar at $12\frac{1}{2}$ cents a pound, and sells it at 10 cents, what $\%$ loss does he suffer?

Analysis. — Since he pays $12\frac{1}{2}$ cents for the sugar, and sells it at 10 cents, he loses the difference between $12\frac{1}{2}$ cents and 10 cents, which is $2\frac{1}{2}$ cents.

Since on $12\frac{1}{2}$ cents the loss is $2\frac{1}{2}$ cents, or $\frac{1}{5}$ of the cost, the loss $\%$ is $\frac{1}{5}$ of 100%, or 20%. Hence, etc.

Find the rate of profit or loss on

12. Tea bought at \$.75 per pound, and sold at \$.87½.

13. Tea bought at \$.87½ per pound, and sold at \$.75.

14. An investment of \$2500 that realized but \$2000.

15. Paper bought at \$3.60 a ream, sold at \$.20 a quire.

16. 8 bbl. of apples at \$3.50 a bbl., that were sold for \$21.

17. What % does a grocer lose by buying crushed sugar at 15 cents a pound and selling it at 12 cents a pound?

18. What % is gained or lost by selling an article for ¾ of its cost? For ⅘ of its cost? Double its cost? ½ its cost?

331. A merchant gained 20% by selling velvet at a profit of \$2 a yard. What was the cost?

Analysis. — Since the gain was 20%, then $\frac{20}{100}$, or ⅕, of the cost equals the gain, which is \$2. If \$2 is ⅕ of the cost, etc.

Find the cost of

19. Sugar sold at 1 cent above cost at 8⅓% gain.

20. A sewing-machine at 12½% loss, amounting to \$8.

21. Lumber sold at a profit of \$3 per M, or 16⅔% gain.

22. A book sold so as to gain \$.25, or 12½% of the cost.

23. 25 lb. raisins sold at a loss of 10%, or 2½ ct. per lb.

24. A broker invested in stocks, and lost \$1000, this sum being 6¼% of his capital. What sum had he invested?

25. If 6¼% of the cost of muslin is 2 cents, what is the cost? If 8⅓% is 2 cents? If 12½% is 1½ cents? If 2½% is ½ cent? If ½% is .1 cent? If 1¼% is ¼ of a cent?

332. By selling a watch for \$87½, a jeweler cleared 16⅔%. What was the cost of the watch?

Analysis. — Since the gain was 16⅔%, then $\frac{16\frac{2}{3}}{100}$, or ⅙ of the cost, equals the gain, which, added to ⅚ of the cost, is ⅞ of the cost, or \$87½. If \$87½ is ⅞ of the cost, ⅛ of the cost is, etc.

Find the cost of

26. *Corn* sold at \$.62½ at a gain of 11⅑%; 25%; 12½%.

27. A farm-wagon sold for $112 at a loss of $16\frac{2}{3}\%$.

28. A ton of iron sold for $41\frac{1}{4}$ at a gain of 10%.

29. Fruit sold for $20 at a gain of 20%; at a loss of $12\frac{1}{2}\%$.

30. A gold bracelet sold for $9 at a gain of $12\frac{1}{2}\%$; at a loss of $33\frac{1}{3}\%$; at a gain of 8%.

31. A dealer paid $12 for 16 bu. of corn. For what must it be sold per bushel to realize $16\frac{2}{3}\%$ profit? $6\frac{2}{3}\%$?

32. A produce dealer bought 20 bbl. of apples for $60, and sold them at $3\frac{3}{8}$ per bbl. What % did he clear?

33. An agent sold a map for $14, and thereby cleared $16\frac{2}{3}\%$. What % would he have gained or lost by selling it for $8?

34. What % is gained by buying an article for $\frac{7}{8}$ of the market price, and selling it at $12\frac{1}{2}\%$ above market price?

35. How many yards of cloth at $3 a yard must a merchant buy, that by selling it at a profit of $16\frac{2}{3}\%$ he may gain $6?

36. A drover sold two horses at $200 each, one at a gain of 20%, and the other at a loss of 20%. Find his entire gain or loss %.

2. *Commission.*

333. An ***Agent, Factor,*** or ***Commission Merchant,*** is a person who buys or sells merchandise, or transacts other business for another.

334. ***Commission*** is the sum or percentage allowed an agent or commission merchant for transacting business.

335. A ***Broker*** is a person who buys and sells gold, stocks, bonds, etc., for a commission called *Brokerage.*

336. The corresponding terms of Commission or Brokerage and Percentage are —

1. The ***Amount of Sales*** or ***Sum Collected*** is the *Base.*

2. The ***Rate of Commission*** or ***Brokerage*** is the *Rate.*

3. The ***Commission*** or ***Brokerage*** is the *Percentage.*

4. The ***Amount of Sales*** or the ***Sum Collected,*** plus the commission, is the *Amount.*

The ***Amount of Sales*** or the ***Sum Collected,*** minus the commission, is the *Difference,* or *Net Proceeds.*

Note. — The ***sum remitted*** is the *amount,* and includes the sum to be invested plus the commission.

1. If a horse dealer sells a horse for $200, and receives 5% of the value as his commission, what is the amount of his commission?

337. Analysis. — Since the rate of commission is 5%, the amount of his commission is 5%, or $\frac{5}{100}$, of $200, etc.

Find the commission or brokerage on

2. The sale of flour for $250 at 3%; at $2\frac{1}{2}$%; at $3\frac{1}{4}$%.

3. The purchase of $300 worth of stock at $\frac{1}{8}$%; $\frac{1}{4}$%; $\frac{3}{8}$%.

4. The sale of 10 bbl. flour at $7\frac{1}{2}$ per bbl., at $2\frac{1}{2}$%; $1\frac{3}{4}$%.

5. A debt of $1500, collector's fees $3\frac{1}{4}$%; $4\frac{1}{2}$%; $5\frac{1}{8}$%.

6. A factor bought 100 lb. of tea at $.75, and charged his principal $2\frac{1}{2}$%. What did the tea cost the principal?

