SLang - the Next Generation

Tutorial

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0.1 Random process with given power spectral density

Consider a random process $F(t)$ defined by power spectral density

$$S_{FF}(\omega) = \frac{S_0}{1 + \left(\frac{\omega}{\omega_0}\right)^4} \tag{1}$$

We want to generate sample functions $F^{(k)}(t)$ with a time interval Δt. This is achieved by first generating i.i.d standard Gaussian variables a_ℓ and b_ℓ. The sin and cos components of the Fourier transform are defined as products of a_ℓ and b_ℓ with the power contained at frequency ω_ℓ within a frequency interval $\Delta\omega$, i.e.

$$c_\ell = \sqrt{2S_{FF}(\omega_\ell)\Delta\omega}\; a_\ell;\; s_\ell = \sqrt{2S_{FF}(\omega_\ell)\Delta\omega}\; b_\ell \tag{2}$$

Then an inverse FFT is applied to $c\ell, s_\ell$. This is shown in the following listing.

```
--[[
SLangTNG
Simple test example for simulation of random processes
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-- This function defines the two-sided PSD of the process
function PSD (S, a, b)
  local p = S/(1+(b/a)^4)
  return p
  end

-- Define process parameters
  S0 = 10
  om0 = 3
  om_max = 20
  nOmega = 500
  dOmega = om_max/nOmega

-- Fill an array with PSD values
  spec = tmath.Matrix(nOmega)
  var = 0
  for i=0,nOmega-1 do
    spec[i] = PSD(S0, om0, (i+.5)*dOmega)
    var = var + 2*spec[i]*dOmega
    end

  a = stoch.Simulate(nOmega,1)
  b = stoch.Simulate(nOmega,1)
  c = tmath.Pow(spec*2*dOmega, 0.5):CW()*a
  s = tmath.Pow(spec*2*dOmega, 0.5):CW()*b

  help = c:AppendCols(s)*math.sqrt(nOmega/2)
  f, dt = spectral.IFT(help,dOmega)

-- Check actual PSD
  f1 = f:GetCols(1)
  psd = spectral.AutoSpectrum(f1, dt)

-- Append target values for comparison
  psd = psd:AppendCols(spec)

-- Plot the result
  vis=tnggraphics.TNGVisualize(520,20,800,450, "Process")
  vis:SetLabels("Random process sample", "Time", "Process")
  vis:Plot(f:GetCols(0), f:GetCols(1))
  vis:File("process.pdf")

  vis2=tnggraphics.TNGVisualize(520,540,800,450, "PSD")
  vis2:SetLabels("Power spectral density", "Circular frequency", "PSD")
  vis2:Plot(psd:GetCols(0), psd:GetCols(1,2))
  vis2:File("PSD.pdf")
```

The resulting process sample $f(t)$ is shown in Fig. **??**. The power spectrals density as estimated from this sample function is compared to the target in Fig. **??**.

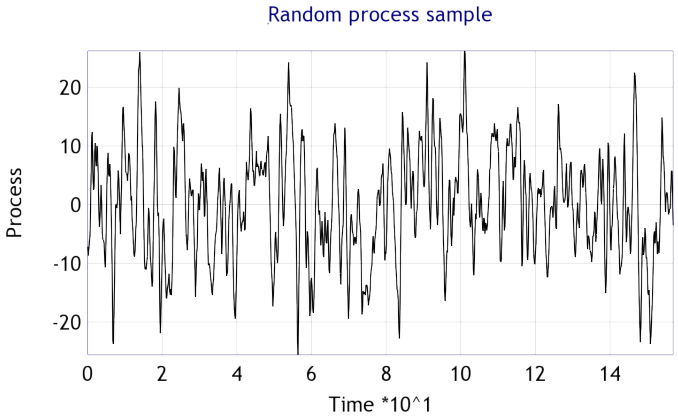

Figure 1: Sample function of a random process

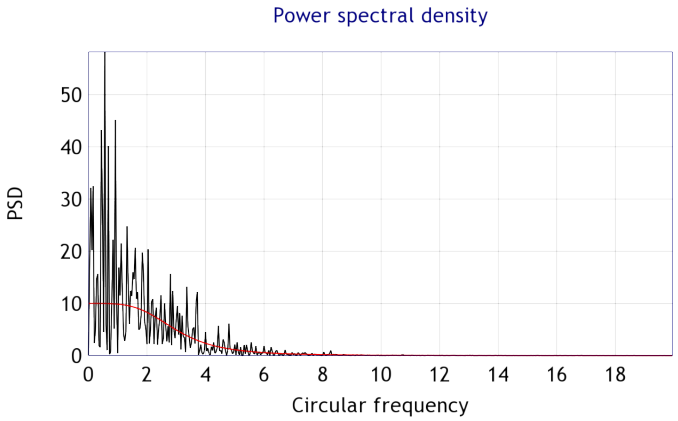

Figure 2: Comparison of sample PSD to target PSD