7. An attorney collected 75% of a note for $400 at $6\frac{1}{4}$%. What was his commission? What did he pay over?

338. An auctioneer received $15 for selling a set of furniture valued at $300. What was his rate of commission?

Analysis. — Since he received $15 for selling furniture valued at $300, the rate of commission was $\frac{15}{300}$ of the amount of sales, or $\frac{1}{20}$ of 100%, which is 5%. Hence, etc.

Find the rate of commission or brokerage

8. If $20 are paid for collecting bills at 4%; at 5%; $2\frac{1}{2}$%.

9. If a broker charges $1.25 for buying stock with $1000.

10. If $12\frac{1}{2}$ are paid for buying 200 lb. of wool at $.25.

11. If I am charged \$1.25 for 10 shares R.R. stock at \$90.

12. An agent collected \$500, retained his commission, and paid over \$425. What was the rate of commission?

13. A produce merchant sold 20 bbl. of apples at \$3½ a bbl. If his commission was \$1¾, what was the rate % ?

339. I paid a collector \$7½ for collecting a debt at the rate of 3¾%. What was the amount collected?

Analysis. — Since the rate of commission is 3¾%, \$7½ are $\frac{3\frac{3}{4}}{100}$, or $\frac{1\frac{5}{0}}{4\,0\,0}$, of the amount collected. $\frac{1}{4\,0\,0}$ of the amount is $\frac{1}{1\,5}$ of \$7½, etc.

340. A tax-receiver charged 5% commission, and paid over \$665. What amount had he collected?

Analysis. — Since the amount paid over, or the net proceeds, is the sum collected, 100%, less the commission, 5%, \$712½ are 95%, or $\frac{9\,5}{1\,0\,0}$, of the sum collected; and 1%, or $\frac{1}{1\,0\,0}$, etc.

Find the amount of sales or sum collected

14. If \$10 are paid as commission at 2½%; at 3⅓%.

15. If \$6¼ are charged as brokerage at ⅛%; at ¼%.

16. If the commission is 2½ %, and the sum paid over \$390.

17. If the net proceeds is \$1975, and the rate 1¼%.

18. At 2½%, a wool buyer received \$11 for buying some wool at \$.44 a pound. How many pounds did he buy?

19. An agent received 20% for selling washing-machines at \$5, and paid over one week \$300. How many had he sold?

341. An agent received \$630 with which to buy goods after retaining 5% of the cost. How much did he expend for goods?

Analysis. — Since he received 100% of the sum expended plus 5% of the sum for his fee, he received 105% of the sum expended.

If 105% of the sum expended is \$630, 1% of the sum is, etc.

Find the sum or amount invested

20. If \$206 are remitted, 3% com. to be deducted.

21. If \$410 are received, 2½% com. to be retained.

22. If \$1604 are received, ¼% brokerage to be retained.

23. An agent receives $618 with which to buy books after deducting 3% of the cost. How much can he lay out in books?

24. How many pounds of coffee at $.25 per pound can be bought for $205 after retaining $2\frac{1}{2}\%$ commission?

25. A broker received $1001.25 to invest in stock at $50 a share. How many shares did he buy after retaining $\frac{1}{8}\%$ brokerage? If he had received $500.62\frac{1}{2}$?

26. What is an agent's commission, if he receives $624 to invest in hops at $.20 a lb., after deducting 4% commission?

27. What amount of accounts must an agent collect in order to pay over $900, and retain $6\frac{1}{4}\%$ commission?

28. After deducting $2\frac{1}{2}\%$ com. and $7.50 for freight and expenses, a commission merchant remitted to his principal $675. What was the amount of sales?

3. *Interest.*

342. ***Interest*** is the sum paid for the use of money.

343. The ***Principal*** is the sum for the use of which interest is paid.

344. The ***Rate of Interest*** is the rate per cent. of the principal paid for its use for one year.

345. The ***Amount*** is the sum of the principal and the interest.

346. The corresponding terms of Interest and Percentage are —

1. The ***Principal*** is the *Base.*

2. The ***Rate % per Annum*** is the *Rate.*

3. The ***Interest*** is the *Percentage.*

4. The ***Amount,*** or the sum of the principal and the *interest, is* the *Amount.*

1. At 5%, what decimal part of money borrowed equals the sum paid for its use? At 6%? At 7%? At 8%?

2. At 6% for 2 yr., what decimal and what fractional part of any principal equals the interest? At 7%? At 8%?

347. What is the interest of $50 for 1 year at 6%?

Analysis. — Since the interest of any principal at 6% for one year is $\frac{6}{100}$, or $\frac{3}{50}$, of the principal, the interest of $50 for 1 year at 6% is $\frac{3}{50}$ of $50, or $3.

Find the interest of

3. $50 for 1 year at 4%; at 5%; at 7%; 8%; 9%; 10%.

4. $60 for 1 year at $2\frac{1}{2}$%; at $3\frac{1}{3}$%; at $3\frac{1}{2}$%; $3\frac{1}{4}$%; $3\frac{1}{8}$%.

5. $80 for 1 year at $4\frac{1}{2}$%; at $4\frac{1}{4}$%; at $5\frac{1}{3}$%; $5\frac{3}{8}$%; $6\frac{1}{4}$%.

348. What is the interest of $50 for 2 years at 5%?

Analysis. — Since the interest of any principal at 5% for 1 year is $\frac{5}{100}$, or $\frac{1}{20}$, of the principal, the interest of $50 for 1 year at 5% is $\frac{1}{20}$ of $50, or $2\frac{1}{2}$; and for 2 years the interest is 2 times $2\frac{1}{2}$, or $5.

Find the interest of

6. $30 for 2 years at 3%; at 4%; at 5%; at 6%; 7%.

7. $40 for 3 years at 4%; at $4\frac{1}{2}$%; at 5%; $5\frac{1}{2}$%; 6%; 7%.

8. $50 for 4 years at 5%; at $6\frac{1}{2}$%; at 7%; at 8%: $8\frac{1}{2}$%.

349. What is the interest of $50 for 2 yr. 6 mo. at 8%?

Analysis. — Since 12 months are 1 year, 6 months are $\frac{6}{12}$, or $\frac{1}{2}$, of 1 year; and 2 yr. 6 mo. are $2\frac{1}{2}$ yr.

Since the interest of any principal at 8% for 1 year is $\frac{8}{100}$, or $\frac{2}{25}$, of the principal, the interest of $50 at 8% for 1 year is $\frac{2}{25}$ of $50, etc.

Find the interest of

9. $60 for 2 yr. 3 mo. at 5%; for 3 yr. 3 mo.; 4 yr. 9 mo.

10. $500 for 3 yr. 2 mo. at 7%; for 4 yr. 6 mo.; 5 yr. 8 mo.

11. $200 for 4 yr. 3 mo. at 8%; for 5 yr. 9 mo.; 6 yr. 10 mo.

350. What is the interest of $100 for 2 yr. 3 mo. 20 da. at 6%? What is the amount?

Analysis. — Since 30 da. are 1 mo., 20 da. are $\frac{2}{3}$ mo., which, added to 3 mo., gives $3\frac{2}{3}$ mo.; and since 12 mo. are 1 yr., $3\frac{2}{3}$ mo. are $\frac{11}{36}$ yr., etc.

Since the interest of any principal at 6% is $\frac{6}{100}$, or $\frac{3}{50}$, etc. And The amount of \$100 for 2 yr. 3 mo. 20 da. at 6% is the sum of \$100 and \$13$\frac{5}{8}$, or \$113.83$\frac{1}{3}$.

Find the interest and the amount of

12. \$100 for 18 da. at 6%; 24 da.; 30 da.; 60 da.; 90 da.

13. \$200 for 30 da. at 7%; 36 da.; 60 da.; 72 da.; 90 da.

14. \$300 for 45 da. at 5%; 54 da.; 66 da.; 90 da.; 120 da.

15. \$400 for 2 mo. 12 da. at 10%; 3 mo. 6 da.; 4 mo. 15 da.

16. \$500 for 5 mo. 10 da. at 6%; 5 mo. 20 da.; 7 mo. 6 da.

17. \$700 for 1 yr. 2 mo. 12 da. at 5%; 2 yr. 6 mo. 20 da.

18. \$1000 for 3 yr. 3 mo. 18 da. at 7%; 4 yr. 4 mo. 15 da.

19. A man borrowed \$150 for 1 yr. 8 mo. at 6%. How much did he owe at the end of that time?

20. What annual income is derived from the investment of \$5000 in stocks that pay 7$\frac{1}{2}$% interest?

21. How much interest has accrued on a mortgage for \$400, which has been overdue 1 yr. 6 mo. 15 da., at 6%?

22. A mechanic borrowed \$1000 to buy a house. He paid it, with interest at 7%, in 9 mo. 10 da. What amount did he pay?

23. At $\frac{3}{4}$% a mo., how much will a banker receive from the use of \$500 for 2 yr. 3 mo. 26 da.?

24. A owes a \$1000 mortgage due 2 yr. 4 mo. ago, with interest at 6%. What is due if he pays \$800 on account?

351. What sum of money will gain \$20 in 2 yr. at 5%?

Analysis. — Since the interest of \$1 for 2 yr. at 5% is \$.10, \$20 is the interest of as many dollars as the number of times \$.10 are contained in \$20, or \$200. Hence, etc.

352. What sum will amount to \$224 in 2 yr. at 6%?

Analysis. — Since the interest of \$1 for 6% for 2 yr. is \$.12, the amount of \$1 is \$1 plus \$.12, or \$1.12; and \$224 is the amount of as many dollars as the number of times \$1.12, etc.

What principal will

25. Gain \$25 in 4 yr. at 6$\frac{1}{4}$%? \$45 in 3 yr. at 7$\frac{1}{2}$%?

26. Gain \$56 in 1 yr. 9 mo. at 8%? In 3 yr. 6 mo.?

27. Gain \$42 in 2 yr. 4 mo. at 6%? In 4 yr. 8 mo.?

28. Amount to \$230 in 2 yr. 6 mo. at 6%? At 4%?

29. Amount to \$250 in 3 yr. 1 mo. 15 da. at 4%? 8%?

30. What principal gives an annual income of \$120 at 6%?

31. What sum of money at $\frac{1}{2}$% a month will give \$51 in 5 yr. 8 mo.? Will amount to \$369 in 3 yr. 10 mo.?

32. If the semi-annual interest on a mortgage is \$75, at $7\frac{1}{2}$%, what is the face of the mortgage?

33. What sum must a father invest for his son, at 6%, to give him a semi-annual income of \$200?

34. How much must I invest in bonds paying $7\frac{3}{10}$% to give \$73 due every 3 months? \$36.50 per month?

353. At what rate will \$80 gain \$20 interest in 5 years?

Analysis. — Since the interest of \$80 for 5 yr. at 1% is \$4, \$20 is the interest at as many per cent. as the number of times \$4 are contained in \$20, or 4%. Hence, etc.

At what rate per cent. will

35. \$40 gain \$6 in 3 yr.? \$12 in 3 yr.? \$16 in 4 yr.?

36. \$50 give \$11 in 2 yr. 9 mo.? \$13 in 3 yr. 3 mo.?

37. \$80 amount to \$104 in 5 yr.? \$110 in 3 yr. 9 mo.?

38. \$100 amount to \$121 in 3 yr. 6 mo.? To \$128? \$$124\frac{1}{2}$?

39. If I pay \$81 for the use of \$400 for 3 yr. 4 mo. 15 da., what rate of interest do I pay for it?

40. If a man loans \$300, and at the end of 3 yr. 3 mo. 10 da. receives \$359, what rate of interest does he receive?

41. At what rate of interest will \$20 gain \$20 in 1 yr.? In 10 yr.? In 20 yr.? In 12 yr.? In $16\frac{2}{3}$ yr.?

42. At what rate % will any principal double itself in $8\frac{1}{3}$ years? In 10 yr.? In 12 yr. 6 mo.? In 16 yr. 8 mo.?

43. What rate of interest does a man receive who invests \$5000, and receives from it an annual income of \$365?

354. In what time will \$50 gain \$8 at 6%?

Analysis. — Since the interest of \$50 at 6% for 1 yr. is \$3, \$8 must be the interest for as many years as the number of times \$3 are contained in \$8, or $2\frac{2}{3}$ yr., or 2 yr. 8 mo. Hence, etc.

In what time will

44. \$80 gain \$11 at 5%? Gain \$16 at 6%? \$20 at 8%?

45. \$100 gain \$21 at 6%? Gain \$38 at 8%? \$42 at 9%?

46. \$90 amount to \$108 at 6%? \$139 at 7%? \$119$\frac{1}{4}$ at 7$\frac{1}{2}$%?

47. \$150 amount to \$228 at 6$\frac{1}{2}$%? \$213 at $\frac{3}{4}$% a month?

48. If I paid \$87 for the use of \$300 at 6% a year, how long had I the use of the money?

49. A man borrowed \$400 at 7% a year, and paid it back when it amounted to \$477. How long had he used it?

50. In how many years will \$20 gain \$20 interest at the rate of 1% a year? At 5% a year? At 10% a year?

51. In what time will any principal double itself at 4%? At 6%? At 7%? At 8%? At 9%?

52. A father left to his son \$5000 to remain on interest at 6% until it should amount to \$7500. How soon did his son receive it?

4. Discount.

355. ***Discount*** is a percentage deducted from the price of goods, or from the face of a debt.

356. Discount is of two kinds: *Commercial Discount* and *True Discount.*

357. ***Commercial Discount*** is a percentage deducted from the price of goods without regard to time.

358. The corresponding terms of Commercial Discount and Percentage are —

1. The ***Selling Price,*** or the ***Face of an Obligation,*** *is the* ***Base.***

2. The ***Rate of Discount*** is the *Rate.*

3. The ***Commercial Discount*** is the *Percentage.*

4. The ***Net Price,*** or ***Net Proceeds,*** is the *Difference.*

1. The regular price of a book was \$2, but the bookseller allowed me a discount of 25%. What did it cost me?

2. If a bookseller buys books that retail at \$1.50 at 20% off, what does he pay for them? What % does he make?

3. The regular price of a sewing-machine is \$50. If I buy at $\frac{1}{5}$ off, and sell at the regular price, what % do I clear?

4. A stationer buys note-paper that sells wholesale at \$.20 per quire, at $\frac{1}{5}$ off, and retails it at \$.24. What % does he make?

5. If I buy muslin by the piece at 15 cents a yard, 20% off, and sell it at 16 cents, what % do I clear?

6. What is the cash cost of goods bought for \$2000 on 6 mo. credit, on which a discount of 5% off is allowed for cash?

7. A bill of goods was invoiced at \$500 on 30 days' time, less discount of 10% off, and 5% off for cash. Find the net price.

8. Find the net cash price of flour invoiced at \$7.50 a barrel on 30 days' time, or 5% off for cash.

9. I bought some books at wholesale price, which was $\frac{1}{4}$ off from list or retail prices, and was allowed a further discount of 5% for cash. Find the net cash price of books worth \$100 retail.

10. What per cent. of the retail price of books do I pay by getting them at $\frac{1}{4}$ off from retail or list price, and a further reduction of 5% for cash?

359. The ***Present Worth*** of a debt payable at a future date without interest is such a sum as being placed on interest will amount to the given debt when it becomes due.

Thus, *$100* is the *present worth* of *$106* due in a year at 6%, since $100 on interest for 1 year at 6% amounts to $106.

360. True Discount is a sum deducted for the payment before it becomes due of a debt or note not drawing interest.

Time is *always an element* in True Discount.

Note. — The *true discount* is the *interest* on the *present worth* for the time between the date of discount and the date of the maturity of the note or debt.

361. The corresponding terms of True Discount and Interest are —

1. The ***Face of the Debt*** is the *Amount.*

2. The ***Present Worth,*** or ***Proceeds,*** is the *Principal.*

3. The ***True Discount*** is the *Interest* on the Present Worth.

11. If I put $1 on interest for 1 year at 6%, to how much will it amount at the end of the year?

12. If $1 on interest for a year at 6% amounts to $1.06, what is the present or cash value of $1.06 due a year hence?

13. What is the cash value or present worth of $106 due a year hence, when money is worth 6% a year?

14. If I take a note promising to pay $212 a year hence, what is it worth now? How much less than its face?

15. What sum of money on interest for 4 mo. at 6% will amount to $153? How much is the interest?

362. Find the present worth and the true discount of $153 payable in 4 months without interest.

Analysis. — Since the interest of $1 for 4 mo. at 6% is $.02, the amount of $1 is $1 plus $.02, or $1.02. Now, since $1 is the present worth of $1.02, the present worth of $153 is as many dollars as the *number of times* $1.02 is contained in $153, or $150. Hence, etc.

Find the present worth and true discount of

16. \$81 due in 3 mo. at 5%. \$103 in 6 mo. at 6%.

17. \$153 due in 90 da. at 8%. \$202 in 60 da. at 6%.

18. \$212 due in 9 mo. at 8%. \$307 in 120 da. at 7%.

19. \$357 due in 4 mo. at $\frac{1}{2}$% a mo. \$436 in 1 yr. 6 mo.

20. What amount of cash is equivalent to \$243 due 1 yr. 4 mo. hence, when money is worth 6% a year?

21. What discount should be allowed me if I pay a debt of \$442 1 yr. 9 mo. before it is due, money being worth 6%?

22. A merchant offered to sell goods for \$608 on 60 da., or for \$590 cash. Which was the better offer, money at 8%?

23. A broker bought a note for \$530 due in 4 mo. at a discount of $1\frac{1}{2}$% a month. What did the holder lose?

24. A broker bought a debt of \$621 due in 3 mo. 15 da. at a discount of 12%. How much cash did he pay for it?

5. Insurance.

363. Property Insurance is indemnity for loss or damage of property by fire, by accident, by the sea, etc.

364. The ***Valuation*** is the amount for which property is insured.

365. The ***Premium*** is the sum paid for the insurance.

366. The corresponding terms of Property Insurance and Percentage are

1. The ***Valuation*** is the *Base.*

2. The ***Rate of Premium*** is the *Rate.*

3. The ***Premium*** is the *Percentage.*

367. What must be paid for insuring goods for \$400 at $1\frac{1}{2}$%?

Analysis. — Since the rate of premium is $1\frac{1}{2}$%, or $\frac{3}{200}$ of the valuation, the premium on \$400 is $\frac{3}{200}$ of \$400, or \$6. Hence, etc.

Find the premium for insuring

1. Some furniture for \$500 at 1%; $\frac{1}{2}$%; 2%; $2\frac{1}{4}$%.

2. A case of dry-goods for \$600 at $\frac{1}{4}$%; $\frac{1}{2}$%; $\frac{3}{4}$%; $2\frac{1}{4}$%.

3. A house worth \$6000 for $\frac{2}{3}$ of its value, at $2\frac{1}{2}$% premium.

4. What must be paid to effect an insurance of \$4000 on a stock of goods, at $1\frac{1}{2}$% premium? At $1\frac{3}{4}$%? At $2\frac{1}{2}$%?

5. At $\frac{1}{8}$ of 1% a month, what will be the cost of insuring goods worth \$3000, which remained in store 3 months?

368. If \$$7\frac{1}{2}$ are paid for insuring \$300, what is the rate %?

Analysis. — Since on \$300 the premium is \$$7\frac{1}{2}$, the rate of premium is $\frac{7\frac{1}{2}}{300}$, or $\frac{1}{40}$, of 100%, or $2\frac{1}{2}$%. Hence, etc.

Find the rate of premium

6. If an insurance of \$600 costs \$6. If it costs \$9.

7. If \$15 are paid for an insurance of \$600 on goods.

8. If an insurance of \$2000 costs \$5. If \$$2\frac{1}{2}$. If \$$7\frac{1}{2}$.

9. If the annual premium for insuring a factory worth \$5000 is \$$37\frac{1}{2}$, what is the rate of premium? If \$$62\frac{1}{2}$?

10. A premium of \$10 is paid for insuring a stock of goods worth \$6000, for $\frac{2}{3}$ of their value. Find the rate of premium.

369. At $2\frac{1}{2}$%, what sum can be insured for \$10?

Analysis. — Since the rate of premium is $2\frac{1}{2}$%, $\frac{2\frac{1}{2}}{100}$, or $\frac{1}{40}$, of the sum insured equals the premium, \$10. If \$10 is $\frac{1}{40}$, etc.

Find the sum insured

11. For \$8 at 1%. For \$9 at $2\frac{1}{4}$%. For \$$10\frac{1}{2}$ at $1\frac{3}{4}$%.

12. For \$12 at 2%. For \$15 at $2\frac{1}{2}$%. For \$$18\frac{3}{4}$ at $1\frac{7}{8}$%.

13. For \$18 at $\frac{1}{2}$%. For \$20 at $\frac{1}{4}$%. For \$21 at $\frac{7}{8}$%.

14. If the annual premium for insuring a house at $\frac{5}{8}$% is \$20, what amount of insurance is covered?

15. If it costs \$$22\frac{1}{2}$ at $\frac{3}{8}$% to insure a cargo of grain from *Chicago to Buffalo*, what is the amount insured?

6. *Stocks.*

370. ***Capital Stock*** is the property or funds invested in the business of a company or corporation.

371. The ***Par Value*** of stock is the original value, or value mentioned in the certificate.

The par value of shares of stock is usually $100 or $50.

372. The ***Market Value*** of stock is the sum for which it can be sold.

Stock is *at par* when it sells for its original or face value, or 100%; it is *above par*, or *at a premium*, when it sells for more than its face value, or is above 100%; it is *below par*, or *at a discount*, when it sells for less than its face value, or is below 100%.

373. ***Stock Quotations*** are published statements of the market value of stock.

Thus, if stock is at par, it is quoted at *100;* if at 5% above par, it is quoted at *105;* and if 5% below par, it is quoted at *95*.

374. The principal bonds or securities issued by the United States are the following: —

The 6's of '81, the 5-20's, and the 10-40's.

The 6's of '81 and the 5-20's bear interest at the rate of 6%, payable in gold; the 10-40's, at the rate of 5%, in gold.

375. ***Stocks*** is a general term applied to corporation stock, and to various Government securities, State bonds, etc.

376. In all ordinary transactions, the corresponding terms of Stocks and Percentage are —

1. The ***Par Value*** is the *Base.*

2. The ***Rate of Premium*** or ***Discount*** is the *Rate.*

3. The ***Premium*** or ***Discount*** is the *Percentage.*

4. The ***Market Value*** is the *Amount* or the *Difference.*

377. What is the market value of 5 shares of bank stock quoted at 104? What is the cost, brokerage being ¼%?

Analysis. — Since at 104% the market value of one share is $\frac{104}{100}$ of \$100, or \$104, and the market value of 5 shares is 5 times \$104, or \$520; and since at $\frac{1}{8}$% the brokerage is $\frac{1}{800}$ of \$520, or \$.65, the entire cost is, etc. Hence, etc.

Find the market value or entire cost of

1. 8 shares of R. R. stock at par, brokerage $\frac{1}{8}$%; at 90.

2. 10 shares Penna. 6's, quoted at 104, brokerage $\frac{1}{8}$%.

3. Four \$1000 U. S. 5-20's at 120, brokerage $\frac{1}{8}$%; at $118\frac{1}{2}$.

4. A broker bought 20 shares of bank stock at $2\frac{1}{2}$% discount, and sold them at $101\frac{1}{2}$. What did he gain?

5. An insurance company declared a dividend of $4\frac{1}{2}$%. How much did a stockholder owning 10 shares receive?

6. When gold is quoted at $12\frac{1}{2}$% premium, what is the currency value of \$500 in gold? When quoted at $113\frac{1}{4}$?

378. If 4 shares of stock cost \$381, including brokerage at $\frac{1}{4}$%, at what rate does it sell?

Analysis. — Since the par value of 4 shares is 4 times \$100, or \$400, at $\frac{1}{4}$% the brokerage is $\frac{1}{400}$ of \$400, or \$1. \$381 less \$1 is \$380, the market value of 4 shares; and $\frac{1}{4}$ of \$380 is \$95, which is .95, or 95%, of the par value.

Find the market value and rate of premium or discount

7. If 8 shares of R. R. stock cost \$801, brokerage $\frac{1}{8}$%.

8. If 10 shares \$50 stock cost $\$451\frac{1}{4}$, brokerage $\frac{1}{4}$%.

9. If a \$1000 bond U. S. 5-20's sells for \$1185; for \$1177.50.

10. If 20 shares bank stock sell for \$105 above par, including brokerage $\frac{1}{4}$%; for \$205 discount; for \$50 premium.

11. A company with a capital of \$50000 receives \$2500 more than it expends. Find the dividend on ten \$50 shares.

379. When stock is at 5% premium, how many shares, each \$50, can be bought for \$421, brokerage $\frac{1}{4}$%?

Analysis. — Since the market value is 5% above par, each share with brokerage costs $105\frac{1}{4}$% of \$50, or $\$52.62\frac{1}{2}$; and for \$421 there *can be bought* as many shares, etc.

Find the par value, or number of shares

12. Of bank stock bought for \$975, at $97\frac{1}{4}$, brokerage $\frac{1}{4}\%$.

13. Of a U. S. bond that cost \$1172.50, at $117\frac{1}{8}$, bro. $\frac{1}{8}\%$.

14. Of Lehigh R. R. bought for \$824, at $102\frac{3}{4}$, bro. $\frac{1}{4}\%$.

15. When gold is at a premium of $12\frac{3}{4}$, how much can be bought for \$451? For \$902? For \$676.50?

16. A broker bought mining stock at $2\frac{1}{2}\%$ discount, and sold it at $101\frac{1}{2}$, and cleared \$60. Find the number of shares.

380. What must I invest in 6% stock, at 95, to yield \$300?

Analysis. — Since the interest of 1 share of 6% stock is \$6, to yield \$300 there must be invested as many dollars, etc. And 50 shares at \$95 a share cost \$4750, the amount of investment.

What sum must be invested

17. In 5% stock, at $102\frac{1}{2}$, to yield an income of \$500?

18. In U. S. 5's of '81, at $115\frac{1}{8}$, brokerage $\frac{1}{8}\%$, to give \$600?

19. In R. R. 7's, at 90, brokerage $\frac{1}{4}\%$, to give \$350?

20. U. S. 10-40's are quoted at $118\frac{3}{8}$, brokerage $\frac{1}{8}\%$. What must be invested to give a yearly income of \$1000?

21. What sum must I invest in stock at 110, paying 10% annual dividends, to realize \$500 a year?

381. What $\%$ do 8% bonds yield when bought at 120?

Analysis. — Since each share at \$120 yields \$8 interest, the rate of income is $\frac{8}{120}$, or $\frac{1}{15}$, of 100%, or $6\frac{2}{3}\%$. Hence, etc.

Find the rate of income realized from

22. 6% bonds bought at 90; at 102; at 96.

23. Stock paying 10% annual dividend, bought at 120.

24. U. S. 5-20's bought at $116\frac{7}{8}$, brokerage $\frac{1}{8}\%$.

25. What $\%$ of income is realized from an investment paying 10%, bought at $112\frac{3}{8}$, brokerage $\frac{1}{8}\%$?

26. If I buy for \$100 State 6's, at $97\frac{3}{4}$, what interest do I receive? What rate on the investment, brokerage $\frac{1}{4}\%$?

382. At what price must 6% stock be bought to yield an income of 8% on the investment?

Analysis. — Since \$6 is the interest on each share, \$6 must be 8%, or $\frac{8}{100}$, of the price; and if \$6 is 8% of the price, etc.

At what price must

27. 6% stock be bought to yield an income of 9%?

28. 8% stock be bought to yield 6% interest? 7%?

29. U. S. 6's of 1881 to yield 5% in gold? 6%?

30. What premium must be paid on stock paying 10% dividend, to realize 7½%? 8%?

31. Which is the better investment and how much %, U. S. 5-40's at 112½, or U. S. 5-20's at 120?

Miscellaneous Problems.

1. What is the relative value of 8 yards compared with 24 yards? What is the relation of 8 to 24? The ratio of 8 to 24?

2. What is the relation of 12½ to 75? Of 12½ ft. to 6¼ ft.? \$½ to \$4? The ratio of 5 to 10? Of 2½ ft. to 7½ ft.? Of \$9 to \$2¼? Of 16.5 yd. to 5 5 yd.?

3. Name two numbers having a relation or ratio to each other equal to the ratio of \$6 to \$1½. Of 3 yd. to 7½ yd.

4. What number divided by 7½ gives the same quotient as 8 divided by 2? The same quotient as 6¼ divided by 1¼?

5. What number has the same ratio to 3⅓ that 12 has to 4? That 4 has to 12? That 1½ has to 9? 9 to 1½?

6. The relation of 5 ft. to 15 ft. is the same as the relation of \$4½ to how many dollars? The ratio of 4½ mi. to 9 mi. equals the ratio of 5½ days to how many days?

7. *If 2 oranges* cost 10 cents, what will 8 oranges cost at

the same rate? The ratio of 2 oranges to 8 oranges equals the ratio of 10 cents to how many cents?

8. If 6 men do a piece of work in 8 days, how many men would be required to do it in 1 day? In 4 days?

9. If 2 yards of muslin are worth $25\frac{1}{2}$ cents, how much are $2\frac{1}{2}$ yards worth? How much are $3\frac{1}{3}$ yards worth?

10. If $\frac{4}{5}$ of a barrel of flour cost $\$6\frac{2}{3}$, what must be paid for $\frac{3}{4}$ of a barrel? For $2\frac{1}{2}$ barrels?

11. If $\$11\frac{1}{4}$ will pay for $2\frac{1}{4}$ cords of wood, how many cords can be bought for $\$17\frac{1}{2}$? For $\$21\frac{2}{3}$?

12. If 4 men do a piece of work in $6\frac{1}{2}$ days, working 10 hours a day, in how many days can they do the same amount of work, if they work 12 hours a day?

13. When $\$2\frac{2}{3}$ will pay for $\frac{8}{5}$ of a barrel of flour, how much money is required to pay for $2\frac{1}{4}$ barrels?

14. If $3\frac{1}{2}$ bushels of potatoes cost $\$3\frac{1}{16}$, what will be the cost of $5\frac{1}{3}$ bushels at the same rate? Of $\frac{3}{4}$ of $6\frac{2}{3}$ bu.?

15. How many tons of hay will 4 horses eat in $3\frac{2}{3}$ months, if 3 horses eat $1\frac{1}{2}$ T. in 1 month?

16. How many days will it take 7 men to earn $49, if 5 men earn $\$\frac{7}{8}$ in $\frac{1}{5}$ of a day?

17. Suppose 3 furnaces consume $15\frac{3}{4}$ tons of coal in 6 days, how long will $20\frac{1}{4}$ tons supply 5 furnaces?

18. If $80 gain $\$4\frac{4}{5}$ in 12 months, in how many months will $50 gain $\$2\frac{1}{4}$ at the same rate?

19. If a man can cut a field of grass in $2\frac{1}{2}$ days, what part of the field can he cut in one day?

20. If B can plow $\frac{2}{5}$ of a field in one day, in how many days can he plow the whole field?

21. If B can do a piece of work in 4 days, and C in 6 days, what part can each do in one day? Both?

22. If B and C together can do $\frac{5}{12}$ of a piece of work in one day, in what time can they do all of it?

23. A shoemaker can fill a certain order in 3 days, and his apprentice in $3\frac{3}{4}$ days. In what time can both fill it?

24. If a cistern can be filled by one pipe in half an hour, and by another in 20 minutes, in what time can it be filled by both pipes running together?

25. A man saws a cord of wood in $\frac{3}{4}$ of a day, and his son can saw it in $1\frac{1}{2}$ da. In what time can both saw it?

26. If A can do $\frac{1}{3}$ of a piece of work in one hour, B $\frac{1}{4}$ of it, and C $\frac{1}{8}$ of it, what part can they all do in one hour?

27. If A can do a piece of work in 2 days, B in 3 days, and C in 4 days, what part can each do in 1 day? What part can all do? In what time can all do it together?

28. A can do a piece of work in 3 days, B in 4 days, and C in 6 days. In what time can all do it together?

29. If A and B together can mow $\frac{1}{8}$ of a field in a day, and B alone mows $\frac{1}{20}$ of it, what part does A mow? In what time could A alone mow all of it?

30. A man and his son make a fence in 4 days. The son alone can do it in 12 days. In what time can the father do it?

31. Three men can reap a field of wheat in 4 da. A can do it alone in 8 days, and B in 12 days. In what time could C alone reap the field?

32. A, B, and C could dig a ditch in 4 days, A and B in 6 days, and B and C in 9 days. In what time can each alone finish the work? In what time can A and C do it?

33. A and B can build a wall in 16 days, but with the aid of C they can build it in 10 days. In what time can C alone do it?

34. A, B, and C can dig a ditch in 6 days, A and B in 8 *days, and B in* 12 days. In what time could each dig it?

35. If 2 oxen or 3 cows can eat 3 tons of hay in a month, how much can 2 oxen and 3 cows eat?

36. If 4 horses or 6 oxen plow 8 acres of land in 2 days, in what time can 6 horses and 9 oxen plow 20 acres?

37. If 10 lb. of cheese are worth as much as 4 lb. of butter, and 6 lb. of butter are worth as much as 2 bu. of corn, how many pounds of cheese will pay for 6 bu. of corn?

38. If the sum of 4 and 5 were ten, what would be the sum of 6 and 9? Of $8\frac{1}{2}$ ft. and $12\frac{3}{4}$ ft.?

39. Two men bought a ton of coal for \$7, the first paying \$3, and the second \$4. What part belongs to each?

40. Divide \$24 into two such sums that the second shall be double the first; the second, one-half the first.

41. Divide 42 into three such parts that the second shall be double the first, and the third double the second.

42. Two boys divided 21 apples in the ratio of 3 to 4. How many apples did each of them receive?

43. Two kinds of tea were mixed in the ratio of 4 lb. of green to 3 lb. of black. How many pounds of each are in a mixture of 42 lb.?

44. An estate of \$5000 was divided between two children in the ratio of their ages, which were 10 years and 15 years. How much did each receive?

45. If \$45 be divided among three boys so that their shares shall be as 2, 3, and 4, how much will each receive?

46. A and B trade in company, furnishing money in the ratio of \$4 to \$5. If they lose \$1800, what is the loss of each? Find the ratio of B's loss to A's.

47. Two men engaged in a business transaction, the first investing twice as much as the second. If they cleared \$900, what share of the gain was due to each?

48. Three men hired a pasture for \$75, and as often as

the first paid $4, the second paid $5, and the third $6. What did each of them pay?

49. A, B, and C traded together, B investing twice as much as A, and C three times as much as A. If they gained $900, what was each man's share?

50. Three men rented a farm for $300, A paying $150, B $100, and C $50. Their profits were $1500. What was each man's share?

51. Two men rented a pasture for $36, one putting in 3 cows for 4 weeks, and the other 2 cows for 3 weeks. How much of the rent should each man pay?

52. A and B engaged to make a section of a road for $500. A furnished 4 men for 10 days, and B furnished 5 men for 12 days. What sum should each have received?

53. A man failing in business has property valued at $9000. If he owes A $3000, B $4000, and C $5000, what part of his debts can he pay? How much to each creditor?

54. If a grocer buys eggs at 20 cents a dozen, for what should he sell them to gain 100 per cent.?

55. If a grocer buys eggs at 20 cents a dozen, for what should he sell them to lose 100 per cent.?

56. If a grocer sells eggs at 20 cents a dozen, and thereby gains 100 per cent., what did he pay for them?

57. If a grocer sells eggs at 40 cents a dozen, and thereby gains 100%, for what should he sell them to lose 100%?

58. A druggist bought some opium for $5, and sold it in doses so as to double his money. What % did he gain?

59. A wholesale druggist sold some opium that cost him $5, for one-half its cost. What % did he lose?

60. A druggist put drugs worth $5 in prescriptions for the poor, and received nothing in payment. What % did *he lose?*

61. If sample drugs are sent to a retail druggist and he sells them for $5, what % does he gain?

62. If a grocer buys eggs at the rate of 10 for 8 cents and sells 8 for 10 cents, what % does he gain or lose?

63. If a grocer buys eggs at the rate of 8 for 10 cents and sells 10 for 8 cents, what % does he gain or lose?

64. A farmer had 100 bu. of wheat, and sold 25% of it to one man, and $\frac{1}{3}$ of the remainder to another. What per cent. of his crop remained unsold?

65. What % does a grocer gain or lose by selling cheese at $\frac{4}{5}$ of its cost? At $\frac{4}{3}$ of its cost? For double its cost?

66. If a dealer sold 20 bbl. of flour at $6\frac{1}{4}$ a barrel, at a loss of $6\frac{1}{4}$%, what was the entire cost of the flour?

67. A man bought goods at 20% below par, and sold them at 20% above par. If he gained $90, how much had he invested? What % did he clear?

68. If a merchant asks for goods 20% more than they cost, and takes $\frac{4}{5}$ of the price asked, what % does he gain or lose? What % of the price asked does he receive?

69. A drover bought a horse for $100. What must he ask for it that, after falling 25%, he may still gain 20% on cost?

70. What must be asked for cloth costing $4 a yard, so that after falling 10% I may still clear $12\frac{1}{2}$%?

71. If a druggist retails at a gain of 50%, and sells at wholesale for 25% less than at retail, what % does he gain at wholesale? What % of the retail price does he receive?

72. If my gain at wholesale is 20%, and my price at retail is 25% more than my wholesale price, what % do I gain at retail? What % of the wholesale price is the retail price?

73. If goods are marked at an advance of 40% on cost, what % is gained on cloth marked 49 cents net, if sold at a discount of 10% net?

74. What % on cost is made on goods marked 25% above cost, and sold at 10% discount, and 5% off for cash?

75. A merchant bought cloth at the rate of $3 for 4 yards, and sold it at the rate of $4 for 3 yards, thereby gaining $7. How many yards did he buy?

76. What number increased by $\frac{1}{2}$ and $\frac{1}{8}$ of itself is 22?

77. A coat and a vest cost $20, and the vest cost $\frac{1}{4}$ as much as the coat. What was the cost of each?

78. If a yard-stick is broken into two parts, and one part is $\frac{4}{5}$ of the length of the other, how long is each?

79. Frank and William have $75, and Frank has $\frac{2}{3}$ as much as William. How much has each?

80. If my money increased by $\frac{1}{4}$ of itself and then diminished by $5 leaves $45, how much money have I?

81. A watch and chain together cost $64, the cost of the chain being $6 less than $\frac{1}{4}$ of the cost of the watch. What was the cost of each?

82. Two boys have 48 cents, and one of them has but $\frac{5}{7}$ as many as the other. How many cents has each?

83. A man has three sons; the second being twice as old as the youngest, and the eldest twice as old as the second. If the sum of their ages is 28 years, how old is each?

84. Divide $164 into four such parts that the second shall be four times the first, the third three times the second, and the fourth twice the third.

85. If $\frac{1}{2}$ of Harry's age equals $\frac{1}{4}$ of Charles's age, and the sum of their ages is 15 years, how old is each?

86. Two pieces of muslin contain 51 yards, and $\frac{2}{3}$ of the longer piece equals $\frac{3}{4}$ of the shorter. How long is each?

87. A and B invested $500, and 3 times A's investment equals 7 times B's. What amount did each invest?

88. *A drover* paid $264 for two horses, $\frac{1}{6}$ of the cost of one

being equal to $2\frac{1}{2}$ times the cost of the other. What did each cost?

89. A and B start from the same place, but A travels 12 miles an hour, while B travels but 1 mile. In how many hours will A overtake B? How far will each have gone?

90. The hour-hand and the minute-hand of a watch are together at 12 o'clock. When will they be again together?

91. If $\frac{1}{4}$ of the time past midnight equals the time past noon, what is the time of day?

92. What is the time of day, if $\frac{1}{2}$ of the time past midnight equals the time to noon?

93. If $\frac{1}{3}$ of the time past noon equals $\frac{1}{8}$ of the time to midnight, what o'clock is it?

94. If $\frac{2}{3}$ of the time past midnight equals $\frac{2}{7}$ of the time past noon, what is the hour?

95. At what time between 3 and 4 o'clock will the hour-hand and the minute-hand of a watch be together?

96. At what time will the hour-hand and the minute-hand of a watch be together between 8 and 9 o'clock?

97. If $\frac{2}{7}$ of the time to midnight equals $\frac{2}{3}$ of the time to noon, what is the hour?

98. When the time of sunrise is $\frac{5}{9}$ of the time from midnight to noon, at what time does the sun rise?

99. If $\frac{3}{8}$ of the time from midday to midnight is the time of sunset, at what time exactly does the sun set?

100. Harry is 6 years old, and his father is 30 years old. In how many years will the father be only 4 times as old?

101. Frank is four times as old as James, and the sum of their ages is 25 years. In how many years will Frank be only two times as old as James?

102. I had \$50, and after spending some of it, what I had left was $\frac{1}{2}$ of what I had spent. How much had I left?

103. A fish weighs 9 pounds. The head weighs twice as much as the tail, and the body weighs three times as much as the head. What is the weight of each part?

104. If a fish weighs 18 pounds, the head weighing twice as much as the tail, and the body three times as much as the head and tail together, how much does each part weigh?

105. A pole 16 feet long stands in the mud, water, and air. The length in the mud is $\frac{1}{3}$ the length in the water, and the length in the air equals the length of the part in the mud and water together. Find the length of each part.

106. A hare had 40 yards the start of a hound, but while the hare ran 5 yards the hound ran 10. How many yards did the hound run to catch the hare?

107. Frank is 60 steps ahead of Harry, and takes 4 steps to Harry's 6. If their steps are equal, how many steps must Harry take before he can overtake Frank?

108. A hare takes 2 leaps while a hound takes 1 leap, but 1 of the hound's leaps equals 3 of the hare's. How much does the hound gain on the hare every leap?

109. James is 10 of his own steps behind Edward, and takes 3 steps while Edward takes 4; but James's steps are twice as long as Edward's. How many steps must James make to overtake Edward?

110. A mother is 36 years old, and her daughter $\frac{1}{3}$ as old. In how many years will the mother be but twice as old as her daughter? What will then be the age of each?

111. If a son is 35 years old and his father is 60, how long since the son's age was one-half the age of the father? What was then the age of the son and of the father?

THE END.